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Comparison of the "theoretical" formula and a new formula for the calculation of intraocular lenses

by

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Vergleich zwischen "theoretischer" Formel und einer neuen Formel bei der Berechnung der Brechkraft intraokularer Linsen

Synopsis: Zur Berechnung der Brechkraft intraokularer Linsen wird eine neue vereinfachte Formel vorgestellt, die mit Hilfe einer linearen TAYLOR-Reihe abgeleitet wird. Bei 140 Augen wird die Restrefraktion mit einer theoretischen Formel und mit der neuen Formel vorausberechnet und mit der tatsächlichen postoperativen Refraktion verglichen. Es zeigt sich, daß bei irisgetragenen Linsen die Vorausberechnung nach beiden Formeln gleich gute Ergebnisse bringt, bei Hinterkammerlinsen bringt die neue Formel bessere Ergebnisse.

Introduction:

Formulas for the calculation of the refracting power of intraocular lenses have been presented by various authors [1, 2, 3, 4]. These "theoretical" formulas appear almost identical when the appropriate mathematical transformation is applied. A new formula is presented in this paper derived from the formulas mentioned above using a linear TAYLOR-line. To prove the exactness of the calculation the results of the precalculations obtained with both formulas are compared with the postoperative residual refractions.

Methods of calculation and derivation of the new formula:

Formula A: Formula A based on the method described by GERNET [3] for the calculation of lens refraction. The refracting power of a natural and the artificial lens is described by WERNER [5] as follows:

$$D_L = \frac{n}{a - d} - \frac{n}{\dfrac{n}{D_C + D_B} - d} \qquad (1)$$

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D_L = Refracting power of the intraocular lens (D), a = Axial length of the eye (mm), d = Distance from the principal plane of the lens to the corneal vertex (mm), D_C = Refracting power of the corneae (D), D_B = Vertex power of spectacles (D), n = Refractive index of aqueous and vitreous (1.336), D = diopter.

The refracting power of the cornea is given as a result of:

$$D_C = \frac{n_H - n_L}{r} \tag{2}$$

n_L = Refractive index of air (1.000), n_H = Refractive index of the cornea (1.332), r = Radius of curvature of the anterior surface of the cornea (mm). The remaining symbol as in formula (1).

For the calculation in mm you have to multiply n, n_H and n_L with the factor 1000. For the calculation of the residual refraction, formula (1) is transformed into D_B.

Formula B:

a) Emmetropizing Lens:

With the use of a TAYLOR-line in a linear form [6] one obtains the following derivations from formula (1) ($D_B = 0$):

$$\Delta D_L (a) = \frac{-1.336}{(a - d)^2} \cdot \Delta a \tag{3}$$

$$\Delta D_L (r) = \frac{+1.336 \cdot 4.024}{(4.024\, r - d)^2} \cdot \Delta r \quad \frac{n}{D_C} = 4.024 \tag{4}$$

$$\Delta D_L (d) = \frac{1.336}{(a - d)^2} - \frac{1.336}{(4.024\, r - d)^2} \cdot \Delta d \tag{5}$$

The previously mentioned refractive indices are used. $\Delta D_L(a)$, $\Delta D_L(r)$ and $\Delta D_L(d)$ correspond to a change of the refracting power of the lens when one single variable is altered.

With the use of the medians $\bar{a}$ = 23.57 mm, $\bar{r}$ = 7.67 mm, $\bar{d}$ = 3.5 mm [7, 8] the following are obtained from equations (3), (4) and (5):

$$\Delta D_L (a) = -3.31 \cdot \Delta a \tag{6}$$

$$\Delta D_L (r) = +7.18 \cdot \Delta r \tag{7}$$

$$\Delta D_L (d) = +1.53 \cdot \Delta d \tag{8}$$

Δa, Δr and Δd are to be in millimetres, ΔD_L results in diopters. The median $\overline{D_L}$ for an emmetropizing lens is 17.75 D. When the changes in refracting power [caused by single variables as shown in equations (6), (7) and (8)] are added, the result is

$$D_L = 17.75 + 3.31\,(23.57 - a) - 7.18\,(7.67 - r) - 1.53\,(3.50 - d) \tag{9}$$

thus,

$$D_L = 35.34 - 3.31a + 7.18r + 1.53d \tag{10}$$

$$\Delta D_L (D_C) = \frac{-\frac{n^2}{D_C^2}}{\left(\frac{n}{D_C} - d\right)^2} \cdot \Delta D_C \tag{11}$$

