TREATISE

ON

MINERALOGY.

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TREATISE

ON

MINERALOGY,

OR THE

NATURAL HISTORY OF THE MINERAL KINGDOM.

BY

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Translated from the German, with considerable Additions,

BY

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PREFACE.

The spirit and zeal with which Mineralogy and the kindred sciences are cultivated in Britain and America, and the numerous opportunities afforded to the inhabitants of these countries of visiting the most remote regions of the globe, have made the author of the present work anxious to contribute his share to the more general diffusion of Mineralogical Science, by publishing in the English language the elements of a method which places Mineralogy within the reach of those who wish to become acquainted with minerals, without the assistance of lectures or extensive collections. With the view therefore of fulfilling the promise of Mr Mohs, given in the Introduction to his *Characteristic*, p. vi., a translation of his *Grundriss der Mineralogie* is now laid before the English public. The original Work appeared in two volumes, the first in 1822, and the second in the autumn of 1824. A considerable portion of it was translated from the manuscript under the eye of the author, and the remainder from the printed sheets which were sent over during the progress of publication. In consequence of a continued correspondence with Professor MOHS, and the present rapid progress of the science itself, the translator found it necessary to make many alterations, improvements, and additions; so that this Treatise on Mineralogy may be considered in many respects as a second edition, rather than as a mere translation of the original work.

The principles according to which Mineralogy is here treated, are so different from those generally received, that, in order to prepare the public for the reception of his method, the author found it necessary to give a full developement of his ideas in a Preface of considerable length; and this was the more indispensable, as the second volume was not published along with the first. In conformity with the views of Mr MOHS, the translator has endeavoured to attain the same object, by publishing in the Transactions of the Royal Society of Edinburgh, and in the scientific journals of that city, several papers drawn up in strict accordance with these principles, and shewing their application in particular cases. These papers were designed to convey a just idea of the leading principles of the present work, from which even the substance of some of them is extracted. From these considerations, it would be superfluous to give here the translation of that part of the German Preface which regards the exposition of the principles, notwithstanding its high importance. It will only be necessary to advert to those passages in it, which refer more particularly to the arrangement of some of the departments of the work itself.

In the systematic nomenclature, introduced by Professor Mohs, and employed in the present work, the compound names and denominations express the degree of connexion in which the species stand to each other, and faithfully represent their resemblance. In the trivial nomenclature, the name applied to the species does not express any thing of that connexion, and it must be a single word, if it shall be convenient for use, in cases where we do not intend to apply it in Natural History to any scientific purpose; consequently those are selected which, according to the rules of §. 241., may be considered as unexceptionable, and are added to the specific characters in the Characteristic, referring at the same time to the page of the second and third volume, where the species is more amply described, and other synonymes added. Where no good trivial names existed, the names or denominations used by Professor Jameson in the third edition of his system, or those adopted by other Mineralogical authors, or by Chemists, have been introduced in their place.

The actual employment of the Characteristic to the purpose for which it is intended, the determination of minerals occurring in nature, cannot be too strongly recommended to the beginner. This alone will make him accustomed to observe with his own eyes the characters upon which depend the identity or difference between several species. The present work is the only one hitherto published, which enables those who have studied the Terminology, to determine every mineral by a philosophical process, although they should never have seen it before.

The synonymes quoted in the General Descriptions of the species are confined to a very few works. Among those in the English language, the works of Professor JAMESON are no doubt the principal ones. The synonymes selected for the use of the present publication are contained in the third edition of his valuable *System of Mineralogy*, and *Manual*, in which he has adopted the system of the method of Natural History. To these synonymes are added the names in the third edition of Mr PHILLIPS' *Elementary Introduction to the knowledge of Minerals*, which appeared too late to be attended to in the German edition. The German works noticed, are the System of WERNER, as contained in the *Handbuch der Mineralogie* by HOFFMANN, continued by Mr BREITHAUPT, and the System of Professor HAUSMANN, these works being framed according to the most original views. The former, in particular, has met with a very general reception for a long series of years. The System of Mr VON LEONHARD is of a later date, and is recommended by its comprehensive references to mineralogical works, and other interesting notices. Among the French works on Mineralogy, those of the Abbé HAÜY have been selected, as being most generally received and understood. The nomenclature used in both editions of his *Traité de Minéralogie*, and in the *Tableau Comparatif*, may of itself be considered as an abstract of the history of his system.

The works and memoirs of Messrs BROOKE, LÉVY, and PHILLIPS, have been consulted in regard to numerous and highly valuable crystallographic observations on several substances, which had not before been examined with sufficient exactness, and which were unknown to Professor MOHS, at the publication or the composition of the German original.

It was not till the publication of the first volume of this work, in 1822, that the axis of any mineral was ascertained, by actual measurement, to be in an inclined position towards the base: and although that fact, which was first introduced into crystallography by Mr MOHS, is there indicated in the characters of some of the species, it had not yet been so generally ascertained, nor could it be so fully developed in the crystallographic department of the work, as it requires. The formulæ which in the German are given in the Preface, are inserted in this translation in their proper place, but without changing the numbers and distribution of the paragraphs. The names and denominations also remain; in the characters alone the necessary changes are made, the expression *Prismatic* in a more general signification being employed as including the prismatic, hemi-prismatic, and tetarto-prismatic forms. In regard to such hemi-prismatic forms as have their axis inclined to the base, it should be observed, that the angles of horizontal prisms indicated, are those which the face of the prism includes with the inclined axis, like BAM and B′AM, Vol. I. Fig. 41.

Those simple forms which have been observed in nature, are noted with an asterisk. For these, and also for each of the combinations, some locality is mentioned in which they have been discovered. In a few instances another variety is substituted in the translation for one in the original, when a certain locality could be obtained for an uncertain one, by a comparison with the specimens in the cabinet of Mr Allan.

The letters inclosed and printed in italics refer to the figures, or to the works of Haüy, sometimes also to particular papers, and in this case the title of the latter has been mentioned. Some of the figures represent the combinations, where they are quoted themselves; others are only similar to them, in so far as they have the same general appearance, but different angles.

To distinguish the compound varieties from the simple ones, is a matter of the highest importance; and they must therefore be kept perfectly separate. This is the point where Mineralogy ends and Geology begins, two sciences which have nothing common with each other. Geology presupposes Mineralogy; but it considers the productions of the Mineral Kingdom in quite another point of view, and according to different principles; without which it would not be a distinct science.

With the enumeration of the compound varieties, the General Description of the species is completed. But there exists besides, a great variety of information in regard to the productions of the Mineral Kingdom, belonging in part to Natural History, but partly also foreign to this science; the latter nevertheless is generally deemed an essential part of a work on Mineralogy. Something of this information is contained in the *Observations* added to every species, and which may require here a few explanations.

The first of them properly belongs to the province of Natural History. It comprehends some remarks on crystallographic subjects, or on the history of the species. Here the species are also compared with the determinations and divisions into sub-species and kinds, as contained in the Wernerian system, which will enable the reader not only to understand the principles of these divisions, but also to form an idea of their contents in reference to the varieties occurring in nature. These distinctions are not susceptible of strictness and precision; the only purpose, therefore, in treating of them, is to convey the ideas with brevity and distinctness.

Then follow some of the chemical properties of the species, as exhibited before the blowpipe, or when acted upon by acids, &c., and one or more analyses, instituted by the most celebrated chemists, in many cases accompanied by the formulæ and corresponding proportions among the ingredients, as proposed by Berzelius. To Professor Mitscherlich the translator is indebted for several interesting facts regarding the circumstances under which certain species still continue to be formed or may be produced at will, in laboratories and furnaces.

In the third place, something of the geological position of the species is mentioned; it does not contain every thing known in this respect, but only so much as will suffice for giving a general idea of the modes of occurrence in nature, peculiar to the species.

The geographic distribution of mineral species is of far less importance than the distribution of plants or animals, in which so much depends on the geographic position, climate, soil, the particular place of growth or residence, and other accidental circumstances. It is the subject of the fourth class of observations, which are confined to the statement of a few localities only, since it cannot fall within the scope of an elementary work to enumerate all the localities of the different varieties of the species.

Under the fifth head, some of the applications of the species are mentioned, and sometimes a sixth number is added, containing notices of species, nearly allied to the one just treated of, but which have not yet been received into the system. Sometimes one or several of these classes of observations are wanting, or joined in a single number.

The first Appendix, which follows immediately after the system, contains such minerals as will probably, when farther examined and compared, be received into the system as particular species. They are arranged in alphabetical order; and in some of them the order, or even the genus, is mentioned, to which they will probably be found to belong. Their great number cannot surprise those who are aware how imperfectly many minerals which were long ago known, have been hitherto examined and described; consequently it is less advisable to receive at once newly discovered minerals into the system, when we see that even those determinations, which were usually considered most firmly established, have frequently been found, on more accurate examination, to be erroneous. The species contained in this appendix must be viewed exactly in the same light as the *plantæ incertæ sedis* in the natural system in Botany (not in artificial systems, which cannot admit of any appendix), which are not included in any of the systematic unities, notwithstanding the advantage that the examination of one, or of two individuals at the most, should here be sufficient for knowing the species to its full extent.

The properties of the minerals contained in the second Appendix are such, that we cannot expect that they will ever form particular species, since they are not susceptible of a natural-historical determination. Some minerals of this description have been enumerated in the Observations annexed to those species, to the decomposition of which they owe their existence, as, for instance, Porcelain-Earth, which is noticed under the head of Prismatic Feldspar.

The copper-plates, which have been extremely well executed by Mr Miller, are intended not merely to represent the figures quoted in the general descriptions of the species, but also for producing a general survey of the combinations most commonly occurring among the simple forms found in nature. They are disposed in the order of the systems of crystallisation, and provided on the opposite page with the explanations given in crystallographic signs. The figures in the last five plates are not arranged in this order, since they were added only in the course of printing the work. They refer either to some remarkable varieties of species described in the system, or they have been rendered necessary by the reception of several species into the first Appendix, which had been described by various authors, the greater part of them since the publication of the German original.

In comparing many of these with nature, the cabinet of Mr Allan, as will appear from the frequent references made to it, has afforded the translator the most important assistance; and he trusts it will not be found out of place, if he embraces the present opportunity of expressing to that gentleman, the deep sense he must ever entertain of the many marks of kindness he has received from him. To him he has not only been indebted for a home in a foreign land, but also for much assistance, and many valuable observations, in the progress of this work. To Dr Brewster he is under the greatest obligations, both for many interesting additions, concerning the optical and other properties of minerals, and for the perseverance and patience with which he has aided him in the correction of the press. Dr Turner also has favoured him occasionally with his valuable assistance.

The translator feels it an agreeable duty to acknowledge, in the present place, that the additions to this work, and likewise the papers, which he composed previous to its publication, owe the greater share of any merit they may possess to Professor Mohs, whose constant tuition in Mineralogy, since the year 1812, he has had the good fortune to enjoy, and of whose continued friendship he has every reason to be proud.

The following words of Professor Mohs, at the end of his Preface to the first and to the second volumes of the original, will form the best conclusion of these prefatory observations. "The present Treatise on Mineralogy is founded on principles so "different from those generally received in treating "of minerals, being in part in direct opposition to "them, that it is not without hesitation that I have "determined to lay it before the public. I have "endeavoured to unite accuracy, correctness, and "perspicuity, with as much of precision as I could "command; yet I am perfectly aware that I have "not everywhere succeeded, and that this Trea-"tise is in many respects imperfect. The task I "had to perform was nothing less than to apply "with consistency to a whole science, a method "which, though not new in itself, had yet been dis- "regarded in Mineralogy, and to remove all the "difficulties arising from deficiencies I had to "supply, and from errors I had to correct. This "problem, however, requires so much time and "labour, that the person who undertakes to re- "solve it, must leave many parts to subsequent "investigation, while those who judge of the merits "of a first attempt of this kind, will be disposed "to relax in the severity of their criticism. Yet "I wish that this work may be subjected even "to the strictest examination, provided it be can- "did, well grounded, and does not omit to con- "sider, that at the present moment the disposi- "tion of the whole must be of greater importance "than the minuter details of the various depart- "ments of the work. I know none of the im- "perfections still to be met with, which could "not be removed by future labours, and which "will not soon disappear, if I have been for- "tunate enough to call the attention of natu- "ralists towards the exact knowledge of the phy- "sical qualities of minerals, and to induce them "to investigate these more closely and accurately "than has hitherto been customary. Like every "other department of Natural History, Minera- "logy is a charming science. But its charms are "grounded only upon its exactness; and nothing "has a more baneful influence on the science itself, "than a superficial view of it. My only intention "is to forward the scientific progress of Mineralogy, "which is chiefly dependent on the purity of "method necessary in every science; on the cor- "rectness of principles already demonstrated in the "other departments of Natural History; and on "the consistency of the different parts of the science "among themselves, objects which I have en- "deavoured to attain. In this condition, Minera- "logy answers every purpose that can be reason- "ably expected from any part of Natural History. "I trust that the results already obtained, however "insufficient they may be, will induce naturalists "to take advantage of the first step towards the "construction of that edifice, of which I have laid "down the plan in the present Treatise."

Charlotte Square, EDINBURGH,
25th March, 1825.

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INTRODUCTION.

§. 1. NATURE.

In the sciences, the word *Nature* is used under three different significations. In the *first*, it denotes the general idea of the natural bodies altogether, or the compass of natural existence; in the *second*, the assemblage of the properties of a single body, or the constitution and appearances of things; in the *third*, it is used for expressing the power or cause which produces them.

These significations are contained in the following examples. "There are bodies in Nature very much resembling, and yet different from, each other." "It is in the nature of gold to be ductile, heavy, &c." "Nature produces different species of animal, vegetable, and mineral bodies."

§. 2. NATURAL HISTORY.

The object of *Natural History* is Nature considered as the assemblage of all *material* bodies.

The name of Natural History, does not express the essential properties of the science to which it is applied, and has therefore been used in a very improper sense. Natural History is by no means a historical science; it has no business with accidents or facts, but refers to objects, of which it is indifferent whether they exist contemporaneously or consecutively; and it considers these objects either singly, or in such relations as they are brought into, by the application of the science itself. Of this kind is the connexion produced amongst the natural objects by the Theory of the System, or the second principal head of the present work. Natural History by no means considers those connexions among different bodies, in which some of them produce alterations in the others, or contain the causes of certain events.

The peculiar character of History, consists in being a narrative or a relation of facts, arranged according to the succession of time. Natural History has nothing to relate, and takes no notice of the succession of events.

The impropriety of the words Natural History, has exercised a prejudicial influence upon the developement and the progress of the science itself, and has given rise to many misconceptions. All these misconceptions disappear as soon as the notion of Natural History is circumscribed within proper limits. Supposing the existence of a definition of this kind, the name of Natural History can be retained, particularly since it has not, till now, been superseded by another more appropriate expression.

§. 3. NATURAL-HISTORICAL PROPERTIES.

The properties of a body, in as far as they are considered and made use of in Natural History, are called *Natural-Historical Properties.*

The Natural-Historical properties are those with which Nature has endowed the bodies which it produces, provided these properties, as well as the bodies themselves, remain unaltered during their examination. A body is said to be in its natural state, while it continues to shew these properties. The natural state of a body is either constant, or it is variable during a certain period of time. In the first case, Natural History at once selects such of the invariable properties, as may serve its purpose agreeably to the principles of the science. In the second, it determines before hand the state of the highest perfection, or of the full developement of these bodies, and then makes the same choice. Properties thus selected are the natural-historical properties of a body. Hence every natural-historical property is one of those appertaining to a body in its natural state; but every one of the properties to be met with in a body is not on that sole account also a natural-historical property. The assemblage of all the natural-historical properties of a natural production, is its Natural, or Natural-Historical quality.

Properties not subservient to the use of Natural History, are considered in other sciences, which, in respect to their fundamental principles, entirely differ from that of Natural History.

§. 4. NATURAL PRODUCTIONS.

Material Bodies, in consequence of their being produced by Nature, are called *Natural Productions.*

It is Nature alone which produces bodies. Art only modifies certain properties of the bodies produced by Nature. Natural productions, modified or altered by the assistance of art, are called *artificial productions.* A tree is a natural production; a table, into the form of which the wood of the tree has been fashioned, is an artificial production. If a gem undergoes a chemical analysis, it ceases by that process to be a natural production. If it is cut and polished, abstraction being made of its artificial form, it still must be considered as a natural production; whilst, in respect to the form itself, it becomes an artificial one.

§. 5. DESIGN OF NATURAL HISTORY.

Natural History considers the natural productions as they are, and not how they have been formed.

Natural History does not inquire into the mode of formation of natural productions, but only into their natural-historical properties, because herein consists the only object of its consideration. Yet this is not on account of the difficulties, which attend the explanation of the mode in which natural productions have been formed, but because it acknowledges principles, which entirely exclude *all explanations of this kind*. Thus the principles of Natural History fix the extent and the limits of that science; limits which it cannot transgress without inconvenience. Yet it is not thereby too much confined, since whatever may thus be excluded, does not belong to Natural History itself, but to other sciences. Every addition from these would only serve to contaminate the fundamental principles of Natural History. It is a matter of the highest importance to keep the sciences perfectly distinct from each other, and strictly within their respective limits, in order to become acquainted with their stronger and their weaker parts, and to assist wherever it should be necessary; but the philosopher must not possess them separately. The sciences might be compared to working tools set in different handles, and subservient to different purposes. The *intelligent* naturalist is like an *able* artist, who knows how to employ them conformably to his design.

§. 6. INDIVIDUALS.

A natural production, in as far as it is a single body, and, as such, by itself fit to be an object of natural-historical consideration, is termed a *Natural-Historical Individual.*

Natural productions, which are not individuals, or whose individuals are no more recognisable, may, nevertheless, be objects of examination, according to the principles of Natural History. The idea of individuality implies unity of form; and by this an individual becomes an independent whole, whose natural-historical consideration does not presuppose the existence of, or at least not the connexion with, another individual. In Natural History, a tree is an individual; not the trunk, nor a branch, nor the fruit of that tree. For the first of these is, by itself, an object of natural-historical consideration, the others only in as far as they are parts of the tree.

In water and other fluid bodies, individuals are at least not observable. Water and other fluid bodies produced by Nature, though whole masses of them (which may, nevertheless, consist of individuals) are without individuality, on all accounts remain natural productions, and, as such, objects of Natural History.

§. 7. ORGANIC AND INORGANIC NATURAL PRODUCTIONS.

The most conspicuous difference which presents itself in Natural History, is that which exists between bodies either *organic* or *organised*, and *inorganic*.

An organised body is composed of organs; that is to say, of vessels and instruments suitable to the subsistence, increment, and reproduction of themselves and of the whole. During a certain variable period of time, called *Life*, the organised body is beyond the reach of those powers which affect inanimate matter, if removed from that condition. Matter, in as far as it forms a part, or is the product, of an organised body, is called *organic* matter; and a body consisting of it, an *organic* body. An *inorganic* body consists of inanimate (not organic) matter. Here the powers actuating it, have finished their effect, and are therefore in equilibrium; it exists in itself in an invariable state, and can be altered only by external force. Certain products of organised bodies, as resins, &c. are not organised themselves; that is to say, do not consist of organs; yet they do not cease to be organic bodies, because they consist of organic matter. However, they are not by themselves objects of a natural-historical consideration (§. 6.).

§. 8. ANIMALS AND PLANTS.

A farther difference takes place among the organised natural productions, depending upon their mode of generation, subsistence, growth, propagation, and upon the quality and utility of their organs. One part of them are termed *Animals*, the other *Plants*.

§. 9. MINERALS.

There is no such difference among the inorganic bodies. The inorganic productions of Nature are altogether comprehended under the name of *Minerals*.

Some naturalists have attempted to introduce a distinction among the inorganic productions of Nature, similar to that mentioned above in respect to the organised bodies; yet the characters upon which this distinction was founded, do not refer to those bodies themselves, or to their natural-historical properties; but arise merely from their connexion with each other, from local relations, &c.; and hence the distinction itself is foreign to Natural History.

Those inorganic productions of Nature which have been separated from the minerals, and provided with a particular name, are the *Atmospherilia*, or those bodies which constitute the atmosphere, in the same way in which the others form the solid parts of the globe. Agreeably to the preceding considerations, this difference, the only one between the two classes of natural bodies, is quite inadmissible in Natural History; for Natural History does not consider the natural productions in so far as they constitute the solid mass of the globe, or the fluid mass of the atmosphere, but in so far as, taken separately, they possess certain natural-historical properties. Hence the atmospherilia cannot be separated from the minerals. In a subsequent paragraph it will appear, that a distinction of this kind would be contrary to the very idea of a mineral.

