Ber. nat.-med. Verein Innsbruck	Band 73	S. 7 — 15	Innsbruck, Okt. 1986

Nomographs for the determination of meltwater from snow- and ice surfaces

by

Walter AMBACH *)

(Institut für Medizinische Physik, Innsbruck,
Vorstand: Univ.-Prof. Dr. W. Ambach)

Nomogramme zur Bestimmung des Schmelzwassers von Schnee- und Eisoberflächen

Synopsis: Es werden Nomogramme vorgestellt, aus denen das Schmelzwasser durch fühlbare Wärme, latente Wärme und durch die kurzwellige und langwellige Strahlungsbilanz in einfacher Weise bestimmt werden kann. Für die Ablesung des Schmelzwassers durch fühlbare und latente Wärme werden als Meßgrößen, Temperatur, Windgeschwindigkeit, relative Feuchte und die Seehöhe benötigt. Die Nomogramme basieren auf der Anwendung der Prandtl'schen Grenzschichttheorie für adiabatische Schichtung. Das Schmelzwasser durch kurzwellige und langwellige Strahlungsbilanz ergibt sich in einfacher Weise aus der Globalstrahlung, Albedo und der Bewölkung.

1. Introduction:

At Post-Graduate Courses on snow- and ice hydrology, the question of the discharge of meltwater from snow-and ice surfaces represents a central topic (AMBACH, 1985). Generally the discharge of meltwater is determined by means of energy balance considerations. For isothermal snow- and ice layers of 0°C and for periods without precipitation it holds true that

$$Q_M = Q_{SR} + Q_{LR} + Q_S + Q_L \tag{1}$$

The meaning of the individual terms is energy per unit of area and unit of time, and can be seen from section 5.2. For practical considerations, and in order to facilitate a comparison with precipitation values, the terms are converted into mm water equivalent per day by means of the specific heat of melt, when

$$W(\frac{mm}{d}) = 2.98\ Q\ (\frac{MJ}{m^2d}) \tag{2}$$

*) Anschrift des Verfassers: Univ.-Prof. Dr. phil. W. Ambach, Institut für Medizinische Physik, Müllerstraße 44, A-6020 Innsbruck, Österreich.

2. Nomograph for the determination of the water equivalent of melting by sensible and latent heat:

The calculation of the sensible and latent heat is performed by means of Prandtl's boundary layer theory for adiabatic stratification (HOINKES and UNTERSTEINER, 1952), assuming logarithmic profiles for wind velocity, temperature, and vapor pressure.

2.1. Sensible heat:

The sensible heat depends on the wind velocity, the temperature, the atmospheric pressure, and the roughness parameter of the surface. It holds true that

$$Q_S = c_p A \frac{dT}{dz} \tag{3}$$

$$A = \kappa \rho u_* z \tag{4}$$

$$\rho = \rho_o \frac{b}{b_o} \frac{T_o}{T} \sim \rho_o \frac{b}{b_o} \tag{5}$$

$$u_* = \frac{v(z)}{5{,}75 \log (z/z_{oW})} \tag{6}$$

$$\frac{dT}{dz} = \frac{k_T}{z} \quad \text{with} \quad k_T = \frac{T(z)}{\ln(z/z_{oT})} \tag{7}$$

and therefore it results with $z = 200$ cm

$$Q_S = K_S\, b\, v_{200}\, T_{200} \tag{8}$$

$$K_S = \frac{c_p \kappa \rho_o}{b_o\, 5{,}75 \log (200/z_{oW}) \ln (200/z_{oT})} \tag{9}$$

The numerical formulation of equ. 8 is given in section 5.1. (i), the meaning of the terms is explained in section 5.2. By taking the logarithm, equ. 8 is formulated