With the use of the above mentioned medians one obtains ($D_B = 0$):

$$\Delta D_L (D_C) = -1.27 \cdot \Delta D_C \quad (12)$$

and

$$D_L = 17.75 + 3.31 (23.57 - a) + 1.27 (43.28 - D_C) - 1.35 (3.5 - d) \quad (13)$$

thus,

$$D_L = 145.37 - 3.31a - 1.27 D_C + 1.35 d \quad (14)$$

The formula for the postoperative ametropia is formula (23) and for the residual refraction formula (25).

The refracting power of an emmetropizing lens can be calculated by using this formula (Fig. 1), see also the foot note*).

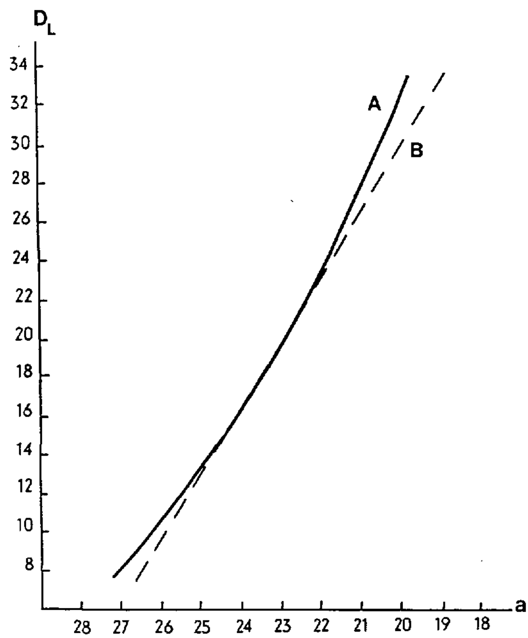

Fig. 1: Graph of the theoretical formula (A) and the new linear formula (B). Ordinate: lens power D_L in diopters, abscissa: axial length a in millimetres. refracting power of the cornea D_C is 43.28 D (r = 7.67 mm) and the postoperative anterior chamber depth is 3.5 mm.

A simplification is possible:

The conversion factor from microseconds into millimetres for the time measured by ultrasound can be included in the equation (10). This factor 0.774 results from division of the axial acoustic velocity (1548 m/sec, [9]) and the conversion of seconds into microseconds and metres into millimetres.

*) If the corneal radius is measured in dioptres, the derivation has to be done after ΔD_C.

The following results:

$$D_L = 35.34 - 2.56t + 7.18r + 1.53d \tag{16}$$

$t = \mu sec.$

b) Postoperative ametropia:

In the case of desired postoperative ametropia the procedure should be as follows and as shown in formula (1). The refracting power of spectacles is added to the refracting power of the cornea [3]. The connection between the radius of the cornea r and the refracting power of the cornea D_C is shown in formula (2). The refracting power of spectacles can also be expressed in a hypothetical change of the radius of the cornea [7].

The linear form of a TAYLOR-line can be derived from formula (2):

$$\Delta D_C = \frac{-n}{r^2} \cdot \Delta r \tag{17}$$

with the use of the median $\bar{r} = 7.67$ mm one obtains

$$\Delta D_C = -5.6 \cdot \Delta r \tag{18}$$

Whereby ΔD_C conforms to the emmetropizing spectacles D_B and procedure a change of the corneal radius:

$$\Delta r = \frac{D_B}{-5.6} \tag{19}$$

In order to determine the refracting power of the lens in desired postoperative ametropia it is necessary to add or substract the quantity, that results from formula (19) to or from the measured radius in formula (10).