The Wernerian school has applied the term *Fossil* to those minerals, which constitute the solid part of our globe. Commonly this name is given to the remains of organic bodies, which are dug out from the earth, as "fossil wood, fossil bones, &c." and this is indeed the right use of the expression. The name of a fossil becomes entirely inapplicable, if, agreeably to the principles of Natural History, the atmospherilia are united with the minerals. Moreover, the meteoric masses of iron, being the only varieties we know, of the species of octahedral Iron, cannot be called fossil bodies.

§. 10. NATURAL KINGDOMS.

Natural History considers the differences mentioned in §. §. 8. and 9., as the foundation of dividing the natural productions. Each member of this division is called a *Kingdom*. That division which comprehends the animals is termed the *Animal;* that which contains the plants, the *Vegetable;* and that which comprises the minerals, the *Mineral Kingdom.*

§. 11. DIVISION OF NATURAL HISTORY.

The distinction among the natural productions, in §. 10., has occasioned a division of Natural History, according to these three Kingdoms. That part of Natural History which considers the Animal Kingdom, is called *Zoology*; that which considers the Vegetable Kingdom, *Botany;* and that whose object is the Mineral Kingdom, *Mineralogy*, or the Natural History of the Mineral Kingdom.

This division of Natural History, is founded upon the difference of the objects, to which the single parts of that science refer. It has no influence upon the principles and upon the method; or, properly speaking, it is not a consequence from these, which are identical for all the three parts of Natural History.

There is, however, another division required in Natural History, which does not depend upon differences among the objects considered, but is founded upon the being of the science, and is therefore equally applicable in Zoology, in Botany, and in Mineralogy. This is the division in *Determinative* and *Descriptive* Natural History, which will be explained more fully hereafter. It appears from the preceding observations, that respecting the mineral kingdom, *Anorganography* does not signify the same as Mineralogy, but applies merely to the descriptive part of it. *Oryctognosy*, however, means the doctrine of what is dug out of the earth, that is to say, according to the mode of expression mentioned in §. 9., of the fossils, and cannot therefore be applied with more propriety than the other, to the Natural History of the Mineral Kingdom.

Mineralogy, or the Natural History of the Mineral Kingdom, does not allow of any other subdivision than that which has just been considered. Hence Geology is not a part of Mineralogy, but of Physical Astronomy; Mineralogical Chemistry is not a part of Mineralogy, but of Chemistry; Economical Mineralogy is not a part of Mineralogy, but of Economy; nor is Mineralogical Geography a part of Mineralogy, but of Physical Geography, which belongs to Physical Astronomy.

§. 12. PRINCIPAL HEADS OF NATURAL HISTORY.

The method of Natural History in general, and each of its departments in particular, is developed under the following heads: 1. *Terminology*, 2. *Theory of the System*, 3. *Nomenclature*, 4. *Characteristic*, 5. *Physiography*.

If this be effected in a general way for all the three kingdoms, it produces the method of Natural History *in general;* if applied to each of them, it gives the method of the Natural History of the kingdom concerned.

As yet, the method of Natural History in general, has not been treated of separately, nor is it an object which requires to be more circumstantially developed in the present place. This method would be for the whole compass of the productions of Nature, what the Philosophia Botanica of Linnæus is for the vegetable kingdom.

The method of the Natural History of any particular kingdom, is contained in that of Natural History in general, and differs from the method of the other kingdom, only by its being applied to different bodies. This will be perfectly evident, if we reflect that the different parts of Natural History could not be parts of one and the same science, should their methods be different. Indeed, the method according to which the aggregate of various information, commonly called Mineralogy, has hitherto been treated, is different from the method of Natural History in general. Mineralogy, however, treated in this manner, is not the Natural History of the Mineral Kingdom, but is a compound not contained within a single science, and which altogether cannot be traced to constant principles, by any regular process of reasoning.

§. 13. TERMINOLOGY.

Terminology is the explanation of the natural-historical properties, in as far as they are employed in recognising, distinguishing, and describing the productions of Nature, and in developing those general ideas, which the method requires.

Terminology teaches the language adapted to the peculiar use of the science, and explains the meaning of what has been called the *Technical Terms.*

In this scientific language, fixed expressions are connected with accurately determined ideas, and, *vice versa,* accurately determined ideas with fixed expressions. It is as necessary in Natural History, as it is in Geometry; and it may be said to be in respect to the former, what the Definitions are in respect to the latter. In Natural History, however, the Terminology has to surmount many more difficulties than in Geometry, since it refers principally to *empiric* notions. Hence, the more geometrical ideas can be introduced into the mineralogical Terminology, the greater advantage will be obtained; because, by this means, its explanations will approach the nearer to the character of geometrical definitions. None of the other parts of Natural History allow of the introduction of geometrical ideas to so great an extent as the Natural History of the Mineral Kingdom.

§. 14. THEORY OF THE SYSTEM.

The Theory of the System determines the idea of the *Species* in Natural History. It fixes the principle of classification; and upon the idea of the species, it founds, according to this principle, the ideas of the *Genus*, the *Order*, the *Class*, and the *Kingdom*, in both the *natural* and the *artificial systems*; the difference of these it likewise indicates and explains. Lastly, by applying all these ideas to Nature, the outline of the system thus constructed, is furnished with its contents, in conformity to our knowledge of the productions of Nature, as obtained from immediate observation.

The Theory of the System contains the reasoning, or philosophical part of the science, and consists in the production of ideas of a greater extent, than those derived immediately from observation. These are the ideas mentioned above. The fundamental proposition, in this part of the science, is the following: *All things are identical* *which, in their natural state* (§. 3.), *do not differ from each other in any of their properties;* and this may be considered as an Axiom in Natural History. This mode of reasoning is common to all the three parts of the science. There occur, however, differences in respect to the application of these ideas to Nature. They arise out of the different qualities of the natural productions contained in each of the Kingdoms.

The possibility of introducing mathematical ideas into the Terminology of the Mineral Kingdom, is particularly beneficial to the establishment of these systematic ideas, in as much as their precision, in some measure, extends to the latter; and imparts to the most important of them all, to the Idea of the Species, a degree of evidence, which seems to be wanting in the other kingdoms, both vegetable and animal, and which it is scarcely possible to supersede by any other considerations. In this part of Natural History, the Theory of the System takes the place of the Axioms and Theorems of Geometry.

The name of *Classification* has been sometimes applied to the systematic reasonings in Natural History. Yet, properly speaking, classification is only that part of the Theory of the System which refers to the idea of genera, orders, &c. under which the species shall be finally arranged, and in their application to Nature.

§. 15. NOMENCLATURE.

Nomenclature, is the assemblage of rules, according to which *Names* and *Denominations* are applied to natural productions. By these names and denominations, the ideas of the system are fixed; or the one can be substituted as representatives instead of the other.

The scientific nomenclature in Natural History is *systematical.* Any nomenclature, not systematical, is termed a *trivial* nomenclature, and does not belong to the science.

The necessity of a systematic nomenclature in Natural History needs no demonstration. *Fundamentum Botanices duplex est: Dispositio et Denominatio.* LINN. Phil. Bot. 151. The systematical nomenclature is the base upon which the existence and the progress of the science is founded, which, without it, must fall into confusion. This is more obvious, indeed, in Zoology and Botany, than in Mineralogy, yet by no means less true in this part of Natural History, as is sufficiently proved by long continued experience.

No systematical nomenclature has hitherto existed in Mineralogy; and even the fragments of it, to be met with here and there, do not deserve our attention, because they refer to systems foreign to Natural History.

Trivial names* are not fit for any scientific use, but they are very convenient for common usage, particularly if they are well chosen.

§. 16. CHARACTERISTIC.

The Characteristic furnishes us with the peculiar terms or marks, by which we are able to distinguish objects from each other, in so far as they are comprehended in the ideas established by the Theory of the System. The Characteristic is peculiar to the *Determinative part of Natural History* (§. 11.).

The Characteristic presupposes the general notions or ideas of Natural History to have been developed and applied to the data of observation, and therefore is not the source of these ideas, nor of any other. The natural-historical properties, or those assemblages of them, by which we can *distinguish* the different species of one genus, the different genera of one order, the different orders of one

* What Linnæus calls trivial names, will be explained in its proper place.

class, the different classes of one kingdom, are termed *Characters;* while the single properties made use of, or contained in them, are called their *Characteristic terms* or *marks.* The Characteristic is intimately connected with, and indeed presupposes the existence of the system. A character referring to a natural system is called a *natural Character*, and one which refers to an artificial system, an *artificial Character.*

Hitherto there has never existed a Characteristic in Mineralogy, nor was it even possible to be successful in the attempt of its construction; because there never has been a system, to which a Characteristic could have been applied.

§. 17. PHYSIOGRAPHY.

Physiography is the description of natural productions, and consists in the enumeration of all their natural-historical properties. Physiography is peculiar to the *Descriptive part of Natural History* (§. 11.).

Descriptio est totius plantæ character naturalis, qui describat omnes ejusdem partes externas. LINN. Phil. Bot. 326.

Physiography is intended to produce, by its descriptions, a distinct image of the natural productions. If considered as a mere description, the object of Physiography is the individual (§. 6.); and these descriptions do not require any thing but Terminology, and the correct idea of Natural History itself. No systematical ideas are wanted, and any names may be employed, if they be only fit to be kept in a constant, though in itself arbitrary, reference to the object described. Very little advantage, however, is derived from such descriptions, for the Natural History of the Mineral Kingdom. In order to answer the purposes of Physiography, their object must be the Species; and the result obtained by that means, is the *Collective* or *General Description* of the species, which unites in itself the description of all its individuals or varieties. Under this supposition, it requires also a correct notion of the natural-historical species. But since Physiography is entirely independent of the system, and consequently also of the systematic nomenclature; the general descriptions will be applicable to any system, provided the terminology employed be sufficiently accurate. Any nomenclature can be made use of in this part of the science, because the arbitrary names in every instance can easily be exchanged for the systematic denominations. The Descriptive part of Mineralogy has been hitherto the only one, towards which the labours of naturalists have been directed; and it is solely to them that we are indebted for the progress of our information respecting the products of inorganic Nature.

To the Descriptive part of Mineralogy must be referred all those representations of the objects, as drawings, models, &c. which are executed with the view of giving a more comprehensive idea of these objects themselves; but they belong to Terminology, if they are intended to elucidate certain particular properties of the minerals, as the drawings and models in Crystallography, which are employed for the sake of developing the whole theory of Crystallisation.

§. 18. IDEA OF NATURAL-HISTORY.

Natural History is the science, which enables us to find the Systematic Denomination of any natural production (§. 3.), if its Natural-Historical Properties be given or known; and, *vice versa*, from the Denomination being given, to find the Natural Quality of a body. Mineralogy being one of the three departments of Natural History, is the same to the Mineral Kingdom (§. 10.), as Natural History in general, is to the whole material Nature (§. 2.).

Lege artis mutuo noscatur planta ex nomine et nomen ex planta; utrumque ex proprio charactere; in illo scripto, in hac delineato; tertius non admittatur. LINN. Phil. Bot. 261.

Hence the application of Natural History, to the objects of observation, essentially consists in the process of connecting the natural-historical properties of the natural productions with their systematic denominations; or, on the contrary, in that of joining the denominations with the individual or collective descriptions (§. 17.). The first requires the assistance of the System and of the Characteristic; the other can be immediately effected, and does not require the application of a particular proceeding.

From the manner in which Mineralogy has hitherto been treated, it was impossible to obtain any other but an *empirical* knowledge of Minerals, which consists in the recollection of having already met with a similar object, to which a certain arbitrary name had been given.

It is very difficult to attain a correct knowledge of the productions of the Mineral Kingdom, if we confine ourselves to empiricism. Besides, it is a waste of time, and the information thus acquired, is at the best uncertain. The bad consequences of having chosen an unscientific mode of proceeding of this kind, increases with the actual enlargement of our information, in respect to the productions of inorganic Nature.

§. 19. METHOD OF STUDYING THE NATURAL HISTORY OF THE MINERAL KINGDOM.

The only scientific way of studying Mineralogy is, to proceed according to the principles, and conformably to the method of the science itself. This requires some practice in several observations, relative to certain properties of Minerals; it presupposes some acquaintance with the mathematics; and a little tuition will greatly facilitate its application.

Every person who intends to acquire solid information in Mineralogy, must endeavour to become conversant with the principles of Natural History; without which, it is but too easy to miss the way to its attainment. Another very material object is the correct application of these principles to the Mineral Kingdom; for the best and most perfect instrument is of no utility to those who are not acquainted with its employment.

Lastly, a certain degree of skill is required in recognising and finding out the connexion of those forms which, in Mineralogy, are called regular. In order to facilitate the acquisition of this, some mathematical knowledge is necessary.

After having become sufficiently acquainted with Terminology, the surest and shortest way for the beginner to proceed, is to apply at once to Nature itself. This may be effected by means of the Characteristic, which, according to the rules laid down, under the fourth Head, must be employed in order to acquire a certain degree of practice, in the determination of individuals occurring in Nature. This leads to an intimate acquaintance with the minutest details, and thus becomes the basis of information of greater extent.

If the student has an opportunity of examining well arranged collections, he will be enabled to acquire general ideas, and form general views, in a much shorter period of time than would be possible by the comparatively slow, yet detailed and sure processes of the Determinative part of Mineralogy. In collections of this kind, the determination of the species must be correct, and their arrangement conformable to the general principles of Natural History. Collections otherwise arranged, can be of little use to the beginner; on the contrary, they may be prejudicial to his future progress, in as much as they confound his ideas; and indeed they may be said to be useful only to those, who wish to enlarge their information, by observation and inquiry.

There exist but few Mineralogical works, which can properly be recommended to a beginner. The following enumeration contains those most useful for this purpose.

FOR THE STUDY OF TERMINOLOGY.

Von den æusserlichen Kennzeichen der Fossilien, von A. G. Werner. Leipzig. 1774. This work has been translated into several languages. It has been translated into English, under the title of—

A Treatise on the External Characters of Fossils, of Abraham Gottlob Werner, by Thomas Weaver. Dublin. 1805.

A Treatise on the External, Chemical, and Physical Characters of Minerals, by Robert Jameson. Second Edition. Edinburgh. 1816.

Cristallographie, ou Description des Formes Propres à tous les Corps du Règne Minéral. Avec Figures et Tableaux Synoptiques de tous les Cristaux connus. Par M. de Romé de l'Isle. 2de Edition. Paris. 1783.

Traité de Mineralogie, par le Cen. Haüy, &c. En cinq volumes, dont un contient 86 planches. Paris. 1801.

De la Cristallisation considérée géométriquement et physiquement ; ou Traité abrégé de Cristallographie, &c. Par A. J. M. Brochant de Villiers. Strasbourg. 1819.

Versuch eines ABC Buchs der Krystallkunde von Karl von Raumer. Berlin. 1820.

Nachträge zu dem ABC Buche der Krystallkunde von Karl von Raumer. Berlin. 1821.

Various Memoirs in the Journal and the Annales des Mines, &c. by Messrs Haüy, Monteiro, &c. Also in the Memoirs of the Academy and the Society of Berlin, in the Journal für Chemie und Physik of Schweigger, &c. by Messrs Weiss, Bernhardi, &c.

Since the original publication of this work in German, there has appeared—

A Familiar Introduction to Crystallography ; by Henry James Brooke. London. 1823.

FOR THE STUDY OF THE THEORY OF THE SYSTEM.

Caroli Linnæi Philosophia Botanica, &c. Holmiæ. 1751.

Des Caractères Extérieurs des Minéraux, ou existe-t-il dans les Substances du Règne Minéral des Caractères qu'on puisse regarder comme spécifiques, &c. Par M. de Romé de l'Isle. Paris. 1784.

Most of the works relating to the subject of the Mineral System, and Classification in general, require the utmost attention on the part of the beginner, who intends to peruse them.

As to the principles of Nomenclature and the Characteristic, the study of the works of Linnæus is particularly to be recommended, and, above all, of his Philosophia Botanica and Critica Botanica.

In the second volume of this Treatise, those works are quoted, which regard the Descriptive part of Mineralogy, and which partly also have been referred to in the descriptions of the single species.

PART I.

TERMINOLOGY.

GENERAL CONSIDERATION OF MINERALS. THEIR DISTINCTION INTO SIMPLE, COMPOUND, AND MIXED. DIVISION OF THEIR NATURAL-HISTORICAL PROPERTIES.

§. 20. POWER OF CRYSTALLISATION, AND ITS PRODUCTS.

That power which produces the individual (§. 6.) in the Mineral Kingdom, is termed the *Power of Crystallisation.*

This name has been applied, because the most eminent and perfect productions of that power are Crystals (§. 26.). The power of Crystallisation is included in the general idea of the *Individualising* power, which tends to produce individuals in all the three kingdoms of nature; and which refers equally well to regular crystals, and to such individuals of the mineral kingdom, as are produced by the power of crystallisation, although they are not crystals themselves, as will be shewn in another part of this work.

Individuality does not require regularity, but it implies unity of form (§. 6.). An individual, whatever may be its form, fills the space occupied by that form, with a certain matter (§. 23.), and thus it represents a *whole*, being coherent in itself, and limited towards the outside. For this reason, the individual is a single body, and, as such, by itself a fit object for the consideration of Natural History.

When minerals pass into the state of individuality, they at the same time are endowed with the rest of those natural-historical properties which are peculiar to them in that state of distinct existence; and hence these properties must likewise be considered as produced by the power of crystallisation. The assemblage of those properties, is the mineral, or the natural production itself; at least, in as far as it is an object of Natural History.

Minerals, upon which the power of crystallisation has not exercised its action, are without individuality, and therefore do not possess any of the properties connected with this state of existence. They want unity of space; they are not single bodies, and, as such, by themselves fit objects of Natural History. As mere shapeless masses, with certain inherent properties, they can be considered as objects of Natural History, only because they are natural productions (§. 4.).

Temperature has a great influence over the power of crystallisation. Several minerals, as water, fluid mercury, &c. pass into the state of individuality, and become solid, if the degree of temperature be sufficiently reduced; on the contrary, by an increase of temperature, hexahedral silver, octahedral bismuth, &c. leave this state, and become liquid, and others elastic.* For that reason, in treating of Natural History, it is necessary to fix the degree of temperature in which the productions of the mineral kingdom are considered; and this is the *ordinary* temperature, at which water is fluid, and the most fusible crystals are solid.†

§. 21. MINERALS DECOMPOSED AND IMPERFECTLY FORMED.

The productions of the power of crystallisation continue to be objects of natural-historical consideration, so long as they retain the properties peculiar to them, which they have derived from the action of this power. By the loss of some or of seve-

* If the change produced on a mineral by the application of heat, affects more than the mere state of aggregation, the consideration of this change makes part of another science, and has no reference to Natural History.

† This is the reason why water, and not ice, has received a place in the system.

ral of these properties, they *cease* to be suitable objects for the consideration of Natural History.

A mineral possessed of the properties it received from the power of crystallisation, is in its natural or original state (§. 3.). A mineral which has lost these properties more or less, is decomposed, and ceases to be an object of natural-historical consideration.

Minerals thoroughly decomposed commonly appear in the form of powder, or as shapeless masses, without presenting any regular structure, or lustre, or determined and constant degrees of hardness or specific gravity; and the cohesion of their particles is destroyed. They form part of the *friable* or *earthy* minerals. An example of an earthy mineral we have in Porcelain-earth, a substance produced by the decomposition of prismatic Feld-spar.* The decomposition of minerals, however, does not in all cases proceed so far. Some minerals retain their form, whilst colour, lustre, hardness, &c. are changed; as in several varieties of hexahedral and prismatic Iron-pyrites. All, even the slightest, of these alterations, exercise an influence upon the natural-historical consideration of those bodies. It is in direct opposition to the principles of Natural History, to consider decomposed varieties of one species, as varieties of another; but this, nevertheless, has been but too often the case in Mineralogy. Thus, decomposed varieties of hexahedral and prismatic Iron-pyrites, and of brachytypous Parachrose-baryte, have been taken for varieties of prismatic Iron-ore. In most cases it is possible to determine what the decomposed minerals have been in their natural or original state, though indeed, for that purpose, we have often to recur to considerations foreign to Natural History.

It seems that the substance of several minerals has, in the period of their formation, not arrived at that state of perfection which distinguishes the finished productions of

* Another class of friable minerals consists of very small fragments of crystals, and grains of fresh or not decomposed minerals. Such are fine sand, &c.

crystallisation. In respect to Natural History, they must be classed with those which are decomposed. Minerals imperfectly formed, may be compared to animals or plants *mutilated*, *defective*, *monstrous*; while those that are decomposed, having ceased to retain their original state, may be compared to the animal or the plant which has *ceased to live*. They may elucidate facts, both in Zoology and Botany, though in that state they are not in themselves objects of inquiry in Natural History. It is therefore perfectly evident, that the distinction introduced by some naturalists among minerals, into crystallised, crystalline, and amorphous, depends upon accidental circumstances in the formation of these bodies; and, therefore, is not essential.