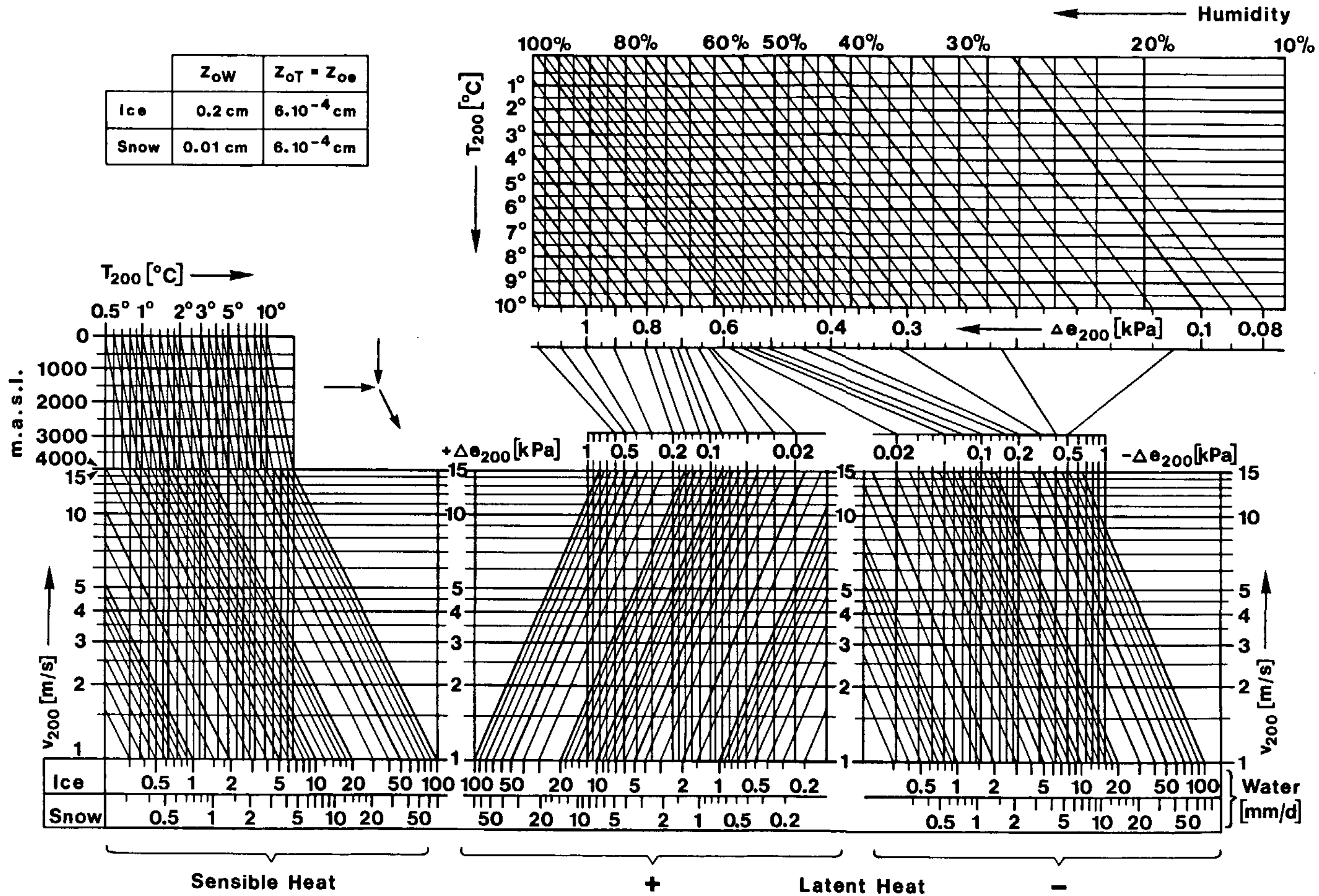
$$\log Q_S = \log K_S + \log b + \log v_{200} + \log T_{200} \tag{10}$$

Equ. 10 is represented in two diagrams with logarithmic scales on both axes. The values $\log Q_S$ = constant are given by parallel straight lines (fig. 1).