$$D_L = 35.34 - 3.31a + 7.18(r + \Delta r) + 1.53d \tag{20}$$

Formula (20) can be transormed to Δr and this results in:

$$-7.18\Delta r = [35.34 + 7.18r + 1.53d - 3.31a] - D_L \tag{21}$$

As can be seen in equation (21) the symbols in brackets in equation (10) conform to an emmetropizing lens from equation (10) is inserted instead of the parenthetical expression and equation (19) inserted instead of Δr in equation (21), the result is

$$1.28\ D_{B\ (desired)} = D_{L\ (emmetrop)} - D_L \tag{22}$$

or

$$D_L = D_{L\ (emmetrop)} - 1.28 \cdot D_{B\ (desired)} \tag{23}$$

The emmetropizing lens will be called D_L (emmetrop) and the lens to be implanted will be called D_L, D_B will be called $D_{B\ (desired)}$.

c) Determination of the residual refraction on a given lens refraction:

When the calculated emmetropizing lens is not available or for some other reason a non-emmetropizing lens has to be implanted, a postoperativ residual refraction that can be precalculated appears.

There for formula (22) is transformed. The residual refraction is called $D_{B\ (optional)}$ and the non-emmetropizing lens is called $D_{L\ (optional)}$.

$$D_B = \frac{D_{L\,(emmetrop)} - D_{L\,(optional)}}{1.28} \quad (24)$$

or as multiplication

$$D_B = [D_{L\,(emmetrop)} - D_{L\,(optional)}] \cdot 0.78 \quad (25)$$

i.e. the difference between an emmetropizing lens and a lens with a certain refracting power whose implantation is intended has to be multiplied by 0.78 to receive the prospective postoperative residual refraction.

Patients:

In 70 eyes of iris supported lenses (d = 3.5 mm) and 70 eyes of posterior chamber lenses (d = 4.0 mm) the postoperative refraction is compared with the precalculated residual refraction. Measurement of the corneal radius was performed with the Zeiss-Ophthalmometer ($n_H = 1.322$).

If there is astigmatism the median of the horizontal and vertical radius is used. For the measurement of the axial length the A-scan istrument 7200 MA (8 MHZ) of Kretz technic is used. The following acoustic velocities are taken [9]:
Aqueous and vitreous 1532 m/sec., optic axis 1548 m/sec.
The preoperative residual refraction is calculated with formula (1) – A – and formula (25) – B.

In all cases postoperative vision is better than 0.5 and postoperative astigmatism less then 4 diopters. For the statistic calculation a minicomputer was established. The significances shown in the tables of Sachs [10] were used.

Results:

The results are shown in the tables 1a und 1b.

Tables 1a, 1b: Comparison of the results of the preoperative calculated residual refraction by formula A and B and the actually postoperative residual refraction. n.s. = no significance.

residual refraction	correlation coefficient	probability	median (D)	standard deviation (D)
theoretical formula (A)	0.414	$p < 0.001$	–0.59	0.78
new formula (B)	0.447	$p < 0.001$	–0.56	0.79
postoperative refraction	–	–	–0.43	1.03

Table 1a: 70 patients with iris-supported lenses

residual refraction	correlation coefficient	probability	median (D)	standard deviation (D)
theoretical formula (A)	0.094	n.s.	–0.14	0.61
new formula (B)	0.264	$p < 0.5$	–0.27	0.62
postoperative refraction	–	–	–0.06	0.93

Table 1b: 70 patients with posterior chamber lenses

Discussion:

We found, that there is no significant difference in the exactness of the precalculation with both formulas for the calculation of iris supported lenses. With both methods the correlation of pre- and postoperative refraction is significant. This is not the case with posterior chamber lenses. There is no significant correlation using the theoretical formula for the calculation. Using the new formula there is a significant correlation between precalculation and postoperative refraction (Table 1a, 1b).

The new formula represents an easier method for the calculation of the refracting power of intraocular lenses. The formulas used for the calculation are formula (10) — or formula (14) for corneal refraction measured in diopters — for an emmetropizing lens, formula (25) for the residual refraction of the actually implanted lens, and formula (23) for the postoperative ametropia.

Summary: In order to calculate the refracting power of intraocular lenses a new formula is presented, which is derived from a theoretical formula, with the help of a TAYLOR-line. In 140 eyes the residual refractions are calculated with the help of a theoretical and the new formula. We can find that the results are the same in the case of iris supported lenses, but in the case of posterior chamber lenses better results have been achieved with the new formula.

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