§. 22. SIMPLE MINERAL.

A mineral consisting of one single individual, or forming a part thereof, is termed a *Simple Mineral.*

This is the idea of a simple mineral in Natural History. The simple mineral must be distinguished from what is called simple in Chemistry; and, likewise, from what Mineralogists commonly call simple. The last frequently consists of several individuals, and is therefore not simple in the sense of Natural History. Examples of simple minerals are crystals and grains of dodecahedral Garnet, or of octahedral Diamond. The particles of which granular Limestone is composed, are each simple minerals belonging to the species of rhombohedral Lime-haloide; while those of Coccolite are also simple minerals, belonging to the species of paratomous Augite-spar, &c.

§. 23. COMPOUND MINERAL.

A mineral consisting of more than one individual of the same quality,* is termed a *Compound Mineral.*

* The term *Homogeneous* individuals would be more

The compound mineral consists of simple ones. It is produced when several individuals of the same quality are formed in a common space, either at the same time, or one after the other; one being the support of, or at least contiguous to the other. It is, therefore, not *one* simple mineral, but a composition of *several*. If many of these simple minerals come into contact, they prevent each other mutually from assuming their regular form. Compound minerals, therefore, which consist of many simple ones, do not possess regularity.

Examples of compound minerals are frequently met with, as in the above mentioned varieties of rhombohedral Lime-haloide, and paratomous Augite-spar; also the globular masses of hexahedral and prismatic Iron-pyrites, and the stalactitic masses of rhombohedral Quartz, called Calcedony, &c. may serve as examples of compound minerals.

§. 24. MIXED MINERAL.

A mineral, consisting of several individuals of different qualities, is termed a *Mixed Mineral.*

The mixed mineral consists of simple minerals, like the compound. The *mixed mineral*, as such, is not an object of Natural History, because its constituent parts, the *simple minerals*, have already been considered by themselves, and received their appropriate places in the system of Natural History. For the same reason it becomes necessary, from the principles of Natural History, to exclude even *com-*

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exact, if, in the present place, we could avail ourselves of that expression. In order to understand what is meant here, it will be sufficient to consider individuals of the *same quality*, to be such as are contained in the examples quoted in the preceding paragraph of rhombohedral Lime-haloide, and paratomous Augite-spar. Individuals of *different quality*, are such as exhibit notable differences in their natural-historical properties; as, for instance, Granite, where the component individuals of rhombohedral Quartz, prismatic Feld-spar, and rhombohedral Talc-mica, widely differ in appearance and character.

pound minerals from these considerations. It is necessary, however, to distinguish correctly between the simple and the compound minerals; and since this cannot be done otherwise, than by knowing all the details respecting these bodies themselves, their consideration must not entirely be neglected.

The union among the simple minerals in the mixed mineral, is sometimes so close, and the particles of the mixture so diminutive, that it becomes impossible to ascertain their reality by simple ocular inspection. Many Mineralogists in this case consider mixed minerals as simple, and class them as such in their systems. But this is not the only error of the kind, occurring in such arrangements. Both mixed and decomposed minerals are by themselves no longer objects of the method; yet there are even mixtures of decomposed minerals, which have been introduced into the systems, and to which particular places have been assigned.

Examples of mixed minerals we have in many varieties of rocks; in granite, gneiss, porphyry, &c.; also in many of those masses which constitute beds and veins. Examples of close or impalpable mixtures, are found in Iron-flint and Heliotrope, both varieties of rhombohedral Quartz; the first of which is mixed with oxide of iron, the other with Green Earth, a variety of prismatic Talc-mica. Mixtures of decomposed minerals we have in Clay, Yellow Earth, Tripoli, &c.

§. 25. DIVISION OF THE NATURAL-HISTORICAL PROPERTIES.

The natural-historical properties of *minerals* are divided into; 1. such as refer to *simple*; 2. such as refer to *compound minerals*; 3. such as are common to *both*.

The natural-historical properties of minerals comprehend their colour, the different degrees of hardness, the different kinds of lustre, the regular forms, the circumstances and relations, under which the particles of the individuals can be separated from each other, &c.; because these are the properties of minerals exhibited in their natural state, and may be considered without producing any change or alteration on the mineral.

Properties which can only be observed during, or after a change, cannot be employed agreeably to the principles of Natural History, and must therefore be excluded from Mineralogy; because, in observing them, we transfer the object itself from its natural state, into another, in which it ceases altogether to be an object of Natural History. Properties of this kind are, the fusibility of minerals examined before the blow-pipe, or by the assistance of some other apparatus, and the concomitant phenomena;—their solubility in acids;—phosphorescence produced by heat, if, after the first experiment, it cannot be observed any longer;—chemical analysis instituted to ascertain the quality or relative quantity of the component parts, and the results of that process:—every thing, in short, must be excluded, which alters the natural state of a mineral. There are properties to be met with in minerals in their natural state, which, although not altered by examination, yet are of no utility in Natural History; such as the size of crystals; the irregular enlargement, and figure of some of the faces depending upon it; the accidental forms minerals assume by being broken, rubbed down, water-worn, decomposed, &c. Such properties are *accidental*, because the identity (§. 14.) of the individual is not destroyed by their occurrence.

The natural-historical properties include the greater part of the characters commonly called *external*, and some of those called *physical*.

As to the distribution of those properties among the different heads mentioned above, the first will include those which can be observed only in an individual itself, or in a fragment of an individual. These are the *geometrical* properties, or such as refer to *Space*; the relations of *Structure*, those of *Surface*, and the phenomena of *Refraction*, in so far as they depend upon the regular form of minerals.

To the second belong the relations of *Composition*, *the Forms* of compound minerals, the mode of *Junction* of the individuals in these compositions, &c. these properties being such as are only to be met with in compound minerals. The third comprehends those in which the simple or compound state of the mineral has no influence upon the consideration of the properties; as *Colour*, *Lustre*, *Transparency* in general, *Hardness*, *Specific Gravity*, the *State of Aggregation*, *Taste*, *&c.*

Terminology includes, therefore, *three Sections*, within which each of the above mentioned properties is considered in a separate *Chapter*.

SECTION I.

THE NATURAL HISTORICAL PROPERTIES OF SIMPLE MINERALS.

CHAPTER I.

OF THE REGULAR FORMS OF MINERALS.

I. GENERAL CONSIDERATION OF THE REGULAR FORMS.

§. 26. CRYSTAL.

In Mineralogy, the term *Crystal* is applied to a body, which consists of continuous and homogeneous matter, and occupies, from its origin, a regularly limited space.

Crystals assume a regularly limited space in their origin, that is to say, in the very act of their formation. A mineral which appears in a regularly limited space only after a part of its homogeneous matter has been detached from it, is not contained under the preceding definition, and therefore no Crystal.

The matter contained within the regularly limited space, is termed *homogeneous*, if it be everywhere of the same quality; and it occupies or fills the space with *Continuity*, if in its interior it allows no particles to be distinguished from one another, of which the whole mass might be said to be composed. There are minerals occupying a regularly limited space, with homogeneous matter, but without continuity; because in their interior, particles can be observed, which are evidently distinct from each other (§. 186.). Minerals which are found thus to consist of homogeneous matter within a regularly limited space, yet want continuity, and cannot therefore be called crystals in the signification of that term, as defined above.

§. 27. OBJECT OF CRYSTALLOGRAPHY.

The object of the science of *Crystallography*, is to ascertain the regularly limited space, that is to say, the *Form of the Crystals*, not the matter, which occupies that space.

Since the object of Crystallography is nothing but figured *Space*, and in this nothing is to be considered besides geometrical quantities, and their relations to each other; it appears that Crystallography is a pure geometrical science.

§. 28. FORMS AND FACES.

The regularly limited space occupied by a crystal, is termed a *Form of Crystallisation*, and the limits or planes, *Faces of Crystallisation.*

In Crystallography, the faces of crystallisation are considered as perfect *Planes*, although this is not always the case in nature.

They are termed Faces of Crystallisation, in order to distinguish them from certain other faces of minerals, which, though they exhibit regular shapes, yet are no crystals (§. 26.).

The faces of crystallisation receive particular names, according to the forms which they limit, as, for instance, *Faces of the Rhombohedron*, *of the Octahedron*, &c. They are called *Faces*, without any nearer determination, if the form to which they belong is understood to be a form of crystallisation.

§. 29. EDGES.

The limits of the faces, or their intersections with each other, are termed *Edges*.

The Edges of forms are always supposed to be *straight* lines, although in nature they are not always straight.

The Edges are denominated not only according to the forms to which they belong, but also according to their particular situation in respect to these forms. If a form of crystallisation contain edges of only one kind, these bear simply the name of the form; as, for instance, *Edges of the Hexahedron.* If it contain several kinds of edges, they are distinguished from each other by their name, for instance, *Terminal* and *Lateral Edges* of the Rhombohedron, &c.

§. 30. SOLID ANGLES.

The limits or terminal points of the edges are *Solid Angles.*

The solid angles are named according to the forms in which they are found, and receive a nearer determination, by some epithet expressing their particular situation and quality. Thus we say, *Solid Angles of the Hexahedron;* also *rhombohedral, pyramidal, prismatic Solid Angles,* &c.

§. 31. HOMOLOGOUS FACES.

Faces, equal and similar to each other, and similarly situated, are termed *homologous;* such as are not equal and similar, or assume a different situation in the forms, are *not homologous.*

In nature, the homologous faces are not always equal and similar to each other, yet they are always similarly situated. Sometimes a single face of a crystalline form is enlarged, and assumes a figure different from what it should be, and dependent upon this enlargement. Crystallography takes no notice of these irregularities, in as much as they are accidental, and because this science is intended to promote the study of the forms in their peculiar *Regularity and Perfection*, in order to enable us to develope their relations to each other, and to facilitate the application of both to the phenomena of Nature.

§. 32. HOMOLOGOUS OR EQUAL EDGES.

Edges are said to be *of equal magnitude*, or *of equal value*, if the faces meeting in them are equally inclined to each other, or produce an equal angle of incidence; they are said to be *of equal length*, if they are formed by equal sides of the faces; and if they are both of equal magnitude and equal length, and at the same time similarly situated, they are termed *equal* or *homologous*.

The inclination at the edges is invariable in nature; and upon this constancy of the angles of incidence, is founded the application of crystallography to nature. The length of the edges is subject to variation, as well as the figure of the faces. In the crystals themselves, all the edges of equal quantity which are similarly situated, although perhaps not of equal absolute length, are considered as homologous.

§. 33. DENOMINATION OF THE SOLID ANGLES.

Solid angles are denominated according to the number of faces contiguous to them, or according to the quality of the edges produced by the intersection of these faces. Solid angles formed by homologous faces, are said to be *Homologous* themselves.

A solid angle formed by the intersection, or consisting of three, four, five, &c. faces, is said to be a solid angle *of three* faces, a solid angle *of four* faces, &c. A solid angle is *equiangular*, if the plane angles contiguous to it are equal; it is *unequiangular*, if they are different from each other. A solid angle, resulting from the junction of two, three, &c. different *kinds* of edges, is said to be *digrammic*, *trigrammic*, &c.; and a solid angle having all its edges equal, or which possesses only one kind of edge, is, in opposition to the latter, termed a *monogrammic* solid angle.*

§. 34. SIMPLE AND COMPOUND FORMS.

A form contained under homologous faces (§. 31.) is termed a *Simple Form;* one that is contained under faces which are not homologous, a *Compound Form.*

We have examples of the former, in the Hexahedron, the Octahedron, as Geometry considers those solids, Fig. 1. 2., and in several others besides. Of the latter in the same, if their angles or edges, or both, are replaced by faces not belonging to their own form, Figs. 3. 4., or in general, if the form is limited by more and other faces, than is required for a simple form.

§. 35. THE COMPOUND FORMS CONSIST OF THE SIMPLE.

A compound form consists of two or more simple ones. Those faces of the compound, which are

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* *Monogrammic*, single-edged or one-edged, from μόνος, single, and γραμμὴ a line; *digrammic*, double-edged or two-edged, from δὶς, double, and γραμμὴ; *trigrammic*, triple-edged or three-edged, from τρὶς, triple, and γραμμὴ; referring to the number of different *kinds* in those lines or edges, which terminate in the solid angle.

homologous to each other, belong to one and the same simple form.

The hexahedron, whose angles are replaced by equilateral triangles, or by equiangular hexagons, Figs. 3. 4., is a compound form. The faces of a four-sided or eight-sided figure, homologous to each other, are faces of the hexahedron, which is one of the simple forms; the triangles or hexagons, again homologous to each other, are faces of the second simple form, which is the octahedron, and the compound form is said to consist of both.

It is possible, that a compound form may assume the aspect of a simple one, in so far as it may be contained under faces, which, according to the given definition, are homologous. The particular circumstances, under which this happens, and the reasons, why a form of that kind, nevertheless is considered as compound, will be given afterwards.

§. 36. TANGENT PLANES.

A plane, which touches a simple form in one of its edges, is called a *Tangent Plane.*

The edge of the simple form lies in the tangent plane; and the latter is always supposed to be equally inclined to both the faces meeting in the edge of the simple form, unless it be expressly mentioned otherwise.

§. 37. SECTIONS.

A plane, which intersects a simple form, is termed a *Section.* A *Principal Section* divides the form into two equal halves, without dissecting an edge; a *Transverse Section* is perpendicular to a certain line within the solid.

The knowledge of Sections is very useful, in a more detailed examination of the forms themselves; and the Principal sections, in particular, allow of many interesting applications, both in Crystallography and in Optics. In Optics, however, the term principal section is applied only to those planes, which pass through the principal axis.

In the hexahedron, Fig. 1., the principal section ACEG passes through the parallel diagonals AC and EG of two opposite faces, and through the edges joining them AE and CG, forming an *oblong* or *rectangle*. In the rhombohedron, Fig. 7., *one* principal section ABXC passes through those diagonals of two parallel faces AB and XC, which join different solid angles with each other, and through the intermediate edges AC and BX, forming a *rhomboid*. *Another* principal section C′C″B′B″, passes through the diagonals of parallel faces C′C″ and B′B″, joining equal solid angles with each other, and through the intermediate edges C′B″ and C″B′, forming a *rectangle*. Two or more principal sections, of equal and similar figure, and similarly situated, are accounted as one. Some forms, as the rhombohedron, have more than one; others, as the tetrahedron, no principal section at all. The consideration of these sections is not of equal importance in all forms.

It is not necessary to carry the distinction of these sections any farther, than to such as yield regular, or at least equiangular or equilateral figures. If, therefore, sections in general are mentioned, only sections of that description are to be understood.

§. 38. HOMOLOGOUS SECTIONS.

Sections, which either possess similar figures, or which assume them, if reduced to the same distance from the centre of the solid, or in which the junction of certain points, by straight lines, produces similar figures, are termed sections of the same kind, or *Homologous Sections*.

There are two sets of homologous sections in the hexahedron, containing on one side all the squares, as A′B′C′D′,

Fig. 1.; on the other all the equilateral triangles, as RST, Fig. 5. But the equiangular hexagons, as R″R′S″S′T″T′, produced by sections parallel to the triangles, are homologous with them; for triangles of this kind can be inscribed in them, or they are transformed into such triangles, if brought to the same distance from the centre of the form.

The sections of the hexahedron, which appear as oblongs or rectangles, as RSR′S′ Fig. 6., are likewise homologous to each other; for they are similar, if made equidistant from the centre. Let the edge of the hexahedron AD be $= 1$, and AS that part of it through the end of which the section passes, $= \frac{1}{\sqrt{2}}$; the figure of this section will be a square. A section of that kind, however, if made equidistant from the centre with a rectangle, is likewise transformed into a rectangle, and therefore homologous with these figures, and not with the squares above mentioned.

In every oblong, a rhomb can be inscribed, if we join the centres of its sides by straight lines. Hence sections of a rhombic figure are homologous with sections of an oblong or rectangular figure.

Besides the sections described in the hexahedron, there are none to be met with in any other solid whatever; or those which may be met with in other solids, can always be traced to one of these. The different kinds of sections are, therefore:

1. Such as are either *equilateral Triangles* themselves, or in which equilateral triangles may be inscribed; as regular hexagons, or equiangular hexagons, whose alternate sides, or equilateral hexagons, whose alternate angles, are equal; dodecagons of the same description, &c.

2. Such as are either *Squares* themselves, or into which squares may be inscribed, as regular octagons, or equiangular octagons, whose alternate sides are equal, or equilateral ones, whose alternate angles are equal, &c.

3. Such as are *Rectangles* or *Rhombs*, or in which rectangles or rhombs may be inscribed. It must be remarked here, that if among the rectangular sections, there is only one, or two squares, as in the tetrahedron and in the hexahedron,

the rectangular, as the greater number, determine the *kind* of the sections.

The different kinds of sections will be furnished with appropriate and expressive names in the following §§. 50, 52, 53.

§. 39. AXES.

The straight line passing through the centres of two parallel sections, if it be perpendicular to their planes, is termed an *Axis*.

Suppose a hexahedron, Fig. 1., to be intersected by a plane A′ B′ C′ D′ parallel to one of its faces; the section will be a square. The straight line PQ through M and P the centres of this, and of a parallel square, will be an axis. Take from a solid angle of the hexahedron, Fig. 5. equal parts AR, AS, AT upon the edges terminating in this angle, and lay a section through the points thus determined. The straight line AG through the centres M, M′ of this and of a parallel section R′ S″ S′ T″ T′ R″, even though the figure of the latter should be no triangle, is likewise an axis. Take equal parts AR, AS; ER′, ES′ of the parallel edges of a hexahedron, Fig. 6., beginning from two adjacent solid angles A and E, and lay a plane through the points thus determined. Its figure will be a rectangle, or, at a certain distance from the centre of the hexahedron, it will be a square (§. 38.); and the straight line NO through M and M′ the centres of this and of a parallel section, is again an axis.

Every axis passes through the centre of the solid.

In the centre of the solid, all axes, which are perpendicular to homologous sections (§. 38.), intersect each other at equal angles.

§. 40. HOMOLOGOUS AXES.

An axis *belongs* to that section, in the centre of

which it is perpendicular to its plane. Axes belonging to homologous sections, are said to be themselves *homologous*.

The axes belonging to the equilateral triangles, &c. may for the present be called axes of the first; those belonging to the squares, &c., of the second; and those belonging to the rhombs or rectangles, &c., axes of the third kind.

Some forms contain only one, others two, and others three kinds of axes in different number. The number in which the axes of the first kind appear, is *one* or *four*; that in which those of the second kind are found, *one* or *three*; and that in which those of the third kind are contained in the solids, *one*, *three*, *four*, or *six*.

The hexahedron contains four axes of the first, three of the second, and six of the third kind; the tetrahedron four of the first, none of the second, three of the third; the rhombohedron contains only one axis of the first kind.

§. 41. PRINCIPAL AND SUBORDINATE AXES.

Principal Axes are those whose sections are regular, or such figures as allow regular figures to be inscribed into them; *Subordinate Axes* such whose sections are no regular figures themselves, and in which no regular figures can be inscribed. If a form contains no principal axis properly so called, *one* of the subordinate axes is considered as the principal axis.

The axes of the third kind, in whatever number they may appear, are always subordinate axes, if they occur at the same time with others. But if they occur *alone* in a form, their number is in no case greater than three; and then two of them are subordinate; the third is the principal axis.

§. 42. UPRIGHT POSITION.

A form is said to be in its *Upright Position* when one of its principal axes is vertical.

Forms that have only one principal axis are upright but in a single one; such as have more than one are upright in several positions. If a form contains only axes of the third kind, it is upright, when that axis is in the vertical position, which is considered as its principal axis.

In the subsequent inquiries, all forms, simple and compound, are supposed to have been previously brought into an upright position.

§. 43. PARALLEL POSITION.

Two or several forms are in *Parallel Position,* if the axes of the one are parallel to the homologous axes of the other.

Two or several forms are in parallel position, if of the axes of the one, only two are parallel to two homologous axes of the other. For all the homologous axes intersect each other in the centre of the form, at equal angles (§. 39.)

The parallel position cannot in general be perfectly determined in forms which possess only one axis. Several forms, moreover, may be considered in different positions. It will be pointed out hereafter by what means, in these cases, the parallel position must be determined. The different positions of forms are of great importance in all crystallographic researches, if the object of these be more than the consideration of one form at a time.

Similar forms in parallel position, have their faces parallel.

§. 44. HORIZONTAL PROJECTION.

Place any given form in its upright position.

Draw from the angles of this form, perpendicular lines to a horizontal plane, and join all the points thus determined by straight lines. The greatest plane figure obtained by this proceeding is the *Horizontal Projection* of the form.

The horizontal projection belongs to the vertical axis, and is homologous with the sections to which the axis belongs (§. 40.), and to which the projection is parallel.