The sensible heat flux is determinded by the measured data v_{200} and T_{200} as well as by b and the constant K_S. The constant K_S depends on general physical constants and on the roughness parameters of the surface z_{oW}, z_{oT}. The difference between an ice- and a sow surface is given by the value of the constant K_S, because the roughness parameter z_{oW} is a different one for both surfaces. Practically, this results in a displacement of the scale graduation for them reading of the water equivalents for ice- and snow surfaces.

The following data are required as measured values (fig. 1)

- The air temperature T_{200} (°C). The index 200 refers to a height of 200 cm above the surface
- The atmospheric pressure b (Pa), depending essentially on the altitude above sea level of the site of the measurements. Fluctuations of atmospheric pressure due to changing weather conditions have no effect. Therefore a scale for the altitude above sea level is introduced instead of the atmospheric pressure.
- The wind velocity v_{200} ($\frac{m}{s}$). The index 200 also refers to the height of 200 cm above the surface.



	z_{oW}	$z_{oT} = z_{oe}$
Ice	0.2 cm	6.10^{-4} cm
Snow	0.01 cm	6.10^{-4} cm

Fig. 1: Nomograph for the determination of the water equivalent of melting due to sensible and latent heat. For application example see fig. 3

2.2. Latent heat:

The calculation of the latent heat is performed by means of Prandtl's boundary layer theory, analogously to the calculation of the sensible heat (HOINKES and UNTERSTEINER, 1952). It holds true that

$$Q_L = L\,A\,\frac{0.623}{b}\,\frac{dz}{dz} \tag{11}$$

with equs. 4, 5, 6, and

$$\frac{de}{dz} = \frac{k_e}{z} \quad \text{with} \quad k_e = \frac{e(z) - e_o}{\ln(z/z_{oe})} \tag{12}$$

one obtains with z = 200 cm

$$Q_L = K_L\, v_{200}\, \Delta\, e_{200} \tag{13}$$

$$K_L = \frac{L \kappa \rho_o\, 0{,}623}{b_o\, 5{,}75 \log(200/z_{oW}) \ln(200/z_{oe})} \tag{14}$$

and by taking the logarithm is results

$$\log Q_L = \log K_L + \log v_{200} + \log \Delta\, e_{200} \tag{15}$$

The values $\log Q_L$ = constant are given in fig. 1 by parallel straight lines. Generally, the vapor pressure difference $\Delta e_{200} = e_{200} - e_0$ is not available as measured value. e_{200} results from another graph, together with T_{200} and f_{200} (fig. 1). Therefore T_{200} (°C), f_{200} (%), and v_{200} ($\frac{m}{s}$) are re-
quired as measured values, all in a height of 200 cm above the surface.

For the constant K_L it is distinguished between the following cases:

- Ice surface differs from snow surfaces by the value z_{oW}.
- Condensation differs from evaporation by the value L. In case of condensation there is a transition from the gaseous state into the liquid state, in case of evaporation from the solid state into the gaseous state.

The numerical formulation of equ. 13 is given in section 5.1. (ii, iii).

3. Nomographs for the determination of the water equivalent of melting from the radiation balance:

This nomograph is considerably simpler than that for the sensible and latent heat, since the heat available for melting results directly from the measured values.

3.1. Short-wave radiation balance:

The short-wave radiation balance depends on the global radiation and on the albedo of the surface. It is true that

$$Q_{SR} = G\,(1 - \alpha) \tag{16}$$

and

$$\log Q_{SR} = \log G + \log(1 - \alpha) \tag{17}$$

In a diagram with logarithmic scales on both axes, $\log Q_{SR}$ = constant results by parallel straight lines (fig. 2). For numerical formulation see section 5.1. (iv).