A form possesses as many different horizontal projections as it has kinds of principal axes.

The *Side* of the horizontal projection is the unity of most of the subsequent calculations referring to the dimensions of crystalline forms.

§. 45. REGULARITY.

The *Regularity* of simple forms is their greater or lesser agreement with the regular solids of geometry.

Regularity refers only to simple forms. The regularity of the simultaneous existence of these in the compound, is termed the *Symmetry* of combinations, which will be considered more at large in §. 141.

The irregularities so frequently occurring in crystals, must be abstracted, in our consideration of them, and the forms reduced to their peculiar regularity.

§. 46. DEGREES OF REGULARITY.

The regularity of simple forms allows of being arranged in *Several Degrees.*

Geometry considers solids whose angles are not altogether situated in the surface of one sphere, to be less regular than those whose angles are all touched by the surface of a single sphere; and thus, it likewise acknowledges different degrees of regularity. The degrees of regularity in Crystallography are not the same with these. By the peculiar method of treating its object, Crystallography is forced to ascribe the same degree of regularity to forms, the angles of one of which may lie in one, of another in two, of a third in three different spheres, as to the hexahedron, to the monogrammic Tetragonal-dodecahedron (§. 63.), and to the Tetraconta-octahedron (§. 77.); and it ascribes to others, as to the Tetrahedron, a lesser degree of regularity, although its angles altogether should be situated in the face of one and the same sphere.

§. 47. DETERMINATION OF THE DEGREES OF REGULARITY.

The degrees of regularity of simple forms, are determined according to *the Kind and the Number of their Axes.*

There are four degrees of regularity to be distinguished in simple forms.

Forms of the first degree of regularity contain four axes of the first kind, three of the second, and six of the third; of the second degree of regularity, four of the first kind, some of them at the same time three of the second, some three of the third, some none besides those of the first; of the third degree of regularity, only one axis of either the first or the second kind, and an undetermined number of axes of the third kind; of the fourth degree, three axes of the third kind.

Thus,	first	4	3	6
	second	4	3	0
		4	0	3
		4	0	0
	third	1	0	undetermined.
		0	1	
	fourth	0	0	3

With these degrees of regularity, the rest of the quality of simple forms is in the closest agreement, as will be seen hereafter. The first degree of regularity contains two of those forms, and the second one of those forms, which are geometrically regular.

§. 48. CLASSIFICATION OF SIMPLE FORMS.

Simple forms are divided, according to the number of their principal axes, into such as have only *One Principal Axis*, and such as have *Several.*

The forms with one axis are of the third and fourth, those with several axes are of the first and second degree of regularity.

§. 49. NOMENCLATURE OF SIMPLE FORMS.

The forms of one axis receive their names according to the figure of their faces, or according to some general property; those of several axes, according to the number of their faces; and when a more accurate determination is necessary, according to certain peculiarities of these forms themselves.

Systematic nomenclature remedies the want of conformity and precision, which has hitherto prevailed in the method of denominating crystalline forms; and at the same time produces a distinct idea of the forms themselves, since it is in fact their abridged description. This shews the usefulness of the systematic nomenclature, and justifies its introduction.

Forms of a single axis, whose faces are rhombs, are termed *Rhombohedrons;* others, whose faces are triangles, are called *Pyramids.*

A form of several axes, which is contained under *four* faces, is a *Tetrahedron*, or *the* Tetrahedron, because there exists only one, or because all tetrahedrons are similar; a form contained under *six* faces, is *the Hexahedron;* a form contained under *eight* faces, is *the Octahedron;* a form contained under *twelve* faces, is *a Dodecahedron*, because there are several varieties, or because not all dodecahedrons are similar; a form contained under *twenty-four* faces is an *Icositetrahedron;* and a form contained under *forty-eight* faces, a *Tetracontaoctahedron.*

The denominations which denote the different kinds and varieties of simple forms, according to their peculiar properties, are formed from these names by composition, or by the addition of adjectives.

II. OF SIMPLE FORMS IN PARTICULAR.

CONSIDERATION OF SIMPLE FORMS, AND SOME OF THEIR GEOMETRICAL RELATIONS.

§. 50. THE RHOMBOHEDRON.

The rhombohedron, Fig. 7., is a form contained under six equal and similar rhombic faces; or the rhombohedron is contained under six equal and similar *rhombs.*

1. Any six rhombs, which are equal and similar to each other, limit *one*, and if the obtuse angle of their figure is less than 120°, *two* rhombohedrons.

2. All rhombohedrons belong to the same kind of forms.

3. The solid angles A, X, produced by equal plane angles and equal edges (§. 33.) of the rhombohedron, are termed its *Apices.*

4. The straight line AX through the apices, is the *Axis* of the rhombohedron. The Rhombohedron has only *one* axis, and this is of the first kind (§. 40.). Since all forms connected with the rhombohedron possess axes of this kind, these in future will be designated by the denomination of *rhombohedral Axes.*

5. A solid angle, through which a rhombohedral axis passes, is termed a *rhombohedral* solid angle. This applies equally to forms which are not rhombohedrons themselves.

6. The edges CA, C′A, C″A, BX, B′X, B″X, contiguous to the terminal points of the axis, are *Terminal Edges;* while CB′, B′C″, C″B, &c. or those which do not intersect or meet with the axis, are *Lateral Edges.*

7. The diagonals of the faces of a rhombohedron, are commonly said to be the diagonals of the rhombohedron itself. Those which are horizontal, like CC″, C′C″, &c. when the rhombohedron is in its upright position (§. 42.), are termed the *Horizontal Diagonals;* those which, on the same supposition, assume a direction inclined to the axis, like AB, AB′, &c. are called the *Inclined Diagonals* of that form.

8. The rhombohedron has two principal sections. The first and most useful is a *rhomboid*, bounded by two parallel terminal edges, and the inclined diagonals contained between them, as ABXC; the second is a *rectangle*, as C′C″B′B″. The other sections are of the first kind (§. 38.), and termed *Rhombohedral Sections.* That through the centre of the form, or the transverse section, is a *regular Hexagon.*

9. The horizontal projection of the rhombohedron is a *Regular Hexagon*, equal to that circumscribed about the transverse section.

10. Of two rhombohedrons, that with a greater plane angle at the apex, C′AC″, is termed the *more obtuse;* that with a lesser, the *more acute* of these forms. The same distinction applies also to pyramids.

11. The sections CC′C″ and BB′B″, through contiguous horizontal diagonals, are perpendicular to the axis, and divide it in three equal parts, AP, PQ, and QX.

12. If, as it is supposed in all calculations concerning the rhombohedron, the side of the horizontal projection is $= 1$; the horizontal diagonal is $= \sqrt{3}$.

13. Let the axis AX be $= a$; the angle of inclination at the terminal edge $= x$; we obtain:

$$\cos. x = \frac{2a^2 - 9}{4a^2 + 9}.$$

14. Let α be the plane angle at the apex; we have

$$\cos. \alpha = \frac{2\,a^2 - 9}{2\,(a^2 + 9)}.$$

§. 51. PYRAMIDS IN GENERAL.

A pyramid is contained under equal and similar *triangles.*

1. The whole number of these triangles, as well as its half, is an even number.

2. The triangles are either isosceles or scalene. A pyramid contained under isosceles triangles, is termed an *isosceles pyramid;* one contained under scalene triangles, a *scalene* pyramid.

3. The angles at the vertex of these triangles, are the *Apices* of the pyramid.

4. The straight line through the apices is the *Principal Axis.*

5. The edges contiguous to the terminal points of the axis, are called *Terminal Edges;* they are equal in isosceles, and unequal in scalene pyramids. The remaining edges of the pyramid either lie in a plane perpendicular to the principal axis, or they are situated like the lateral edges of a rhombohedron (§. 50. 6.). They are called *Lateral Edges,* the first of them sometimes *Edges at the Base.*

6. The pyramids are divided according to the whole, and denominated according to half the number of their faces, as follows:

	Number of Faces.	*Denomination.*	*Figure of the Triangles.*
PYRAMIDS.	Eight,	Four-sided,	isosceles. scalene.
	Twelve,	Six-sided,	isosceles. scalene.
	Sixteen,	Eight-sided,	scalene.

7. The term *Pyramid* has not exactly the same signification in Crystallography, as in Geometry. In Geometry, it means a solid, bounded by any number of triangular planes meeting in one point, and terminating at one polygonal plane, as its base. In Crystallography, it must be restricted to simple forms; and, therefore, cannot be applied to any other but those, which have hitherto been called double pyramids. There are no simple pyramids, as simple forms, to be considered in Crystallography. The tetrahedron, which has been called a simple three-sided pyramid, is no pyramid at all, but is a form of several axes, and in the closest connexion with other forms of that kind, particularly with the octahedron. The epithet *double*, therefore, is superfluous, since the crystallographer has on no occasion to distinguish between simple and double pyramids, as two different classes of simple forms.

§. 52. ISOSCELES FOUR-SIDED PYRAMIDS.

The isosceles four-sided pyramids, Fig. 8., are contained under eight *isosceles* triangles.

1. The isosceles four-sided pyramids, have two principal sections, one of which, BCB′C′, is a square, the other, AC′XC, or AB′XB, a rhomb.

2. The remaining sections, belonging to the principal axis, are also squares; and the axis is therefore of the second kind (§. 40.). This axis itself, the solid angles through which it passes, and the sections belonging to it, are termed *Pyramidal*, because all forms in connexion with the isosceles four-sided pyramid, contain axes, solid angles, and sections of the same kind. The same denomination applies to every form possessing similar axes, solid angles, and sections, although this form be not an isosceles four-sided pyramid.

3. Besides these, the isosceles four-sided pyramids contain four axes of the third kind, two of which, BB′ and CC′, are the diagonals, the two others HH‴ and H′H″

parallel to the sides, of the base. These axes intersect each other at angles of 45°.

4. The horizontal projection is a square, equal to the base, or to the square principal section.

5. Let the side of the horizontal projection be = 1; the axis = a, the terminal edge = x; the lateral edge, or that at the base = z; we obtain:

$$\cos. x = -\frac{1}{1 + a^2}; \quad \cos. z = \frac{1 - a^2}{1 + a^2}.$$

§. 53. SCALENE FOUR-SIDED PYRAMIDS.

The scalene four-sided pyramids, Fig. 9., are contained under eight *scalene* triangles.

1. The scalene four-sided pyramids have three principal sections, ABXB′, AC′XC and BC′B′C, all of which are rhombs.

2. The remaining sections, all parallel to the principal ones, are likewise rhombs: the axes AX, BB′, CC′ therefore are of the third kind. These axes, the solid angles through which they pass, and the sections belonging to them, are called *Prismatic*, on account of the great number and variety of oblique angular four-sided prisms existing among the forms in connexion with the scalene four-sided pyramid, all of which possess axes, solid angles, and sections of this kind. Those denominations of axes, solid angles, and sections, are likewise made use of in such forms as are not connected with the scalene four-sided pyramid.

3. Any axis of the scalene four-sided pyramid can be assumed as the *principal* one, or any solid angle can be considered as the *apex*. After the principal axis has been obtained, the subordinate axes, apices, solid angles, the base and its diagonals, are thereby ascertained, and remain invariable in all considerations of a determined form of this kind.

4. Among the terminal edges, the greater is said to be the *obtuse* edge, the lesser the *acute* edge; which is likewise the case in the scalene six-sided pyramids (§. 55.), and in the eight-sided pyramids (§. 56.).

5. The horizontal projection is equal and similar to the base, or to that principal section, which is perpendicular to the principal axis.

5. Let the axis AX of a scalene four-sided pyramid be $= a$; BB′ one of its diagonals $= b$; CC′ the other diagonal $= c$; the terminal edge AB contiguous to $b = y$; the terminal edge AC contiguous to $c = x$; the edge BC at the base $= z$: then

$$\cos. y = \frac{a^2 b^2 - (a^2 + b^2) c^2}{a^2 b^2 + (a^2 + b^2) c^2};$$

$$\cos. x = \frac{a^2 c^2 - (a^2 + c^2) b^2}{a^2 c^2 + (a^2 + c^2) b^2};$$

$$\cos. z = \frac{b^2 c^2 - (b^2 + c^2) a^2}{b^2 c^2 + (b^2 + c^2) a^2};$$

and

$$\cos. y + \cos. x + \cos. z = -1.$$

Hence

$$\cos. y = -(1 + \cos. x + \cos. z);$$
$$\cos. x = -(1 + \cos. y + \cos. z);$$
$$\cos. z = -(1 + \cos. y + \cos. x).$$

7. Suppose $\cos. y = \alpha$; $\cos. x = \beta$; $\cos. z = \gamma$: the following ratio among the diagonals will be obtained:

$$a : b : c = \sqrt{[(1 + \alpha)(1 + \beta)]} : \sqrt{[(1 + \alpha)(1 + \gamma)]} : \sqrt{[(1 + \beta)(1 + \gamma)]}.$$

§. 54. ISOSCELES SIX-SIDED PYRAMIDS.

The isosceles six-sided pyramids, Fig. 10., are contained under twelve *isosceles* triangles.

1. The isosceles six-sided pyramids have two principal sections: one of them AHXZ, &c. is a rhomb, the other HORZNT a regular hexagon. The latter is at the same time the base of the pyramid.

2. The remaining sections are rhombohedral (§. 50.) and prismatic (§. 53.), as also the axes. Of the prismatic axes, three HZ, ON and RT pass through the lateral solid angles, and three IS, KU and LV through the centres of the lateral edges.

3. The horizontal projection is equal and similar to the base, or to the rhombohedral principal section.

4. The side of the horizontal projection being = 1 (§. 50. 12.), let the axis be = m.a (the axis of a rhombohedron being designated by a, and a certain *constant* co-efficient by m); the terminal edge = x; the lateral edge = z: we have

$$\cos. x = -\left(\frac{m^2.\, a^2 + 6}{2\, m^2.\, a^2 + 6}\right);$$

$$\cos. z = -\left(\frac{m^2.\, a^2 - 3}{m^2.\, a^2 + 3}\right).$$

§. 55. SCALENE SIX-SIDED PYRAMIDS.

The scalene six-sided pyramids, Fig. 11., are contained under twelve *scalene* triangles.

1. The principal section A′BX′C, &c. is a rhomboid.

2. The remaining sections are rhombohedral; those which pass only through terminal edges are equilateral hexagons of alternately equal angles; that through the centre, or the transverse section, is an equilateral dodecagon, likewise of alternately equal angles.

3. The lateral edges of this form are disposed like the lateral edges of a rhombohedron.

4. The horizontal projection is a regular hexagon.

5. The side of the horizontal projection being = 1; let the axis A′X′ be = m.a (where a signifies the axis AX of a rhombohedron, whose lateral edges coincide with the lateral edges CB, BC, &c. of the pyramid, and m a *variable* co-efficient): the obtuse terminal edge = y; the acute terminal edge = x; the lateral edge = z: the following formulæ will be derived.

$$\cos. y = -\left(\frac{(3\,m^2 + 6\,m - 1)\,a^2 + 18}{2\,[(3\,m^2 + 1)\,a^2 + 9]}\right);$$

$$\cos. x = -\left(\frac{(3\,m^2 - 6\,m - 1)\,a^2 + 18}{2\,[(3\,m^2 + 1)\,a^2 + 9]}\right);$$

$$\cos. z = -\left(\frac{(3\,m^2 - 1)\cdot a^2 - 9}{(3\,m^2 + 1)\,a^2 + 9}\right).$$

$$\cos. y = 1 + \cos. x + \cos. z - 2.\sqrt{[(1 + \cos\ x)(1 - \cos. z)]};$$

$$\cos. x = 1 + \cos. y + \cos. z - 2.\sqrt{[(1 + \cos. y)(1 - \cos. z)]};$$

$$\cos. z = -(1 + \cos. y + \cos. x + 2.\sqrt{[(1 + \cos. y)(1 + \cos. x)]}).$$

§. 56. SCALENE EIGHT-SIDED PYRAMIDS.

The scalene eight-sided pyramids, Fig. 12., are contained under sixteen *scalene* triangles.

1. The scalene eight-sided pyramids have three different principal sections; the first of these B′SC′S′BS‴CS″ is an equilateral octagon, of alternately equal angles; the other two A′C′X′C and A′B′X′B on one side, and A′SX″S‴ and A′S′X′S″ on the other are rhombs.

2. The remaining sections are pyramidal and prismatic; so are likewise the axes. Of the prismatic axes, every two B′B, C′C, and SS‴, S′S″, pass through equal solid angles.

3. Those edges which are not terminal, are edges at the base.

4. The horizontal projection is equal and similar to the base or the pyramidal principal section.

5. Let the axis A′X′ of the scalene eight-sided pyramid be = m. a (where a is = AX, the axis of an isosceles four-sided pyramid, the side of the horizontal projection of which, SS′ is = 1, and m a variable co-efficient, greater than $1 + \sqrt{2}$ (§ 103.)); the acute terminal edge = y; the obtuse = x; the edge at the base = z: we obtain

$$\cos. y = -\left(\frac{2\,(m.\,a^2 + 1)}{(m^2 + 1)\,a^2 + 2}\right);$$

$$\cos. x = -\left(\frac{(m^2 - 1)\,a^2 + 2}{(m^2 + 1)\,a^2 + 2}\right);$$

$$\cos. z = -\left(\frac{(m^2 + 1)\,a^2 - 2}{(m^2 + 1)\,a^2 + 2}\right).$$

And

$$\cos. y = -\tfrac{1}{2}(1 + \cos. z + 2. \surd [-(\cos. x + \cos. z)(1 + \cos. x)]);$$

$$\cos. x = -\tfrac{1}{2}(1 + \cos. z + 2. \surd [-(\cos. y + \cos. z)(1 + \cos. y)]);$$

$$\cos. z = -(3 + 2. \cos. y + 2. \cos. x + 2. \surd [2(1 + \cos. y)(1 + \cos. x)]).$$

§. 57. THE TETRAHEDRON.

The Tetrahedron, Figs. 13. 14., is contained under four equilateral triangles.

1. The plane angles a,a,a of the tetrahedron are = 60°; the angles of incidence, at the edges A,A, &c. (their magnitude) = 70° 31′ 44″.

2. The sections of the tetrahedron are rhombohedral and prismatic; one of the latter, through the centre, is a square.

3. The principal axes are rhombohedral; they join the solid angles with the centres of the opposite faces; their number is four, and they intersect each other at angles of 109° 28′ 16″, and 70° 31′ 44″. *These angles of intersection are general for the rhombohedral axes, whenever more than one occur in the same form.* The subordinate axes are prismatic; they join the centres of opposite edges; their number is three, and they are perpendicular to each other.

4. The tetrahedron is a regular solid of geometry. *

5. This form occurs, either by itself or in combinations, in tetrahedral Copper-glance, in dodecahedral Garnet-blende, &c.

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* The principal sections and horizontal projections of the forms of several axes being of comparatively little use, and besides very easily ascertained, I have thought it superfluous to enter here into a greater detail.

§. 58. THE HEXAHEDRON.

The Hexahedron, Fig. 1., is contained under six squares.

1. All the angles of the hexahedron, those of the faces as well as those of the edges, are = 90°.

2. The sections are rhombohedral, pyramidal, and prismatic: so are the axes.

3. The rhombohedral axes pass through the solid angles; the pyramidal axes, whose number is three, through the centres of parallel faces, and these are perpendicular to each other; *and this again is general to the pyramidal axes, whenever more than one occur in the same form.* The prismatic axes, whose number is six, pass through the centres of parallel edges; those which belong to parallel edges, intersect each other at right angles; those which belong to edges that are not parallel, at angles of 60° and 120°; *and those are again general angles for the prismatic axes.*

4. The hexahedron or cube is a regular solid of geometry.

5. This form is frequently met with in nature, as in octahedral Fluor-haloide, hexahedral Iron-pyrites, &c.

§. 59. THE OCTAHEDRON.

The Octahedron, Fig. 2., is contained under eight equilateral triangles.

1. The plane angles of the octahedron are = 60°; the edges or angles of incidence = 109° 28′ 16″. The angles of incidence of the octahedron and of the tetrahedron are supplemental to each other (to 180°). These angles are the same as those at which the rhombohedral axes intersect each other (§. 57. 3.).

2. The sections and axes are the same as in the hexahedron; only the rhombohedral axes pass through the centres of parallel faces, and the pyramidal axes through the solid angles.

3. The octahedron is a regular solid of geometry.

4. This form occurs very frequently in different species, as in octahedral Corundum, octahedral Iron-ore, &c.

§. 60. DODECAHEDRONS IN GENERAL.

The Dodecahedrons are contained under twelve equal and similar faces, the figure of which determines the kind of the dodecahedrons. A dodecahedron whose faces are triangles, is termed a *Trigonal*-dodecahedron; one whose faces are tetragons, a *Tetragonal*-dodecahedron; and one whose faces are pentagons, a *Pentagonal*-dodecahedron.

1. None of these dodecahedrons are regular in the geometrical sense of the word; for their faces are not regular polygons; besides, they have at least two different kinds of angles, and, one of the dodecahedrons only excepted, they have also at least two kinds of edges.

§. 61. TRIGONAL-DODECAHEDRONS.

The Trigonal-dodecahedrons, Figs. 15. 16., are contained under equal and similar isosceles triangles.

1. The trigonal-dodecahedrons possess the general aspect of the tetrahedron, and their sections and axes are of the same kind, and in the same situation.

2. They contain four solid angles of three, and four of six faces; both of them being equiangular. The first are monogrammic, and correspond to the centres of the faces; the others are digrammic, and correspond to the solid angles of the tetrahedron.

3. Of the two kinds of edges of the trigonal-dodecahedrons, the first, or those joining the angles of six faces, have the situation of the edges of the tetrahedron; the others meet in the solid angles of three faces, upon the centre of the faces of that form.

4. There are two known varieties of these dodecahedrons, whose dimensions are as follows:

	a.*	b.	A.	B.
1.	117° 2′ 8″.	31° 28′ 56″.	109° 28′ 16″.	146° 26′ 33″.
2.	112° 53′ 7″.	33° 33′ 26½″.	129° 31′ 16″.	129° 31′ 16″.

5. Of the first variety of this form we have examples in tetrahedral Copper-glance; of the second variety, in dodecahedral Garnet-blende.

§. 62. TETRAGONAL-DODECAHEDRONS.

The Tetragonal-dodecahedrons are contained under equal and similar tetragons.

1. There are two kinds of these forms.

2. Of the faces of the one, two are always parallel to each other, and they contain two pairs of equal angles. Of the faces of the other, no two faces are parallel, and they contain only one pair of equal angles, the remaining two being also different betwixt themselves. All the edges of the former are equal, while the latter possess two kinds of different edges.

3. From this last mentioned difference, the denominations of the two kinds are derived; the first containing the *monogrammic*, the second the *digrammic* Tetragonal-dodecahedrons.

§. 63. THE MONOGRAMMIC TETRAGONAL-DODECAHEDRON.

The faces of the monogrammic Tetragonal-dodecahedron, or of *the Dodecahedron*, Fig. 31., are rhombs.

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* The small letters a, b, signify the plane angles of the faces, and the large ones A, B, the angles of the incidence at the edges, as referring to the figures.

1. The plane angles of these rhombs are = 109° 28′ 16″ and 70° 31′ 44″, equal to the edges of the octahedron (§. 59. 1.) and of the tetrahedron. They are equal also to the angles of intersection of the rhombohedral axes (§. 57. 1. 3.). The edges are all = 120°.

2. The monogrammic Tetragonal-dodecahedron has eight solid angles formed by three, and six formed by four faces; both of them are equiangular. The first are situated like the solid angles of the hexahedron, the second like those of the octahedron.

3. The sections and axes are as in these. The rhombohedral axes pass through the solid angles of three, the pyramidal axes through the solid angles of four faces, and the prismatic axes through the centres of parallel faces of the solid.

4. There is only one variety of this form, which is commonly expressed by the name of *the Dodecahedron.*

5. The dodecahedron is not a rare form; it is found in dodecahedral Garnet, hexahedral Gold, &c.

§. 64. DIGRAMMIC TETRAGONAL-DODECAHEDRONS.

The faces of the digrammic Tetragonal-dodecahedrons, Figs. 17. 18., possess the outlines of those inscribed in a Trapezium.

1. The digrammic Tetragonal-dodecahedrons have the general aspect of the tetrahedron.

2. They contain two kinds of solid angles formed by three faces, four of each. Both kinds are equiangular. The more acute correspond to the solid angles, the more obtuse to the centres of the faces of the tetrahedron. They possess moreover six solid angles of four faces, which are equiangular, but digrammic, and situated above the centres of the edges of the tetrahedron.

3. Of the two kinds of edges of these forms, the more obtuse join in the obtuse, the more acute in the acute solid angles formed by three faces, and both in the solid angles consisting of four faces.

4. The sections and axes are as in the tetrahedron. The prismatic axes pass through the solid angles of four faces.

5. There is only one variety known of this form, whose dimensions are the following:

a.	b.	c.	A.	B.
90°.	118° 4′ 10″.	75° 57′ 55″.	90°.	152° 44′ 2″.

6. It has been observed in dodecahedral Garnet-blende.

§. 65. PENTAGONAL-DODECAHEDRONS.

The Pentagonal-dodecahedrons are contained under equal and similar pentagons.

1. There are two kinds of Pentagonal-dodecahedrons.

2. In the one, every face has the opposite one parallel to it, a property which is not to be met with in the other.

3. The first have the general aspect of the hexahedron, and are therefore termed *hexahedral;* the other that of the tetrahedron; and accordingly they bear the denomination of *tetrahedral* Pentagonal-dodecahedrons.

§. 66. HEXAHEDRAL PENTAGONAL-DODECAHEDRONS.

The faces of the hexahedral Pentagonal-dodecahedrons, Figs. 19. 20., have two pairs of equal angles, and four equal sides. The single angle is opposite to the single side.

1. All the solid angles of these Pentagonal-dodecahedrons, are bounded by three faces; eight of them are equiangular and monogrammic, and correspond to the solid angles of the hexahedron. The other twelve are formed by two equal angles, and the single angle; they are digrammic, and pairs of them may be conceived to be situated upon the faces of the hexahedron, in the direction of planes, which pass through two pyramidal axes of that form.

2. Of the two kinds of edges, those opposite to the single angle are the *Characteristic Edges* of this form; because

the examination of these yields the best means to distinguish the different varieties of the hexahedral Pentagonal-dodecahedrons. The other edges meet in the monogrammic solid angles.

3. The sections and axes are as in the tetrahedron. The prismatic axes pass through the centres of the characteristic edges.

4. There are three varieties of this form, whose dimensions are the following:

	a.	b.	c.
1.	102° 35′ 40″.	108° 24′ 30″.	110° 17′ 40″.
2.	121° 35′ 18″.	106° 36′ 2″.	102° 36′ 19″.
3.	141° 16′ 50″.	103° 20′ 33″.	96° 1′ 2″.

	A.	B.
1.	112° 37′ 12″.	117° 29′ 11″.
2.	126° 52′ 12″.	113° 34′ 41″.
3.	143° 7′ 48″.	107° 27′ 27″.

5. The first and second variety are found in hexahedral Iron-pyrites, the angles of the third depend upon the third variety of the icositetrahedrons, §. 71. 130.

§. 67. TETRAHEDRAL PENTAGONAL-DODECAHEDRONS.

The faces of the tetrahedral Pentagonal-dodecahedrons, Figs. 21. 22. 23. 24., have no equal angles; but they possess two pairs of equal sides.

1. The tetrahedral Pentagonal-dodecahedrons have three kinds of solid angles, all of which are formed by three faces. The first are equiangular, monogrammic, four in number, and they correspond to the solid angles of the tetrahedron; the second, of the same description, but more obtuse, correspond to the centres of the faces of the same form. The third are not equiangular; they are trigrammic, twelve in number, and pairs of them are situated between the more acute equiangular solid angles.

2. This form contains three kinds of edges; the first meet in the more acute, the second in the more obtuse equiangular solid angles, and the third join those which are formed by three different plane angles.

3. Its sections are rhombohedral; and it does not possess any other but rhombohedral axes, in conformity with the sections.

4. These solids are remarkable, on account of their being as it were twisted, some to the Right, others to the Left. They are equal and similar to each other; but every part of the one, has exactly the reverse situation of the other.

5. The dimensions of the three varieties of this form, are as follows:

	a.	b.	c.	d.
1.	116° 6′ 13″.	111° 50′ 44″.	93° 49′ 21″.	143° 11′ 29″.
2.	113° 21′ 46″.	113° 43′ 28″.	99° 35′ 38″.	130° 12′ 11″.
3.	113° 34′ 41″.	128° 20′ 44″.	97° 59′ 19″.	136° 39′ 57″.
	e.	**A.**	**B.**	**C.**
1.	75° 2′ 13″.	141° 47′ 12″.	94° 5′ 45″.	106° 36′ 2″.
2.	83° 6′ 57″.	131° 4′ 57″.	78° 27′ 46″.	115° 22′ 37″.
3.	66° 25′ 19″.	131° 48′ 37″.	95° 27′ 54″.	121° 35′ 18″.

6. This form has not yet been found in nature; and the angles of the mentioned varieties depend upon those of the tetraconta-octahedrons, §. 77. 134.

§. 68. ICOSITETRAHEDRONS IN GENERAL.

The Icositetrahedrons are contained under twenty-four equal and similar faces, the figure of which determines the kinds of icositetrahedrons. An icositetrahedron, whose faces are triangles, is termed a *Trigonal*-icositetrahedron; one whose faces are tetragons, a *Tetragonal*-icositetrahedron; and one whose faces are pentagons, a *Pentagonal*-icositetrahedron.

1. None of these icositetrahedrons are geometrically regular.

§. 69. TRIGONAL-ICOSITETRAHEDRONS.

The Trigonal-icositetrahedrons are contained under equal and similar, isosceles or scalene triangles.

1. This species of icositetrahedrons comprises three kinds, different from each other by their general aspect, and the situation of their faces.

2. The varieties of the first kind have no parallel faces; they exhibit the general aspect of the tetrahedron, and are therefore said to be *tetrahedral;* the varieties of the second have parallel faces, and the general aspect of the hexahedron; these are termed *hexahedral;* the varieties of the third possess also parallel faces, but the general aspect of the octahedron, and these are termed *octahedral* Trigonal-icositetrahedrons.

§. 70. TETRAHEDRAL TRIGONAL-ICOSITETRAHEDRONS.

The faces of the tetrahedral Trigonal-icositetrahedrons, Figs. 25. 26., are scalene triangles.

1. These forms have four solid angles, and six faces, all of which are equiangular and digrammic. Those contained by four faces, six in number, are situated above the centres of the edges; the more obtuse solid angles of six faces, four in number, above the centres of the faces of the tetrahedron, and the more acute solid angles formed by the same number of faces, also four in number, correspond to the solid angles of this form.

2. There are three different kinds of edges in this form. The longest join those solid angles of six faces which are not similar to each other, the intermediate ones the more acute, and the shortest the more obtuse of these solid angles with those which are bounded by four faces.

3. The sections and axes are as in the tetrahedron. The prismatic axes pass through the solid angles of four faces.

4. There are three varieties, of the following dimensions:

	a.	b.	c.
1.	56° 15′ 4″.	82° 23′ 19″.	41° 21′ 37″.
2.	53° 46′ 42″.	82° 17′ 58″.	43° 55′ 20″.
3.	54° 21′ 34″.	85° 19′ 19″.	40° 19′ 7″.

	A.	B.	C.
1.	110° 55′ 29″.	158° 12′ 48″.	158° 12′ 48″.
2.	122° 52′ 42″.	152° 20′ 22″.	152° 20′ 22″.
3.	124° 51′ 0″.	144° 2′ 58″.	162° 14′ 50″.

5. The third variety of this form has been observed in hexahedral Boracite; the other two depend upon the first and second variety of the tetraconta-octahedron, §. 77. 133.

§. 71. HEXAHEDRAL TRIGONAL-ICOSITETRAHEDRONS.

The faces of the hexahedral Trigonal-icositetrahedrons, Fig. 32., are isosceles triangles.

1. The solid angles consist either of four or of six faces, and are all equiangular. The first, six in number, are monogrammic, and situated above the centres of the faces; the second, eight in number, and digrammic, are situated like the solid angles of the hexahedron.

2. Those edges of the form which correspond to the edges of the hexahedron, join the angles of six faces with each other; the others join the solid angles of six faces with those of four faces.

3. The sections and axes are as in the hexahedron.

4. There are three varieties of these forms, whose dimensions are the following:

	a.	b.	A.	B.
1.	79° 31′ 28″.	50° 14′ 16″.	157° 22′ 48″	133° 48′ 47″.
2.	83° 37′ 14″.	48° 11′ 23″.	143° 7′ 48″	143° 7′ 48″.
3.	86° 58′ 59″.	46° 30′ 30½″.	126° 52′ 12″.	154° 9′ 29″.

5. The second variety occurs in dodecahedral Garnet, the third in octahedral Fluor-haloide; the angles of the first depend upon those of the first variety of the dodecahedrons, §. 66. 130.

§. 72. OCTAHEDRAL TRIGONAL-ICOSITETRAHEDRONS.

The faces of the octahedral Trigonal-icositetrahedrons, Fig. 33., are isosceles triangles.

1. Their solid angles consist of either three or eight faces, and are equiangular. The first, eight in number, are monogrammic, and correspond to the centres of the faces; the second, six in number, are digrammic, and correspond to the angles of the octahedron.

2. Those edges, which have the situation of those of the octahedron, join the solid angles of eight faces with each other; the other edges unite two dissimilar solid angles.

3. The sections and axes are as in the octahedron.

4. There is only one variety known, of the following dimensions:

a.	b.	A.	B.
118° 4′ 10″.	30° 57′ 55″.	141° 3′ 28″.	152° 44′ 2″.

4. Examples of this form are found in octahedral Fluor-haloide, hexahedral Lead-glance, &c.

§. 73. TETRAGONAL-ICOSITETRAHEDRONS.

The Tetragonal-icositetrahedrons, are contained under equal and similar tetragonal faces.

1. This species of icositetrahedrons comprises two kinds, the varieties of which are distinguished from each other by the figures of their faces, and by several properties depending upon them, chiefly by the diversity of their edges, according to which, they also receive their denominations.

2. The varieties of the first kind, contain only two different edges, and are termed *digrammic;* whilst the second,

or *trigrammic* Tetragonal-icositetrahedrons, possess three different kinds of edges.

§. 74. DIGRAMMIC TETRAGONAL-ICOSITETRAHEDRONS.

The digrammic Tetragonal-icositetrahedrons, Fig. 34., are contained under tetragonal faces, which can be divided by one of their diagonals, in two isosceles triangles.

1. These icositetrahedrons possess three different kinds of solid angles, one of which is formed by three, the others by four faces: all of them are equiangular. The first are monogrammic, eight in number, and correspond to the solid angles of the hexahedron. Of the second, six are monogrammic, and correspond to the solid angles of the octahedron; the other twelve are digrammic, and correspond to the centres of the faces of the dodecahedron, (§. 63.).

2. These forms possess two kinds of edges, the one terminating in the solid angles of three faces, the other in those which are produced by four equal edges.

3. The sections and axes are the same as in the hexahedron, the octahedron, &c. The rhombohedral axes pass through the solid angles of three faces, the pyramidal axes through the monogrammic, and the prismatic axes through the digrammic solid angles consisting of four faces.

4. There are two varieties known in nature, of the following dimensions:

	a.	b.	c.	A.	B.
1.	78°27′46″.	82°15′ 3″.	117° 2′8″.	131°48′36″.	146°26′33″.
2.	84°15′39″.	81°25′37″.	112°53′7″.	144°54′11″.	129°31′16″.

5. Examples of the first variety of these forms we have in hexahedral Kouphone-spar and dodecahedral Garnet; of the second, in octahedral Fluor-haloide and dodecahedral Corundum; of both, in hexahedral Iron-pyrites.

§. 75. TRIGRAMMIC TETRAGONAL-ICOSITETRAHEDRONS.

The trigrammic Tetragonal-icositetrahedrons, Figs. 27. 28., are contained under tetragonal faces, which cannot be divided in two isosceles triangles by any of their diagonals.

1. The angles of these forms consist of either three or four faces. The first are monogrammic, equiangular, eight in number, and they are situated like the solid angles of the hexahedron. Of the solid angles of four faces, six are equiangular and digrammic, and they are distributed like the solid angles of the octahedron; the other twelve are unequiangular and trigrammic, and they have the situation of the digrammic solid angles in the hexahedral pentagonal-dodecahedron (§. 66. 1.).

2. Of the three different kinds of edges, the first terminate in the solid angles consisting of three faces; the first and second in the digrammic, and the first, second, and third, in the trigrammic solid angles, bounded by four faces.

3. The mutual inclination NOP of the longest or greatest edges, in the digrammic solid angle, is the *Characteristic Angle* D of the trigrammic tetragonal-icositetrahedron.

4. The sections and axes are the same as those of the hexahedral pentagonal-dodecahedrons; the rhombohedral axes pass through the solid angles of three faces, the prismatic axes through the digrammic solid angles of four faces.

5. There are three varieties of these forms, whose dimensions are as follows:

	a.	b.	c.	d.
1.	106° 59′ 7″.	79° 53′ 50″.	116° 6′ 13″.	57° 0′ 50″.
2.	104° 38′ 25″.	84° 12′ 32″.	113° 21′ 46″.	57° 47′ 17″.
3.	96° 13′ 37″.	83° 46′ 23″.	113° 34′ 41″.	66° 25′ 19″.
	A.	**B.**	**C.**	**D.**
1.	148° 59′ 50″.	115° 22′ 37″.	141° 47′ 12″.	112° 37′ 12″.
2.	160° 32′ 13″.	118° 59′ 9″.	131° 4′ 57″.	118° 4′ 10″.
3.	154° 47′ 28″.	128° 14′ 48″.	131° 48′ 37″.	126° 52′ 12″.

6. All these varieties are met with in hexahedral Iron-pyrites; the first also in hexahedral Cobalt-pyrites.

§. 76. PENTAGONAL-ICOSITETRAHEDRONS.

The Pentagonal-icositetrahedrons, Figs. 29. 30., are contained under irregular pentagonal faces, all the angles of which are different, but which possess two pairs of equal sides.

1. These forms contain three kinds of solid angles, two of which consist of three, and one of four faces. Eight of those formed by three faces are equiangular and monogrammic; these are situated like the solid angles of the hexahedron: the other twenty-four are unequiangular and trigrammic; the situation of these is similar to that of the trigrammic solid angles in the tetrahedral pentagonal-dodecahedron (§. 67. 1.). The six solid angles of four faces are equiangular, monogrammic, and correspond to the solid angles of the octahedron.

2. There are three different kinds of edges; the first terminate in those solid angles which are produced by the concurrence of three equal edges; the second terminate in the solid angles of four faces; and the third join those solid angles with each other, which do not consist of equal plane angles.

3. The sections are rhombohedral and pyramidal, as also their corresponding axes. The pyramidal axes pass through those solid angles which contain four, the rhombohedral axes through those which contain three equal plane angles. These forms possess no prismatic axes at all, and agree with the tetrahedral pentagonal-dodecahedrons, in this particular as well as in the occurring difference between right and left.

4. The dimensions of the three varieties of these forms are as follows:

	a.	b.	c.	d.
1.	77° 26′ 40″.	126° 18′ 53″.	116° 6′ 13″.	93° 49′ 21″.
2.	80° 24′ 22″.	132° 54′ 46″.	113° 21′ 46″.	113° 43′ 28″.
3.	82° 14′ 1″.	115° 18′ 18″.	113° 34′ 41″.	123° 12′ 38″.

	e.	A.	B.	C.
1.	126° 18′ 53″.	130° 0′ 19″.	141° 47′ 12″.	141° 47′ 12″.
2.	99° 35′ 38″.	135° 35′ 43″.	131° 4′ 57″.	145° 57′ 8″.
3.	105° 40′ 22″.	149° 37′ 57″.	131° 48′ 37″.	135° 35′ 43″.

5. This form has not yet been found in nature; the angles of the three varieties depend upon those of the tetraconta-octahedrons, §. 77. 130.

§. 77. TETRACONTA-OCTAHEDRONS.

The Tetraconta-octahedrons, Fig. 35., are contained under forty-eight scalene triangles.

1. The solid angles of these forms are bounded by four, six, or eight faces; they are equiangular and digrammic. Twelve consist of four faces, and are situated above the centres of the faces of the dodecahedron; eight consist of six faces, and correspond to the solid angles of the hexahedron; and the remaining six, which consist of eight faces, are distributed like the solid angles of the octahedron.

2. Of the three different kinds of edges of these forms, the first, being the longest, join the solid angles of six faces with those of eight faces; the second or intermediate join the solid angles of eight faces with those of four faces; and the third, which are the shortest, unite the solid angles of six faces with those of four faces.

3. The sections and axes are as in the hexahedron, in the octahedron, &c. The rhombohedral axes pass through the solid angles of six faces, the pyramidal axes through the solid angles of eight faces, and the prismatic axes through the solid angles of four faces.

4. There are three varieties of these forms, of the following dimensions:

	a.	b.	c.
1.	86° 56′ 25″.	56° 15′ 4″.	36° 48′ 31″.
2.	87° 34′ 49″.	53° 46′ 42″.	38° 38′ 29″.
3.	85° 50′ 23″.	54° 21′ 34″.	39° 48′ 3″.
	A.	B.	C.
1.	158° 12′ 48″.	148° 59′ 50″.	158° 12′ 48″.
2.	152° 20′ 22″.	160° 32′ 13″.	152° 20′ 22″.
3.	162° 14′ 50″.	154° 47′ 28″.	144° 2′ 58″.