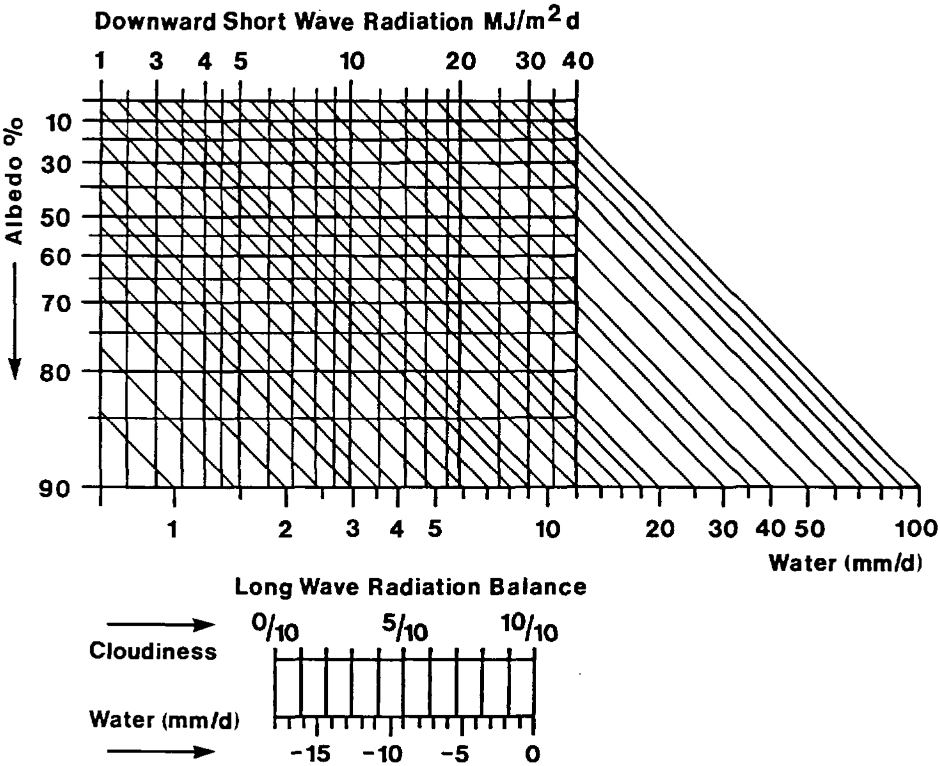


Fig. 2: Nomograph for the determination of the water equivalent of melting due to the radiation balance. Upper part: short-wave radiation balance; lower part: long-wave radiation balance. For application example see fig. 3

3.2 Long-wave radiation balance:

In case of a melting surface, the long-wave radiation balance mainly depends on the cloudiness. A linear relation serves as approximation (AMBACH, 1963)

$$Q_{LR} = Q_0 (1 - kc) \tag{18}$$

For the reading, a scale for cloudiness is used (fig. 2). For numerical formulation see section 5.1 (v).

4. Application of the nomographs:

4.1. Sensibel and latent heat:

The application is shown in fig. 3. For the determination of the water equivalent of melting from the sensible heat, the temperature T_{200} (°C) and the altitude above sea level (m a.s.l.) are inserted in part a, the wind velocity in part b and then connected according to the example. Reading is performed as water equivalent per time unit (mm/d). Separate scale graduations are given for ice- and snow surfaces.

For the determination of the water equivalent of melting from the latent heat, the temperature T_{200} (°C) and the relative humidity f_{200} (%) are inserted in part c; the wind velocity v_{200} ($\frac{m}{s}$) is inserted in the parts d and e and all values are again connected according to the example. Part d indicates the water equivalent per time unit (mm/d) in case of condensation, part e the water equivalent per time unit (mm/d) in case of evaporation. Negative values of the water equivalent imply that no meltwater develops due to evaporation. A negative value is deducted from positive values of the water equivalent, obtained from other processes.