5. The first of these varieties occurs in dodecahedral Garnet; the third in octahedral Fluor-haloide; the second depends upon the second variety of the icositetrahedrons, §. 75. 133.*

CONSIDERATION OF THE CONNEXION AMONG SIMPLE FORMS, AND OF THE RELATIONS, UPON WHICH IT DEPENDS.

§. 78. OBSERVATIONS.

There exists a very remarkable connexion among several simple forms, which depends not only upon the kind, but also upon the relative dimensions of these simple forms.

It is a matter of fact, sufficiently demonstrated by numerous observations, that certain crystalline forms are pe-

* The preceding enumeration of the varieties of tessular forms, as occurring in nature, is by no means complete. Several varieties of the digrammic tetragonal-icositetrahedron (§. 74.), of the tetraconta-octahedron (§. 77.), and of other forms, have already been observed; for instance, in octahedral Fluor-haloide, in dodecahedral Garnet, in hexahedral Iron-pyrites, &c. but not with a sufficient degree of accuracy. It is to be expected, that our knowledge of these forms will be considerably enlarged by a more accurate examination of nature.

culiar to certain mineral species, whilst others are never found in the same substances. Thus hexahedral Gold is found in hexahedrons, but never in rhombohedrons; rhombohedral Lime-haloide in rhombohedrons, never in hexahedrons.

Experience proves quite as generally that varieties of one and the same accurately determined mineral species, may assume several different forms of crystallization; hexahedral Gold, beside the form of the hexahedron, assumes also that of the octahedron, of the dodecahedron, of the digrammic tetragonal-icositetrahedron, &c.; rhombohedral Lime-haloide, besides rhombohedrons, exhibits also several isosceles and scalene six-sided pyramids, and regular six-sided prisms; and we may frequently observe, that even in one and the same individual of such species, several of those simple forms appear at the same time, or in connexion with each other: thus, in hexahedral Gold, the hexahedron occurs in one individual with the octahedron; in rhombohedral Lime-haloide, rhombohedrons are found with pyramids, with prisms, &c.

It is likewise demonstrated by experience, that two or more simple forms, if they appear at the same time, in a species or an individual, do really possess certain dimensions or relations towards each other, and that other forms, though of the same kind with the preceding, are excluded from such species, merely on account of their dimensions. Thus the species of rhombohedral Lime-haloide does not present indiscriminately the forms of any rhombohedron, or of any six-sided pyramid whatever; but we find only such as possess certain dimensions, upon which the symmetry of their combinations depends.

Natural History does not lead us to inquire into the final cause of that remarkable fact, why the crystals of hexahedral Gold should be hexahedrons, octahedrons, &c.; and why those of rhombohedral Lime-haloide should be rhombohedrons and six-sided pyramids of certain dimensions. Such questions, supposing even that they were capable of being answered, are not within the province of Natural

History (§. 5.). But Natural History endeavours to determine the relations under which crystalline forms of certain dimensions appear in the individuals of the same species, or come into connexion with each other. These researches not only form part of the peculiar object of Natural History; but this science derives the greatest advantage from them in its farther developement.

§. 79. DERIVATION.

The method employed in Natural History for determining the kind and the relations of crystalline forms, which occur in the individuals of the same species, or come into connexion with each other, is called the *Derivation.*

To derive one simple form from another, is to shew how, according to a certain general rule, it arises, or is produced from it. The processes of derivation consist in geometrical constructions, which are not gratuitously imagined, but deduced from observation; and their correctness and applicability, though evident from their very origin, is thoroughly confirmed by the exactness with which the phenomena in nature can be explained.

There are several of these rules or methods of proceeding by which the derivation can be effected. Of these different methods, those must be selected which will apply to the quality of the form from which the derivation is to start, and which is termed the *given* form. The product of derivation is called the *derived* form. The derived and given forms are either of the same, or of different kinds. The derived form is a simple form, like the given one; or, should this not be the case, it must be resolved into two or more simple forms. The derived form having thus been developed, the relations existing between this and the given one are to be determined.

§. 80. FIRST PROCESS OF DERIVATION.

The first process of derivation requires tangent planes (§. 36.) to be placed on certain edges of the given form, and enlarged till they limit the space either entirely, or at least as far as the number and situation of the faces will allow.

If the edges to which the tangent planes are applied, be equal or homologous, as, for instance, the terminal edges of the rhombohedrons, and of the isosceles four-sided pyramids, or the acute and obtuse terminal edges of the scalene six or eight-sided pyramids; this process will yield a simple form at once, which is the derived form itself.

If, on the contrary, the edges in which the tangent planes are to be laid, be not homologous, as is the case in the acute and the obtuse terminal edges of the scalene four-sided pyramids; this process will not give a simple form, but a compound one, which is contained under faces not homologous with each other. Compound forms of this kind are not the derived forms themselves (§. 79.), though they either contain them, or at least may be employed for their ulterior derivation. They are considered as *Auxiliary* or *Intermediate Forms.*

All intermediate forms belong to that given form, from which they result by the above mentioned process.

§. 81. SECOND PROCESS.

The second process requires the axis of a form contained under tetragonal faces, to be produced on both sides, to an undetermined but equal length; straight lines to be drawn from the lateral angles of the tetragonal faces towards the terminal points of the lengthened axis, and planes to be laid on every contiguous pair of them. The derived form is contained under these planes.

This process is not limited to forms which, like the rhombohedron, are originally contained under tetragonal faces; but it can be extended to such as are originally contained under triangles, and is therefore applicable to pyramids of every description. In this case, however, the given form requires a certain preparation, the nature of which will be explained in its proper place.

Forms produced in this way, if simple, are the derived forms themselves; if compound, they are, like those in §. 80., considered as intermediate or auxiliary forms, and made use of accordingly.

Of intermediate forms in general, it may here be remarked, that, by enlarging their homologous faces, till the rest disappear, they may be resolved, and by that means the simple forms which they contain, may be extracted.

§. 82. THIRD PROCESS.

The third process requires planes to be laid on the terminal edges of the given form, which may likewise be an intermediate one (§. 80. 81.); their number and inclination being such, that the intersections of the faces from both apices produce a plane figure, similar and parallel to the horizontal projection of the given form. The derived form is contained under these planes.

The number of faces contiguous to every terminal edge, as employed in this process, is either one or two; more than two faces can never be applied to one terminal edge. This process in some cases affords a determined solution of a problem, which it would be impossible to obtain from a process analogous to that of §. 80.

§. 83. FOURTH PROCESS.

The fourth process requires the consideration of those differences which take place in the situation of a *Moveable Plane*, tangent to the uppermost point of a vertical rhombohedral axis in a form of several axes.

The last process refers only to those forms which possess several axes; while the three methods of derivation described above, are more particularly intended for such as have only one axis. The fourth process produces only simple forms.

§. 84. POSITION OF THE DERIVED FORMS.

By the application of these processes of derivation, the derived forms are obtained in such positions in respect to the given one, as will enable them to produce symmetrical combinations, both with the given form, and among each other.

In forms of several axes, this is the parallel position (§. 43.). In those of only one axis, it must be determined in particular, according to the quality of the forms concerned. It is sufficiently demonstrated by all compound forms occurring in the individuals of the mineral kingdom, that the simple forms of which they consist, in every instance are found in such positions as are assigned to them by the derivation and by the connexion which it produces between forms of a certain quality.

§. 85. SERIES.

If one of the processes described above yields a derived form of the same kind as the given form, the same process may be applied also to this derived form; and not only to this, but also to the new product of the derivation, and so on. The assemblage of forms thus produced, and following each other, is termed a *Series*.

Series may also be produced, although the derived form be not of the same kind as the given form; yet this does not take place so immediately as under the circumstances noticed.

These series form a peculiar feature, and are of the greatest importance in the Method of Crystallography developed in this work.

A constant ratio exists between every two subsequent members of those series. The general expression of this ratio is the *Law of the Series.*

Upon the series themselves is founded the method of *Crystallographic Designation* (§. 90.).

§. 86. LIMITS.

The limits of the series of those forms which possess one axis, are *Prisms* of infinite axes.

There is no reason why a series produced by derivation (§. 85.), should stop at a member, as long as another ulterior one can still be derived from it. This is always possible, as long as those dimensions, which are altered by the derivation, remain finite. All members in which this is the case, are termed *finite* members. If a member receives infinite dimensions, the derivation can no longer be continued. The limits of derivation, and consequently the limits of the series arising from it, are therefore attained, if the dimensions of these forms become infinite.

The dimensions of forms which most conveniently may be supposed to grow infinite, or infinitely small, are the axes; if these be infinite, the form becomes a prism; on the contrary, if they be infinitely small, it becomes a plane. Prisms of infinite axes, and planes of infinite extent, are, therefore, limits of all the series of those forms which contain one axis.

Forms of infinite dimensions can never appear by themselves. Those which occur in nature, and consist of terminal and lateral faces, are only segments, or parts of those prisms of infinite axes. The lateral faces of the prism represent the limit of the series on one side; the terminal faces or the base, the limit of the series on the other. Hence they are not simple, but compound forms; and this is the reason why they have not been enumerated among the simple forms. The whole series is comprised within the two limits. The particular mode in which the limits of the series of different forms result, depends upon the quality of those forms themselves, and will be explained in particular in every series.

Forms of several axes cannot have limits of this description. However, if we suppose the different varieties of homogeneous simple forms of variable dimensions (§. 70. 71. 73. 77.) to constitute series; the limits of these series will be represented by those forms of many axes, whose dimensions are constant (§. 58. 59. 63.).

§. 87. FUNDAMENTAL FORM.

That form, which serves as the base of the derivation (§. 81.), is termed the *Fundamental Form.*

The idea of the Fundamental Form in the present method, is well defined, and perfectly determined. Hence mere simple forms, or such as have hitherto been commonly called fundamental, primitive or primary forms, must not be confounded with this idea.

Fundamental forms must possess the following properties. They must be,

1. Simple forms;
2. Forms not derivable from another fundamental form;
3. Forms which do not possess infinite axes; and
4. Forms contained under the least possible number of faces, provided the form itself be not objectionable from other considerations.

According to these characters, the fundamental forms of the mineral kingdom will be,

1. *The Scalene Four-sided Pyramid,*
2. *The Isosceles Four-sided Pyramid,*
3. *The Rhombohedron, and*
4. *The Hexahedron.*

1. DERIVATIONS FROM THE SCALENE FOUR-SIDED PYRAMID.

§. 88. DERIVATION OF MORE ACUTE AND MORE OBTUSE PYRAMIDS, OF SIMILAR BASES WITH THE FUNDAMENTAL FORM.

From every scalene four-sided pyramid may be derived a more obtuse pyramid of the same kind, possessing a similar base with the fundamental form.

First of all, one of the prismatic axes of the given pyramid is fixed upon as its principal axis (§. 41.); according to this the pyramid itself is brought into its upright position (§. 42.). Apply, after the first process (§. 80.), tangent planes to the terminal edges AB, AC, &c. of this form ABCB′C′X, Figs. 37., and enlarge them, till they intersect each other from all sides. There will arise a form AFGIHX, contained under eight isosceles triangles, which, by four and four, are equal and similar to each other. If the form be considered as a four-sided pyramid, its base FGIH has the form of an oblong or rectangle. This is the intermediate form (§. 80.). It will be proper to observe here, that these intermediate forms are not mere geometrical conceptions, but that they are very frequently found in nature, and will be farther explained in §. 97.

Place now, after the third process (§. 82.), *one* plane on each of the terminal edges AF, AG, &c. of this intermediate form, the inclination of this plane to the faces of the form, and to each other, being such as to enable them to intersect each other, after the necessary enlargement, in the plane of the base, and thus to produce a plane figure 𝔅ℭ𝔅′ℭ′, similar and parallel to BCB′C′, the base of the fundamental form. These planes will contain the required form, namely, a scalene four-sided pyramid.

This process may also be applied inversely, that is to say, for any given scalene four-sided pyramid, we may find the one from which it is derivable, according to the method described above. For this purpose inscribe that intermediate form which belongs to the pyramid sought, in that which is given. This is effected by bisecting each of the lateral edges, and joining the points thus determined by straight lines; after which, lines must be drawn from the angles of the oblong figure, which has thus been inscribed in the base of the pyramid, to the terminal points of the axis of the fundamental form. A rhomb is now inscribed in the rectangular base of the intermediate form; the angles of the rhomb coinciding with the centres of the sides of the oblong. This rhomb will be similar and parallel to the base of the fundamental form. But it likewise represents the base of the derived form, which will be completed, if we draw straight lines from the angles of this rhombic figure, towards the terminal points of the axis of the fundamental form, and lay planes on every two of those lines, which are adjacent or contiguous to each other.

§. 89. RATIO BETWEEN THE DERIVED AND THE FUNDAMENTAL FORM.

The axes of two scalene four-sided pyramids, of which the one is derived from the other, according to §. 88., are towards each other in the ratio of $\frac{1}{2}$: 1, if the derived pyramid is more obtuse; in the ratio of 2 : 1, if the derived pyramid is more acute than the given one. The horizontal projections of all these forms are supposed equal; and the axis of the fundamental pyramid = 1.

Let BCB′C′, Fig. 37., represent the base of the fundamental form, which bisects the axis AX in M, the centre of the pyramid; GAF will be a plane tangent to the terminal edge AC, HAF another plane tangent to the terminal edge AB, and consequently AFGIHX the intermediate form, whose base is the rectangle FGIH.

Circumscribe about this rectangle, a rhomb 𝔅ℭ𝔅′ℭ′, similar to the base of the fundamental form, and draw the lines $\mathfrak{B}A$, $\mathfrak{C}A$, &c. These lines $\mathfrak{B}A$, $\mathfrak{C}A$, &c. will be terminal edges; the planes $\mathfrak{B}A\mathfrak{C}$, $\mathfrak{C}A\mathfrak{B}'$, &c. faces, and $\mathfrak{B}\mathfrak{C}\mathfrak{B}'\mathfrak{C}'$ the base of the derived pyramid, which is represented by $A\mathfrak{B}\mathfrak{C}\mathfrak{B}'\mathfrak{C}'X$.

The triangle BMC is equal to the triangle BFC $= \triangle$ $FC\mathfrak{C} = \triangle\ \mathfrak{B}BF$. Hence $\triangle\ \mathfrak{B}M\mathfrak{C} = 4.\ \triangle\ BMC$, and $\mathfrak{B}\mathfrak{C}\mathfrak{B}'\mathfrak{C}' = 4.\ BCB'C'$. Therefore $\mathfrak{B}\mathfrak{C} = 2.\ BC$, and $\mathfrak{B}M = 2.\ BM$.

In the plane $\mathfrak{B}AM$, draw the line BA' parallel to $\mathfrak{B}A$; the triangle $BA'M$ will be similar to the triangle $\mathfrak{B}AM$, and

$$\mathfrak{B}M : BM = MA : MA',$$

hence

$$MA' = \tfrac{1}{2}\, MA.$$

If $A\mathfrak{B}\mathfrak{C}\mathfrak{B}'\mathfrak{C}'X$ be now supposed the fundamental pyramid, AFGIHX will be the inscribed auxiliary form, and $ABCB'C'X$ the more acute derived pyramid, to which the auxiliary form belongs (§. 80.).

In the bases of both these pyramids, the triangle BMC is $= \tfrac{1}{4}\ \triangle\ \mathfrak{B}M\mathfrak{C}$; therefore $BC = \tfrac{1}{2}\,\mathfrak{B}\mathfrak{C}$, and $BM = \tfrac{1}{2}\,\mathfrak{B}M$.

Lengthen now the axis MA to the point $\mathfrak{A}$, and draw the line $\mathfrak{B}\mathfrak{A}$ in the plane $\mathfrak{B}\mathfrak{A}M$. This is the terminal edge of the more acute derived pyramid, if its horizontal projection be supposed equal to that of the fundamental form.

But on account of the similarity of the triangles BAM and $\mathfrak{B}\mathfrak{A}M$, the following proportion takes place:

$$BM : \mathfrak{B}M = MA : M\mathfrak{A},$$

and therefore,

$$M\mathfrak{A} = 2.\ MA.$$

§. 90. SERIES OF SCALENE FOUR-SIDED PYRAMIDS, WHOSE BASES ARE SIMILAR TO THE BASE OF THE FUNDAMENTAL FORM.

If the derivation (§. 88.) be continued, a series of scalene four-sided pyramids of similar bases will arise, whose axes increase and decrease, like the powers of the number 2; the horizontal projections always being supposed equal.

There exist certain constant ratios in the homologous dimensions of any two scalene four-sided pyramids, thus derived from each other; for the sake of an easier comparison, they can be expressed by constant ratios between their axes, if referred to one and the same horizontal projection.

Supposing the horizontal projection to be equal in all the pyramids considered, designate the fundamental form by A; and by B, C, D ... the derived pyramids whose axes are decreasing, by B′, C′, D′ ... those whose axes are increasing: a fragment of the series, containing such members as are nearest to the fundamental form, will be represented by

... D, C, B, A, B′, C′, D′ ...

Let the axis of A be = a, the axis of B will be $= \frac{1}{2}.\ a$, that of $C = \frac{1}{2}.\ \frac{1}{2}.\ a = \frac{1}{4}.\ a$, &c., that of $B' = 2.\ a$, that of $C' = 2.\ 2.\ a = 4.\ a$, &c.; hence the fragment of the series given above, as expressed by the axes of its members:

$$\ldots \tfrac{1}{8}.\ a,\ \tfrac{1}{4}.\ a,\ \tfrac{1}{2}.\ a,\ a,\ 2.\ a,\ 4.\ a,\ 8.\ a \ldots$$

and their ratio to each other =

$$\ldots \tfrac{1}{8} : \tfrac{1}{4} : \tfrac{1}{2} : 1 : 2 : 4 : 8 \ldots$$

that is to say =

$$\ldots 2^{-3} : 2^{-2} : 2^{-1} : 2^{0} : 2^{1} : 2^{2} : 2^{3} \ldots$$

The general member of this series, or the expression of the axis of an indeterminate n^{th} member, will be $= 2^n.\ a$, where a is the axis of the fundamental form, and 2^n the *Law of Progression.* The number 2 is the *Fundamental Number* of the series.

Upon laws of this kind is founded the method of *Crystallographic Designation,* which is comprised under the following rules. The fundamental form is expressed by any arbitrary letter. The same letter serves also for denoting such derived forms as are of the same quality as the fundamental one, as is the case in the present instance, where the derived and the fundamental scalene four-sided pyramids possess similar bases; but if the derived forms are of another kind, the letter is transferred to these derived members, along with the modification or alterations thereby rendered necessary. Under both circumstances, the place of the member in the series to which it belongs, is expressed by the appropriate exponents of the fundamental form, together with + or —, their positive or negative signs. In the fundamental form, the exponent is = 0, and therefore not expressly indicated.

Let P designate the fundamental form of the abovementioned series; the fragment of that series will be

$$\ldots\ P-3,\ P-2,\ P-1,\ P,\ P+1,\ P+2,\ P+3\ \ldots$$

An indeterminate n^{th} member receives the designation $P + n$, the $(n + 1)^{th}$, $P + n + 1$, where the number n may be either positive or negative.

Since the ratio of the diagonals of the base $b : c$ (§. 53.) is known from the dimensions of P, and remains the same in all the members; the dimensions of any required member can be found, if the ratio of its axis to that of the fundamental form be known, and this ratio is indicated by the designation. One of the advantages of this designation consists, therefore, in affording a distinct idea of the forms themselves, speaking as it were to the eye; at the same time, it expresses the connexion existing among them, in as far as they are derived from each other; and, moreover, it contains every thing required for calculating the dimensions of any member, if those of the fundamental form, or of any other member, be known.

The expressions of the cosines of the edges, as given for the scalene four-sided pyramid (§. 53.), refer to those of the pyramid P. If, instead of a^2, $2^{2n}\,a^2$ is substituted in these formulæ, they are changed into the following expressions, which refer to the pyramid $P + n$,

$$\cos.\ y = \frac{2^{2n}\,a^2\,b^2 - (2^{2n}\,a^2 + b^2)\,c^2}{2^{2n}\,a^2\,b^2 + (2^{2n}\,a^2 + b^2)\,c^2};$$

$$\cos.\ x = \frac{2^{2n}\,a^2\,c^2 - (2^{2n}\,a^2 + c^2)\,b^2}{2^{2n}\,a^2\,c^2 + (2^{2n}\,a^2 + c^2)\,b^2};$$

$$\cos\ z = \frac{b^2\,c^2 - (b^2 + c^2)\,2^{2n}\,a^2}{b^2\,c^2 + (b^2 + c^2)\,2^{2n}\,a^2}.$$

In order to find the cosine of any of these angles for a certain determined member of the series, of which P is the fundamental form, there is nothing required but to substitute for n that number which denotes the peculiar place of the member in the series. These general formulæ are very useful in all crystallographic calculations.