4.2. Short-wave and long-wave radiation balance:

The application is shown in fig. 3. For the determination of the meltwater from the short-wave radiation balance, the global radiation (MJ/m^{2}d) and the albedo (%) are inserted in the respective scale in part f and connected according to pattern.

The water equivalent of the long-wave radiation balance results from the cloudiness inserted in part g. A negative value of the water equivalent is deducted from positive values, obtained from other processes.

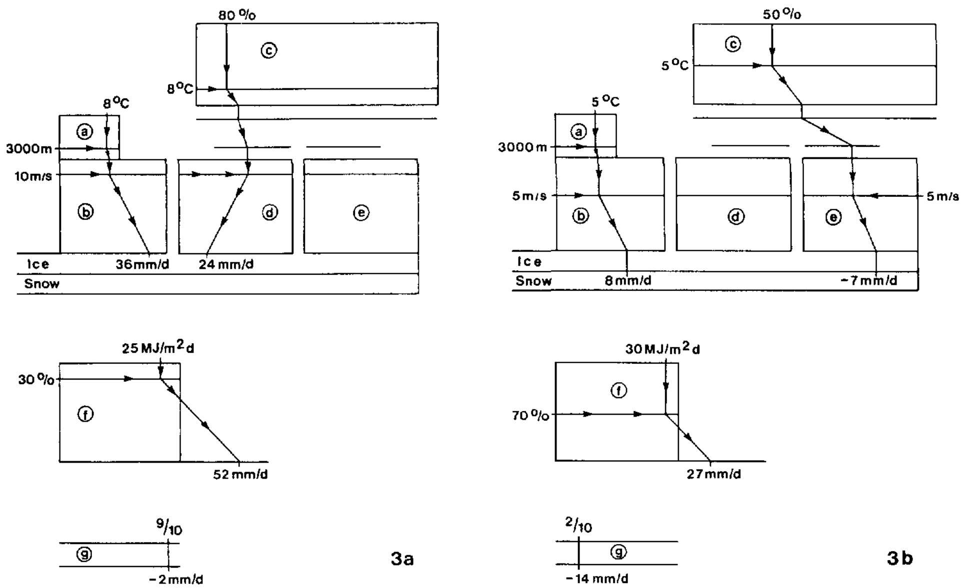


Fig. 3: Scheme for the application of the nomographs.
a) Data applying for the case of ice surface and condensation in accordance with example 1 in section 4.3.
T_{200} = 8°C, h = 3000 m a.s.l., v_{200} = 10 m/s, f_{200} = 80 %, G = 25 MJ/m^{2}d, α = 30 %, c = 9/10
Results: W_S = 36 mm/d, W_L = 25 mm/d, W_{SR} = 52 mm/d, W_{LR} = − 2 mm/d
b) Data applying for the case of snow surface and evaporation in accordance with example 2 in section 4.3.
T_{200} = 5°C, h = 3000 m a.s.l., v_{200} = 5 m/s, f_{200} = 50 %, G = 30 MJ/m^{2}d, α = 70 %, c = 2/10
Results: W_S = + 8 mm/d, W_L = − 7 mm/d, W_{SR} = + 27 mm/d, W_{LR} = − 14 mm/d
The data T_{200}, h, and v_{200} are inserted for the reading of W_S; the data T_{200}, f_{200}, and v_{200} for the reading of W_L. For condensation and evaporation, the values for W_L result in different parts; for snow- and ice surfaces, two scale-graduations are given

4.3. Examples:

The following two examples correspond to the data and results in fig. 3, when the readings were rounded to integral values.