§. 91. LIMITS OF THE SERIES OF SCALENE FOUR-SIDED PYRAMIDS.

The limits of the series, §. 90., are on one side an oblique-angular four-sided prism, whose transverse section is equal and similar to the base of the fundamental form, and its axis infinite; on the other side a plane, perpendicular to that axis.

The series (§. 90.) may be continued on both sides, as long as the members obtained, or as long as their axes, are finite quantities; and there can be no reason why we should consider one of these pyramids as the last, because the derivation always will produce new members of the series. But when the axis becomes infinite, no more new members are produced; and in this case the series breaks off, or arrives at its limits (§. 86.). These limits are therefore scalene four-sided pyramids of known bases and infinite axes.

But suppose now the axis to decrease, the terminal edges will approach to the parallelism with the diagonals of the base, the inclination of the faces in these lines, or the terminal edges, to 180°, and the magnitude of the lateral edges to 0. The final term of these approximations is obtained, when the axis becomes infinitely small; in this case the pyramid is transformed into a plane figure, similar to the base.

The prism of an infinite axis is infinitely distant from P in the series of pyramids; or, in other words, there is an infinite number of members of the series between the fundamental form and that prism. The number n in that form, is therefore $= \infty$. In the same manner, n is $= -\infty$ for the prism of an infinitely small axis. The crystallographic signs for these limits are $P + \infty$ and $P - \infty$, and the series itself is thus represented between its limits:

$$P - \infty \ldots P + n \ldots P + \infty.$$

The algebraic expressions in the preceding §. yield the plane angles of the base of P, which is equal to the transverse section of the prism of infinite axis, if n be supposed $= + \infty$, as follows:

$$\cos. y = \frac{b^2 - c^2}{b^2 + c^2};$$

$$\cos. x = \frac{c^2 - b^2}{c^2 + b^2}.$$

The value of $\cos. z = -1$, indicates that one face of the pyramid contiguous to the upper apex, and one contiguous to the lower apex, coincide in a single plane parallel to the axis, by which the lateral edge z becomes $= 180°$.

Several members of this series, together with their intermediate forms and limits, have been observed in nature. Thus, prismatic Topaz, presents three consecutive members, the intermediate form belonging to the most acute of them, and both the limits of the series. Two consecutive members, with their intermediate forms and limits, are known in several species, as in prismatic Lime-haloide, prismatic Lead-baryte, diprismatic Copper-glance, and others.

§. 92. DERIVATION OF SCALENE FOUR-SIDED PYRAMIDS OF DISSIMILAR TRANSVERSE SECTIONS.

The members of the series of §. 90. serve as a foundation to several other derivations. From every one of them, several *Pairs of scalene four-sided Pyramids* may be derived, the bases of which are *dissimilar* to that of the fundamental form, and partly also amongst themselves.

The derivation is effected by the second process (§. 81.);

but this form being contained under triangular faces, must undergo a preliminary operation, before the process can be applied.

Let AX, Fig. 40., be the axis, BCB′C′ the base of the fundamental form, of which the faces BAC′, C′AB′, &c. are contiguous to the upper, and BXC′, C′XB′, &c. to the lower apex of the pyramid. Enlarge now the planes of these faces upwards and downwards, beyond the edges BC′, C′B′, &c.; and in these enlargements describe the triangles BA′C′, BX′C′, &c. and C′A″B′, C′X″B′, &c. equal and similar to the faces of the fundamental form. This process determines the situation of the points A′, A″, &c. X′, X″, &c. which, being joined by straight lines, will produce rectangular figures, similar and parallel to the base of the intermediate form (§. 90.). These rectangular figures are perpendicular to the axis of the fundamental form, which they intersect in the points A and X. This mode of transforming triangular planes in such as are rhomboidal, is the preparation of forms mentioned above (§. 81.).

After this preparation, let the axis of the fundamental form be produced on both sides to an indefinite but equal length, so as to have A𝔄 = X𝔛 or M𝔄 = M𝔛; and draw straight lines from the points A, A″, &c. of the lower rectangle towards 𝔄, which is the upper point, from the points X′, X″, &c. of the upper rectangle towards 𝔛, which is the lower terminal point of the lengthened axis, and from the angles B, C, B′, C′, of the base of the fundamental form, towards both these extremities. If planes be now laid on every contiguous pair of these lines, those faces which are inclined towards the upper apex, will intersect those which are inclined towards the lower apex, in the lines BS, SC′, C′S′, &c., and thus produce a form contained under sixteen scalene triangles.

The triangles BM𝔄 and C′M𝔄 are rectangular in M, and the line M𝔄 is common to both. But BM is either greater or less than C′M; therefore, also, B𝔄 will be greater or less than C′𝔄. Hence the two faces B𝔄S and C′𝔄S of the derived form, contiguous to the edge 𝔄S, are not homologous with each other; and the form consequently is not a simple one.

It is the intermediate or auxiliary form, mentioned in §. 81. This compound form can be resolved, or the simple forms, contained in it, can be extracted, as follows:—Produce first the lines $C'S$, and CS'', to their intersection in $\mathfrak{B}$; $C'S'$ and CS''' to their intersection in $\mathfrak{B}'$, and draw $\mathfrak{B}\mathfrak{A}$ and $\mathfrak{B}'\mathfrak{A}$: $\mathfrak{B}\mathfrak{A}C'$, $C'\mathfrak{A}\mathfrak{B}'$, &c. will be faces, $\mathfrak{B}C\mathfrak{B}'C'$ the base of the pyramid $\mathfrak{A}\mathfrak{B}C\mathfrak{B}'C'\mathfrak{X}$, which is one of those sought for. On the other hand, produce the lines BS'' and $B'S'''$ to their intersection in $\mathfrak{C}$; BS and $B'S'$ to their intersection in $\mathfrak{C}'$, and draw $\mathfrak{C}\mathfrak{A}$ and $\mathfrak{C}'\mathfrak{A}$: $B\mathfrak{A}\mathfrak{C}'$, $\mathfrak{C}'\mathfrak{A}B'$, &c. will be faces, $B\mathfrak{C}'B'\mathfrak{C}$ the base of $\mathfrak{A}B\mathfrak{C}'B'\mathfrak{C}\mathfrak{X}$, &c. which is the other pyramid contained in the compound form. The transverse sections of this pair of derived pyramids are dissimilar, or differ from each other, as well as from that of the fundamental form.

The former of these two pyramids has the same *short* diagonal CC', the latter the same *long* diagonal BB', as the fundamental form. The latter is therefore said to appertain or to refer to the long diagonal, while the other is said to appertain or to refer to the short diagonal of the fundamental form.

The axis $\mathfrak{A}\mathfrak{X}$, common to these pyramids, may be considered as being $= m.\,AX$, a product of the axis AX of the fundamental form, and a certain number m, which is called the *Number of Derivation.* This number must be positive, and greater than 1, either whole or fractionary. The values of this number most commonly, though not exclusively, occurring in nature, are 3, 4, and 5. The crystallographic signs of the pyramids thus obtained, are composed of the sign of that fundamental or derived member of the series (§. 90.), upon which they depend, which is included in a parenthesis, and of the number of derivation m, added to it in the form of an exponent. The signs $\smile$ and $-$, placed above the letter referring to the fundamental form, denote the diagonal to which the derived pyramids belong. The first indicates the long, the second the short diagonal of the fundamental form. Thus,

$$(\check{P} + n)^m \text{ and } (\bar{P} + n)^m$$

are the crystallographic signs for the derived pair of pyramids.

The axis common to both these forms is $= 2^n. m. a$; $2^n. a$ being the axis of $P + n$, and a the axis of P.

§. 93. THE RATIO OF THE DIAGONALS OF THE BASES IS DEPENDENT ON *m*.

If m be supposed equal, the bases of all $(\check{P} + n)^m$, and, on the other hand, the bases of all $(\bar{P} + n)^m$, are equal and similar to each other.

It is evident from the preceding paragraph, that the figures of the bases, or their dimensions, are determined by the situation of the points S, S′, S″, S‴, or, which is the same, by the length of the lines MS, MS′, &c. Draw the line AA′ which bisects BC′, the lateral edge of the fundamental form in H, and the lines HM and A′X, perpendicular to the axis, it will follow that

$$A'X = 2. HM = \sqrt{(b^2 + c^2)}.$$

From the similarity of the triangles 𝔄SM, and 𝔄A′X, we obtain

$$\mathfrak{A}X : XA' = \mathfrak{A}M : MS; \text{ or}$$

$$(m + 1) a : \sqrt{(b^2 + c^2)} = m. a : MS.$$

Therefore

$$MS = \frac{m}{m+1} \sqrt{(b^2 + c^2)}.$$

From this expression it appears, that for a given ratio of b and c in the base of P, the quantity of the axis a enters for nothing in the determination of the bases of the derived forms, and consequently, that the angles of these depend only upon the number m.

If, according to this process, pyramids of dissimilar bases are derived from several scalene four-sided pyramids, according to a determined m; the bases of all these derived pyramids will be equal and similar to each other, in as far as they belong to one and the same diagonal; because the axes of the fundamental pyramids have no influence upon the dimensions of the bases. It will be exactly the same, if the axes are in the ratio of the powers of the number 2, that is to say, if the fundamental pyramids are members of one series; as, for instance, P, P + 1, P + 2, &c.

§. 94. RATIO OF THE DERIVED AND THE FUNDAMENTAL FORM.

If in the pyramid P, the ratio of the axis, the longer, and the shorter diagonal is expressed by

$$a : b : c,$$

or in P + n, by

$$2^n . a : b : c;$$

the ratio of the analogous lines in the same succession will be,

$$\text{for } (\breve{P})^m = m.a : b : m.c, \text{ or}$$
$$\text{for } (\breve{P}+n)^m = 2^n.m.a : b : m.c;$$
$$\text{for } (\bar{P})^m = m.a : m.b : c, \text{ or}$$
$$\text{for } (\bar{P}+n)^m = 2^n.m.a : m.b : c.$$

Since, Fig. 40., in the ratio of

$$\mathfrak{A}M : M\mathfrak{B} : MC'$$

$\mathfrak{A}M$ and MC' are known quantities, being expressed by a, c and m (for $\mathfrak{A}M$ is $= m.a$, or $= 2^n.m.a$, and $MC' = c$); the only thing still to be effected, is to express $M\mathfrak{B}$ in the same manner, or only by b and m.

Draw the line SN parallel to $\mathfrak{B}M$; the triangles $SC'N$, $\mathfrak{B}C'B'$, will be similar to each other, and

$$C'N : SN = C'B' : \mathfrak{B}B',$$
$$= C'B' : M\mathfrak{B} + MB'.$$

But

$$C'N = C'B' - NB' = C'B' - MS$$
$$= \surd(b^2 + c^2) - \frac{m}{m+1}\surd(b^2 + c^2)$$

$$= \frac{1}{m+1} \sqrt{(b^2 + c^2)}; \text{ and}$$

$$SN = MB' = b.$$

Therefore

$$\frac{1}{m+1}\sqrt{(b^2 + c^2)} : b = \sqrt{(b^2 + c^2)} : (m + 1)\, b;$$

and

$$(m + 1)\, b = M\mathfrak{B} + MB' = M\mathfrak{B} + b,$$

from which follows

$$M\mathfrak{B} = m.\, b.$$

For $(\bar{P} + n)^m$, therefore, will follow that $\mathfrak{A}M : M\mathfrak{B} : MC' = 2^n.\, m.\, a : m.\, b : c$, from which, by the mere permutation of the diagonals, the ratio of the analogous lines of $(\check{P} + n)^m$ is found to be

$$= 2^n.\, m.\, a \ : \ b \ : \ m.\, c.$$

Since

$$AM : BM = \mathfrak{A}M : \mathfrak{B}M;$$
$$AM : C'M = \mathfrak{A}M ; \mathfrak{C}'M;$$

the triangle AMB is similar to $\mathfrak{A}M\mathfrak{B}$, and AMC′ to $\mathfrak{A}M\mathfrak{C}'$, and $\mathfrak{A}\mathfrak{B}$ parallel to AB, $\mathfrak{A}\mathfrak{C}'$ parallel to AC′. If therefore from any member of the series §. 90., according to whatever m, a pyramid of dissimilar base with the fundamental form be derived; the terminal edges contiguous to similarly situated, although unequal diagonals, b and m. b or c and m. c. of the two pyramids, will always be parallel to each other.

The number m may be so great, that m. c becomes greater than b. Nevertheless m. c remains the line corresponding to the diagonal c, which is here supposed to be the short one. The correspondence between two diagonals must not therefore be judged of according to their absolute length, but according to their situation. This will require some attention, in order to avoid being confounded by the apparently different position of such pyramids.

If the ratios obtained just now between the axis and the diagonals of $(\check{P} + n)^m$, and $(\bar{P} + n)^m$ be substituted in the general formulæ for the edges of the scalene four-sided pyramid (§. 51.); the result will be other formulæ, similar to those in §. 90., and as general; and they will refer to scalene four-sided pyramids, derived according to the above mentioned process.

§. 95. SERIES OF DERIVED PYRAMIDS OF A DISSIMILAR TRANSVERSE SECTION, WITH THAT OF P. OTHER METHOD OF DERIVING PYRAMIDS OF THE SAME KIND.

The pairs of scalene four-sided pyramids, derived after one and the same m, from the members of the series §. 90. form *two* series, which proceed according to the law of the series §. 90., and are similarly limited.

The same method which from P produces $(\breve{P})^m$ and $(\bar{P})^m$, if applied to $P + n$, yields $(\breve{P} + n)^m$ and $(\bar{P} + n)^m$. The axis of $(\breve{P})^m$ is therefore to that of $(P + n)^m$ in the ratio of the axis of P to the axis of $P + n$, or in that of $1 : 2^n$. Hence 2 is the fundamental number, 2^n the law of progression of the series.

If the positive and negative value of n becomes infinite, $(\bar{P} + n)^m$ and $(\breve{P} + n)^m$ are changed into $(\bar{P} + \infty)^m$, $(\bar{P} - \infty)^m$ and $(\breve{P} + \infty)^m$, $(\breve{P} - \infty)^m$. According to §. 91. these forms are oblique-angular four-sided prisms, whose transverse sections are equal and similar to the bases of $(\bar{P} + n)^m$ and $(\breve{P} + n)^m$. Their plane angles are obtained by the algebraic expressions in the preceding paragraph, if n is supposed $= \infty$. The signs $(\bar{P} - \infty)^m$ and $(\breve{P} - \infty)^m$ refer to the face perpendicular to the axis, already expressed; which face, however, more generally is designated by $P - \infty$, the sign obtained in §. 91. The complete designation of the two series between their limits, is therefore

$$P - \infty \ldots (\breve{P} + n)^m \ldots (\breve{P} + \infty)^m ;$$
$$P - \infty \ldots (\bar{P} + n)^m \ldots (\bar{P} + \infty)^m .$$

There exists, however, still another method of deriving pyramids of dissimilar bases from the fundamental form; and although this method does not produce any new forms, yet it is very well calculated to shew the agreement between the forms derivable from the scalene, and those derivable from the isosceles four-sided pyramid. This method also tends to preserve the sameness of the value of m in both, at least in respect to those numbers of derivation, which are most commonly met with in nature.

This method of derivation consists in applying the process §. 92., not to the pyramid P itself, but to the intermediate form which belongs to that pyramid. It is exactly the same as that by which the scalene four-sided pyramids of dissimilar bases are obtained from the fundamental pyramids themselves, and therefore requires no particular description.

The first result is a compound form, as obtained above, which, by a further resolution, yields a pair of scalene four-sided pyramids, one of which refers to the long, the other to the short diagonal of the fundamental form; although in the derivation, none of these diagonals remain unchanged. The correspondence of these pyramids to the diagonals of the fundamental form, is determined as in §. 92. Their crystallographic designation, in as far as they are obtained by the application of the last mentioned process, is $(\bar{P}r + n)^m$ and $(\breve{P}r + n)^m$.

The ratio of the diagonals of the bases is entirely dependent upon m. For, considering one and the same m, all the bases of $(\breve{P}r + n)^m$ on one side, and all the bases of $(\bar{P}r + n)^m$ on the other, are equal and similar to each other, as may easily be deduced from §. 93. The two lines MS, MS″, Fig. 39., are in the same ratio as the diagonals c and b. From the consideration of the figure it appears, that

$$MS = \frac{2m}{m+1} . c.$$

$$MS'' = \frac{2m}{m+1} . b.$$

The co-efficients of the three perpendicular lines in this derivation are different from those obtained in §. 94. If the axis, the longer, and the shorter diagonal of P + n be in the ratio of

$$2^n . a \quad : \quad b \quad : \quad c,$$

the ratio of the analogous lines in the same succession will be

$$\text{for } (\breve{P}r + n)^m = \frac{m+1}{2} . 2^n . a \quad : \quad b \; : \; \frac{m+1}{m-1} . c;$$

$$\text{for } (\bar{P}r + n)^m = \frac{m+1}{2} . 2^n . a \; : \frac{m+1}{m-1} . b : \quad c.$$

Let S'𝔅'S, Fig. 39., be half of the base of the derived pyramid $(\bar{P}r + n)^m$; in the ratio of the three lines

𝔄M : M𝔅' : MS

𝔄M and MS are already known, that is to say, expressed by a, c, and m; and the only expression still wanting, is that of M𝔅' by means of a function of m and b.

The triangle C'IS is similar to the triangle M𝔅'S; therefore

C'S : C'I = MS : M𝔅'.

But we have

$$C'S = MS - MC' = \frac{2\, m}{m+1} . c - c = \frac{m-1}{m+1} . c.$$

Therefore

$$\frac{m-1}{m+1} . c \; : \; b = \frac{2\, m}{m+1} . c \quad : \quad \frac{2\, m}{m-1} . b,$$

and

$$M\mathfrak{B}' = \frac{2\, m}{m-1} . b.$$

Since this is the required expression, it follows that

$$M\mathfrak{A} : M\mathfrak{B}' : MS = 2^n \, m . a : \frac{2\, m}{m-1} . b \; : \; \frac{2\, m}{m+1} . c;$$

or if the co-efficient of that diagonal to which the pyramid belongs, is supposed = 1, the same ratio will be expressed by

$$= \frac{m+1}{2} . \; 2^n a : \frac{m+1}{m-1} . b \; : \; c.$$

By exchanging the diagonals b and c, the ratio of the three lines of $(\breve{P}r + n)^m$ will be obtained

$$= \frac{m+1}{2} . \; 2^n \; a : b \; : \frac{m+1}{m-1} . c.$$

The co-efficients of the axis and the two diagonals, as developed here, if substituted in the expressions for the cosines of the edges of a scalene four-sided pyramid (§. 53.), will produce similar expressions for the edges of the derived pyramids, in as far as the process of derivation has been applied to the intermediate form.

The pairs of scalene four-sided pyramids thus produced, according to an equal m, from all the intermediate forms, which belong to the members of the series §. 90., will themselves likewise form two series, which proceed according to the general law of the former, and are limited by planes perpendicular and parallel to the axis. The complete designation of the two series between their limits, is

$$P - \infty \ldots (\breve{P}r + n)_m \ldots (\breve{P}r + \infty)^m;$$
$$P - \infty \ldots (\bar{P}r + n)^m \ldots (\bar{P}r + \infty)^m.$$

It has already been stated, that by these two different modes of derivation, we obtain exactly the same forms. This may be demonstrated, by proving how one form, whose derivation from P is expressed by the sign $(\bar{P} + n)^m$, may likewise be derived from the intermediate form $\bar{P}r + n'$, under the sign $(\bar{P}r + n')^{m'}$, or that $(\bar{P} + n)^m$ may be $= (\bar{P}r + n')^{m'}$.

The ratio of the axis and the diagonals, that is to say, of the three lines perpendicular to each other, is in

$$(\bar{P} + n)^m = \quad m.\ 2^n.\ a : \quad m.\ b \quad : \ c; \text{ in}$$
$$(\bar{P}r + n')^{m'} = \frac{m'+1}{2}.\ 2^{n'}.\ a : \ \frac{m'+1}{m'-1}.\ b \quad : \ c.$$

If we suppose the co-efficients of b in the two pyramids to be equal, we obtain $m' = \frac{m+1}{m-1}$, which being substituted in the ratios for $(\bar{P}r + n')^{m'}$ gives

$$= \frac{m}{m-1}.\ 2^{n'}.\ a : m.\ b \ : \ c.$$

The equality of the three lines supposes therefore also $\frac{m}{m-1}.\ 2^{n'}$ to be $= m.\ 2^n$; and consequently $2^{n'} = (m-1)\ 2^n$.