Example 1: Ice surface, condensation:

h = 3000 m a.s.l., T_{200} = 8°C, v_{200} = 10 m/s, f_{200} = 80 %, G = 25 MJ/m²d, α = 30 %, c = 9/10

Water equivalents	
sensible heat	$+\ 36\ \frac{mm}{d}$
latent heat	$+\ 24\ \frac{mm}{d}$
short-wave radiation balance	$+\ 52\ \frac{mm}{d}$
long-wave radiation balance	$-\ 2\ \frac{mm}{d}$
Total	$+\ 110\ \frac{mm}{d}$

Example 2: Snow surface, evaporation:

h = 3000 m a.s.l., T_{200} = 5°C, v_{200} = 5 m/s, f_{200} = 50 %, G = 30 MJ/m²d, α = 70 %, c = 2/10

Water equivalents	
sensible heat	$+\ 8\ \frac{mm}{d}$
latent heat	$-\ 7\ \frac{mm}{d}$
short-wave radiation balance	$+\ 27\ \frac{mm}{d}$
long-wave radiation balance	$-\ 14\ \frac{mm}{d}$
Total	$+\ 14\ \frac{mm}{d}$

5. Numerical values and terms:

5.1. Numerical values:

$z = 200$ cm, $c_p = 1{,}01$ kJ/kg gd, $\kappa = 0{,}42$, $b_o = 101{,}3$ kPa, $\rho_o = 1{,}29$ kg/m³, $z_{oW} = 0{,}2$ cm (ice), $z_{oW} = 0{,}01$ cm (snow), $z_{oT} = z_{oe} = 6.10^{-4}$ cm (ice and snow), L = 2514 kJ/kg (condensation), L = 2849 kJ/kg (evaporation), e_o = 611 Pa, $Q_o = -\ 6{,}0$ MJ/m²d, k = 1 (1/tenth parts of cloudiness).

Whereas the values of c_p, κ, $b_o\rho_o$, L, e_o are physical constants, z_{oW}, z_{oT} and z_{oe} are obtained from measurements. In the individual case, these values depend on the surface condition as well as on the profiles of the wind velocity, the temperature, and the vapor pressure. Series of measurements do show, however, that representative values can be inserted for z_{oW}, z_{oT} and z_{oe} (AMBACH, 1977).

The height of measurements z = 200 cm was chosen for the input data v, T, e. However, the results W_S, W_L in equs. 19 till 24 are independent from the measuring height. The values Q_o, and k were also obtained from comprehensive measuring series of long-wave radiation balance above a melting surface (AMBACH, 1963).

With the above numerical values for the constants, under consideration of equ. 2 with the units T_{200} (°C), b (Pa), v_{200} ($\frac{m}{s}$), Δe_{200} (Pa) equs. 8 and 13 read as follows:

i Water equivalents of melting by sensible heat:

Ice: $W_S = 6{,}34.10^{-6}\ T_{200}\ b\ v_{200}$ water (mm/d) (19)

Snow: $W_S = 4{,}42.10^{-6}\ T_{200}\ b\ v_{200}$ water (mm/d) (20)

ii Water equivalents of melting by latent heat in case of condensation

Ice: $W_L = 9{,}83.10^{-3}\ v_{200}\ \Delta e_{200}$ water (mm/d) (21)

Snow: $W_L = 6{,}86.10^{-3}\ v_{200}\ \Delta e_{200}$ water (mm/d) (22)

iii Water equivalents of melting by latent heat in case of evaporation

Ice: $W_L = 11{,}14.10^{-3} v_{200}\ \Delta e_{200}$ water (mm/d) (23)

Snow: $W_L = 7{,}77.10^{-3}\ v_{200}\ \Delta e_{200}$ water (mm/d) (24)

For the water equivalents of melting due to short-wave and long-wave radiation balance, the corresponding equs. 16,18 do numerically read as follows. No differentiation between snow- and ice surface has to be made:

iv $W_{SR} = 2{,}98\ G(1 - \alpha)$ water (mm/d) (25)

v $W_{LR} = -\ 17{,}9(1 - c)$ water (mm/d) (26)

The measured value G is indicated in MJ/m^2d, α as ratio, and c in tenth parts. For the constant k in equ. 18, the value of 1 (1/tenth parts of cloudiness) was taken.

Within the accuracy in reading the graphs, the values for W_S, W_L, W_{SR} and W_{LR}, equal the results obtained by the above formulas, as indicated in example 1 and 2 in section 4.3.

5.2. Terms:

A	Adiabatic "Austausch" coefficient
b, b_o	Atmospheric pressure, b_o = standard pressure
c	Cloudiness (in tenth parts)
c_p	Specific heat of air at constant pressure
e, e_o, e_{200}	Vapor pressure, e_o: saturation pressure at 0°C;
Δe_{200}	e_{200}: vapor pressure 200 cm above surface; $\Delta e_{200} = e_{200} - e_o$
f_{200}	Relative humidity 200 cm above surface
G	Short-wave downward radiation (global radiation)
h	Altitude above sea level
K_S, K_L	Factors of proportionality
k	Factor of proportionality
k_e	Constant of logarithmic profile of vapor pressure
k_T	Constant of logarithmic temperature profile
L	Specific latent heat
ln	logarithm base e
log	logarithm base 10
Q_o	Factor of proportionality
Q_L	Latent heat flux, energy per unit of area and unit of time
Q_{LR}	Long-wave radiation balance, energy per unit of area and unit of time
Q_M	Heat used for melting, energy per unit of area and unit of time
Q_S	Sensible heat flux, energy per unit of area and unit of time
Q_{SR}	Short-wave radiation balance, energy per unit of area and unit of time
T, T_o, T_{200}	Actual air temperature (°C), T_o = 0°C, T_{200}: temperature 200 cm above surface
u_*	Shear velocity
v_{200}	Wind velocity 200 cm above surface
W_S, W_L	Water equivalent per unit of time of melting by sensible and latent heat flux
W_{LR}	Water equivalent of melting by long-wave radiation balance
W_{SR}	Water equivalent of melting by short-wave radiation balance
z	Height above surface
z_{oe}	Roughness parameter of logarithmic profile of vapor pressure
z_{oT}	Roughness parameter of logarithmic temperature profile
z_{oW}	Roughness parameter of logarithmic wind profile
α	Albedo
k	Karman's constant
ρ	Air density

Acknowledgment: This manuscript is a part of heat balance studies, which were financially supported by the Österreichische Akademie der Wissenschaften, Wien.

Abstract: Nomographs are introduced, from which the water equivalents of melting by sensible heat, latent heat, short-wave radiation balance, and long-wave radiation balance can be determined in an easy way. For the reading of the water equivalents of melting due to sensible and latent heat, the input data temperature, wind velocity, relative humidity, and altitude above sea level are required. The nomographs are based on the application of Prandtl's boundary layer theory for adiabatic stratification. The water equivalents of melting due to the short-wave and long-wave radiation balance simply result from the global radiation, the albedo, and the cloudiness.

Literature:

AMBACH, W. (1963): Langwellige Strahlungsbilanz der Gletscheroberfläche und Bewölkung. — In: Untersuchungen zum Energieumsatz in der Ablationszone des Grönländischen Inlandeises (Camp IV-EGIG, 69°40'05"N, 49°37'58"W). Expeditionsbericht der Internationalen Glaziologischen Grönlandexpedition. Meddelelser om Grœnland, **174** (4): 68 - 72.

— (1977): Schubspannungsgeschwindigkeit und Rauhigkeitsparameter.— In: Untersuchungen zum Energieumsatz in der Akkumulationszone des Grönländischen Inlandseises. — Ibidem, **187** (7): 16 - 22.

— (1985): Elements of physical glaciology. — International Post-Graduate Course in Hydrology (1985): Edition, Centro Internationale Di Idrologia "Dino Tonini", University Degli Studi di Padova, Monselice, (Pd.), Italy.

HOINKES, H. und N. UNTERSTEINER (1952): Wärmeumsatz und Ablation auf Alpengletschern, I. Vernagtgletscher (Ötztaler Alpen), August 1950. — Geografiska Annaler XXXIV: 99 - 158.