Any scalene four-sided pyramid, which belongs to $P + n$, under the sign of $(\bar{P} + n)^m$, may therefore likewise be considered as deriving from the intermediate form of $P + n$, viz. from Pr + n under the sign $(m-1)(\bar{P}r+n)^{\frac{m+1}{m-1}}$. It depends here upon the value of m, whether the derivation proceeds from an intermediate form belonging to the principal series, or to a subordinate one (§. 96.).

Let m be $= 2$, and the secondary pyramid therefore $= (\bar{P}+n)^2$; we have

$$(m-1)(\bar{P}r+n)^{\frac{m+1}{m-1}} = (\bar{P}r+n)^3;$$

the ratio of the perpendicular lines $= 2.\,2^n.\,a : 2.\,b : c$, exactly as in $(\bar{P}+n)^2$. Let m be $= \frac{3}{2}$, it will follow, that

$$(m-1)(\bar{P}r+n)^{\frac{m+1}{m-1}} = \tfrac{1}{2}(\bar{P}r+n)^5 = (\bar{P}r+n-1)^5,$$

and the ratio of the lines given above $= 3.\,2^{n-1}.\,a : \frac{3}{2}.\,b : c = \frac{3}{2}.\,2^n.\,a : \frac{3}{2}.\,b : c$, as in $(\bar{P}+n)^{\frac{3}{2}}$.

In both these examples, Pr + n belongs to the principal series. But suppose $m = 4$; $(m-1)(\bar{P}r+n)^{\frac{m+1}{m-1}}$ will be $= 3(\bar{P}r+n)^{\frac{5}{3}}$, and the ratio of the three lines will be $= \frac{4}{3}.\,2^n.\,a : 4.\,b : c$, of which the first member still must be multiplied by 3 to make it equal to that of $(\bar{P}+n)^4$, as the crystallographic sign requires. Here Pr + n belongs to that subordinate series, whose co-efficient is the number 3.

In the two first of these examples, it appears that one pyramid may be transformed into another, whose number of derivation is greater; in order to obtain forms in every respect more analogous to the rest of those contained within the compass of forms derived from the scalene four-sided pyramid. The last example shews how a pyramid derived from a member of a subordinate series, may be transformed into another which belongs to a member of the principal one. These two kinds of permutation sufficiently account for the utility in the employment of the double mode of deriving and of designating forms, which, for their absolute dimensions, might be considered as being identical.

The values of m most commonly found in nature are 3, 4 and 5, particularly the first of them; and there is scarcely a species to be found presenting forms in connexion with the scalene four-sided pyramid, which does not afford examples of their occurrence. Prismatic and prismatoidal Hal-baryte, prismatic Topaz, prismatic Chrysolite, may be quoted as examples. Besides these, $(P + n)^6$ and $(P + n)^8$ occur in prismatic Hal-baryte, $(Pr + n)^7$ in prismatoidal Antimony-glance, $(P + n)^{\frac{5}{2}}$ in prismatoidal Manganese-ore.

§. 96. SUBORDINATE SERIES.

There exist several series of forms, homogeneous and of similar bases with that of §. 90. and belonging to it, in reference to which the latter is termed the *Principal* or *Fundamental Series*, while the others are said to be *Subordinate*.

A Subordinate Series is a succession of homogeneous forms, whose bases are equal and similar to those of the members of the principal series, but possessing axes, which, on account of their relative magnitude, are excluded from the principal series: these members, however, may form another series among themselves, which follows the law of progression of the principal one.

The members of the subordinate series may be derived from those of the principal series, either directly, or by the interposition of certain other forms, which are produced from that series. In the present case, the first of these methods being more simple, will find its application.

Let AX, Fig. 38., be the axis, BCB′C′ the base of the fundamental form, and the points A′, A″, &c., X′, X″, &c., be determined, as has been shewn in the preceding derivations.

Through those points lay the two rhombs FGIH and F′G′I′H′, similar and parallel to the base C′B′CB: produce the axis, so as to have $\mathfrak{A}\mathfrak{X} = m. AX$; draw the lines F$\mathfrak{A}$, G$\mathfrak{A}$, &c. F′$\mathfrak{X}$, G′$\mathfrak{X}$, &c. and lay planes into these in such situations that they include a space by themselves. This space will have the form of a scalene four-sided pyramid, the base of which is similar to that of P. The same result is obtained by applying at once the third method of derivation (§. 82.) to the intermediate form in §. 92. This would require to lay planes on the edges formed by the intersection of the faces of $(\breve{P}+n)^m$ with those of $(\bar{P}+n)^m$, the inclination of these planes being such, as to make their common intersection a rhomb, similar and parallel to the base of the fundamental form.

It is now wanted to determine the ratio of the axis of the derived pyramid to that of P, the horizontal projections of the two pyramids being supposed equal.

For this purpose draw the lines: AA′, bisecting the edge BC′ in K; MK in the plane of the base, and XA′ in the plane of the lower rhomb, parallel to MK. From the similarity of the triangles AMK, AXA′ follows

$$XA' = 2.\,MK.$$

Draw now the line A′𝔄, and another K𝔄′ parallel to it; and M𝔄′ will be half the axis of the derived pyramid, its horizontal projection being reduced to BCB′C′. But from the similarity of the triangles 𝔄A′X and 𝔄′KM follows

$$XA' : MK = X\mathfrak{A} : M\mathfrak{A}'$$

or, since

$$\mathfrak{A}X = \mathfrak{A}M + MX = (m+1)\,a:$$

$$2 : 1 = (m+1)\,a : \frac{m+1}{2}.\,a$$

and consequently,

$$\mathfrak{A}'M = \frac{m+1}{2}.\,a.$$

The number $\frac{m+1}{2}$ is termed the co-efficient of the subordinate series, and prefixed in the crystallographic sign of one of its members, to the sign of the member of the principal series, from which the derivation started, so that $\frac{m+1}{2}\,P+n$ is the designation of an indeterminate member of the subordinate series.

If m + 1 becomes a power of the number 2, the derivation yields a member of the principal series itself; and if

$\frac{m+1}{2}$ is a power of the number 2 greater than 1, n is increased for the exponent of this power. Any other m produces one member of a subordinate series from every member of the principal one. In this case $\frac{m+1}{2}$ is divided by that power of the number 2, which, in the series of these powers, differs least from the mentioned number; n is increased for the exponent of that power, or diminished if that power be negative, and the quotient thus obtained is now considered as the co-efficient of the member of the subordinate series.

The expressions for the cosine of the edges referring to those members of the subordinate series, are developed, as has been pointed out before in several similar occasions.

The limits of the subordinate series evidently coincide with those of the principal series.

From the value of $m = 3$, $= 4$ and $= 5$, the co-efficients of the subordinate series are found $= \frac{3}{4}$ and $= \frac{5}{4}$. These and their inverse $\frac{4}{3}$ and $\frac{4}{5}$ have already been found in nature; $\frac{3}{4}$, for instance, in prismatic Hal-baryte, $\frac{5}{4}$ in prismatic Lime-haloide, $\frac{4}{3}$ and $\frac{4}{5}$ in prismatic Sulphur.

§. 97. HORIZONTAL PRISMS.

To every scalene four-sided pyramid, derived from P, as well as to P itself, belong two *Horizontal Prisms*, one of which refers to the long, the other to the short diagonal of the base of the fundamental form.

In any scalene four-sided pyramid, we may suppose one of the diagonals of the base to be increased continually, while the other remains unchanged. The value of the terminal edges changes with the increase of the diagonal. The edge which is contiguous to the unchanged diagonal approaches to 180°. That contiguous to the increasing one approaches to equality with the angle of the principal section through the axis and the unchanged diagonal, and these limits are attained when the increasing diagonal becomes infinite. The pyramid is thus transformed into a prism, the axis of which is the infinite diagonal: its situation is horizontal, and on this account the whole form is termed a *Horizontal Prism.* Since each of the diagonals may thus be supposed to increase till it becomes infinite, there will be two horizontal prisms belonging to every scalene four-sided pyramid, and each of these prisms is referred to that diagonal, which remains unchanged, while the other increases to infinity.

This mode of considering the matter will suffice for giving a general idea of horizontal prisms. But it has no connexion with the relations of forms developed in the preceding paragraphs, where there exists nowhere an absolute increment of a diagonal, this being always a consequence of a simultaneous increment of the axis (§. 93. 95.). In the principal series, whose members also possess their appropriate horizontal prisms, the diagonals do not change at all, while the axes may be increased to infinity.

There are, however, two methods of obtaining horizontal prisms, in connexion with other forms: either the intermediate form (§. 90.) is resolved by enlarging its homologous faces, or tangent planes are laid, not on all, but only on the homologous, terminal edges of the given scalene four-sided pyramid. The result is the same in both processes.

The designation of horizontal prisms is in general $Pr + n$; it is $\breve{P}r + n$, if they belong to the longer, it is $\bar{P}r + n$, if they belong to the shorter diagonal of P. If the faces of $\breve{P}r + n$ and those of $\bar{P}r + n$ appear in combination with each other, and produce the intermediate form, their relative breadth is in the ratio of those diagonals to which their axes are parallel. This intermediate form, in as far as it is a compound form, receives the compound sign $\breve{P}r + n. \bar{P}r + n$; but in as far as it is employed in the derivation of other forms, as in §. 95., the sign $Pr + n$ is applied to it, because the reference to the diagonals is only taken into consideration afterwards.

§. 98. SERIES OF HORIZONTAL PRISMS, AND THEIR LIMITS. INCLINATION OF THE AXIS.

Every series of scalene four-sided pyramids has two concomitant series of horizontal prisms. The limits of these series are planes, which are perpendicular to those diagonals to which they belong, if + n becomes infinite, and perpendicular to the axis of the fundamental form, if — n becomes infinite.

The designation of an indeterminate member in each of these series is $\frac{m+1}{2}\breve{P}r+n$ and $\frac{m+1}{2}\bar{P}r+n$, which, if $m=1$, signifies a member of the principal series. From every other value of m, provided m + 1 be not a power of the number 2, a member of a subordinate series is obtained. If m + 1 be a power of the number 2 greater than 1, yet the co-efficient nevertheless will remain = 1, as in every other member of the principal series; but the n in its crystallographic sign is augmented for the exponent of that power, just as has been mentioned above, in respect to members of subordinate series.

There is no particular designation required for such horizontal prisms as belong to pyramids of dissimilar bases, §. 92. and §. 95., because their principal sections coincide with those of the pyramids, already expressed by the above mentioned signs. This may be proved, for some of them, by considering in general the ratio between the axis and the two diagonals of pyramids, derived according to an undetermined m; and for others, by taking the determined value of m, as obtained from observation.

A horizontal prism is determined by the cosine of that terminal edge, which is contiguous or parallel to the infinite diagonal, or by that angle of the principal section of the pyramid, which lies in the terminal point of the axis.

The values of these cosines are obtained, if in the general expressions of these quantities for $\frac{m+1}{2}$ P + n, one of the diagonals after the other is supposed infinite. In the horizontal prisms belonging to P, the values of the cosines are as follows:

for $\breve{\mathrm{P}}\mathrm{r}$, $\quad \cos. y = \frac{a^2 - b^2}{a^2 + b^2}$;

for $\bar{\mathrm{P}}\mathrm{r}$, $\quad \cos. x = \frac{a^2 - c^2}{a^2 + c^2}$.

As to the limits of the horizontal prisms, it is evident, that in the same proportion in which the axis of the pyramid $\frac{m+1}{2}$ P + n increases, the angle of the horizontal prism at the axis must diminish; and that it must entirely disappear, when the axis becomes infinite. The supplement of this evanescent angle is = 180°, and the horizontal prism therefore is transformed in two unlimited parallel planes, perpendicular to those diagonals to which they belong. If on the other side the axis decreases, the same angle becomes greater and greater, and at last = 180°, if the axis is infinitely small. The supplement of this angle is = 0; the faces of the horizontal prism contiguous to the opposite ends of the axis, coincide with the plane of the basis; and they appear as faces perpendicular to the axis. This is the result of $\frac{m+1}{2}\breve{\mathrm{P}}\mathrm{r} + \infty$ and $\frac{m+1}{2}\bar{\mathrm{P}}\mathrm{r} + \infty$, and of $\frac{m+1}{2}\breve{\mathrm{P}}\mathrm{r} - \infty$ and $\frac{m+1}{2}\bar{\mathrm{P}}\mathrm{r} - \infty$.

The series of horizontal prisms between their limits, are expressed by

$$\mathrm{P} - \infty \ldots \frac{m+1}{2}\breve{\mathrm{P}}\mathrm{r} + n \ldots \breve{\mathrm{P}}\mathrm{r} + n,$$

$$\mathrm{P} - \infty \ldots \frac{m+1}{2}\bar{\mathrm{P}}\mathrm{r} + n \ldots \bar{\mathrm{P}}\mathrm{r} + \infty.$$

The face perpendicular to the axis has received its sign as the limit of the principal series; and in the limits for $n = +\infty$, it is unnecessary to attend to the co-efficients of the different series.

The method of derivation, applied to the scalene four-sided pyramid, is so general, that it will remain unaltered, and yield similar forms and relations, even though the axis of the fundamental form should be inclined at an angle to the plane of the base.

This *Inclination of the Axis* may take place, either in the plane of only one of the diagonals of the rhombic base, as in Fig. 41., or in the planes of both diagonals, as in Fig. 42. In the first two principal sections, ACA′C′ and BCB′C′ are rhombs, and one ABA′B′ is a rhomboid; in the second, only BCB′C′ is a rhomb; and both the others, ACA′C′ and ABA′B′ are rhomboids. A third case is still possible, where all the three principal sections would yield rhomboidal figures, upon which supposition, however, the diagonals CC′ and BB′ themselves intersect each other at oblique angles in the point M. From the want of sufficiently accurate observations, it is at present impossible to decide which of the two last cases, or whether perhaps both of them take place in nature, while the fact of an inclination of the axis in the plane of one of the diagonals has already been established by numerous observations. According to the principles laid down in §. 87. as respects the determination of fundamental forms, it will be impossible to limit the number of these to four, because forms whose axis is inclined, cannot be derived from others whose axis is perpendicular to the base by any of the given processes of derivation. Without entering here into the full developement of the theory of these forms, and without drawing all the necessary consequences from this important fact, it may be useful to mention some of the algebraic formulæ, dependent upon the inclination of the axis in the plane of one of the diagonals.

Let a : b : c : d denote the ratio between the four lines AP, BM, CM, and MP, in Fig. 41., the following formulæ will be obtained:

$$\cos. y = \frac{a^2 (b^2 - c^2) - c^2 (b + d)^2}{a^2 (b^2 + c^2) + c^2 (b + d)^2};$$

$$\cos. y' = \frac{a^2 (b^2 - c^2) - c^2 (b - d)^2}{a^2 (b^2 + c^2) + c^2 (b - d)^2};$$

$$\cos. x = \frac{a^2 (c^2 - b^2) - c^2 (b^2 - d^2)}{\sqrt{[(a^2(b^2+c^2)+c^2(b+d)^2)(a^2(b^2+c^2)+c^2(b-d)^2)]}};$$

$$\cos. z = \frac{c^2 (b^2 - d^2) - a^2 (b^2 + c^2)}{\sqrt{[(a^2(b^2+c^2)+c^2(b+d)^2)(a^2(b^2+c^2)+c^2(b-d)^2)]}}.$$

The angles of the principal sections are found by means of the following formulæ:

$$\cos. CAC' = \frac{a^2 + d^2 - c^2}{a^2 + d^2 + c^2};$$

$$\cos. CBC' = \frac{b^2 - c^2}{b^2 + c^2};$$

$$\cos. BAB' = \frac{a^2 - b^2 + d^2}{\sqrt{[(a^2 + (b + d)^2)(a^2 + (b - d)^2)]}}.$$

In most cases, it will be convenient to have the angles BAP and B′AP separately, as given by the formulæ:

$$\text{tang } BAP = \frac{b + d}{a};$$

$$\text{tang } B'AP = \frac{b - d}{a}.$$

For finding the *Angle of Inclination*, or that at which the axis AA′ is inclined to the line AP, perpendicular upon the base, we have

$$\text{tang } MAP = \frac{d}{a}.$$

The terminal edge of the horizontal prism belonging to the diagonal c, is expressed in the formula:

$$\cos. y = \frac{a^2 - c^2}{a^2 + c^2};$$

that lateral edge of $P + \infty$ which is contiguous to the diagonal b, in the formula:

$$\cos. y = \frac{a^2 b^2 - c^2 (a^2 + d^2)}{a^2 b^2 + c^2 (a^2 + d^2)}.$$

In such forms as Fig. 42., where the axis is inclined in a plane which, if it intersects the base at right angles, passes through neither of its diagonals, the formulæ become more complicated by the introduction of a new variable quantity $e = PR$, yet the general processes of derivation are still applicable to the same extent in this apparently irregular figure, as they are to the scalene four-sided pyramid, whose axes are perpendicular to each other.

2. DERIVATIONS FROM THE ISOSCELES FOUR-SIDED PYRAMID.

§. 99. DERIVATION OF HOMOGENEOUS FORMS.

From every isosceles four-sided pyramid another form of the same kind may be derived, which is more obtuse, and in a diagonal position to the fundamental one.

The derivation is effected by laying tangent planes on the terminal edges AB, AC, &c. of the given pyramid ABCB′C′X, Fig. 45., and enlarging them till they wholly include the space AFGF′G′X (§. 80.). The result of this operation is at once the derived form itself, because the terminal edges of the fundamental pyramid are equal to each other.

In this case, also, the process may be inverted. Draw for this purpose the perpendicular lines from the apices to the lateral edges upon each of the faces of the given isosceles four-sided pyramid, and by planes, laid through every two adjacent ones of these lines, detach those parts of the form which are situated towards their outside. The remaining form is the same isosceles four-sided pyramid, from which, after the process described above, the given more obtuse pyramid has been derived.

The terminal edge of the given pyramid coincides with the perpendicular line, drawn upon the face of the derived form, from the apices towards the lateral edges, as is evident from the circumstance that these planes touch the given pyramid in its edges. The base of the derived pyramid is the square circumscribed about the base of the fundamental form; it is inscribed, if the method has been applied inversely. The two pyramids, and thus every two which are in the same relation, assume such a position towards each other, that the diagonals of the base of the more acute pyramid are parallel to the sides of the base of the more obtuse one, and *vice versa*. This is termed the *diagonal* position. If from the more obtuse pyramid another still more obtuse is derived, and from the more acute one another, still more acute; these new pyramids are diagonally situated to that one from which they are derived; but to the fundamental form they are in such positions as to have their sides and diagonals parallel to the analogous lines of the other, and this is termed the *parallel* position.

§. 100. RATIO BETWEEN THE DERIVED AND THE FUNDAMENTAL FORM.

The axis of an isosceles four-sided pyramid, whose faces touch the edges of another, is to the axis of this latter pyramid, in the ratio of $\sqrt{\frac{1}{2}} : 1$; the axis of that pyramid, whose edges are touched by the planes of another, is to the axis of the latter, in the ratio of $\sqrt{2} : 1$. In both cases, the sides of the horizontal projection are supposed equal.

Let AM, Fig. 45., be half of the axis, BCB′C′ the base of the fundamental form; FAG will be a plane, laid on the terminal edge AB, GAF′ another, laid on the terminal edge AC′, &c.: therefore FGF′G′ will be the base, and FA, GA, &c. the terminal edges of the derived pyramid. The axis AM is common to both.

The square $FGF'G' = 2. BCB'C'$,

hence $FG = GF' = BC'.\sqrt{2}$,

and $FG : BC' = MG : MB = \sqrt{2} : 1$.

Describe from M with the distance MB, the arc BB″; it will follow, that

$$MB'' = MB = \frac{MG}{\sqrt{2}}.$$

Draw the line B″A′ parallel to GA; MA′ will be half of the axis of the derived pyramid, its horizontal projection being equal to BCB′C′. In the similar triangles AGM and A′B″M, is

$$GM : MA = B''M : MA',$$

or

$$GM : MA = \frac{MG}{\surd 2} : MA',$$

therefore

$$MA' = \frac{MA}{\surd 2}.$$

In order to find the ratio of the axis of the more acute pyramid, from which the more obtuse one may be derived, upon the supposition of their horizontal projections being equal, let FGF′G′ be the base of the latter; BCB′C′ will be the base of the former, if the axes of the two pyramids are supposed equal.

But we have $BCB'C' = \frac{1}{2} FGF'G'$.

Therefore $BC' = \frac{FG}{\surd 2}$,

and $BC' : FG = MB : MG = \frac{1}{\surd 2} : 1$.

Produce the line MB, and with the distance MG, describe from the point M the arc GG″, MG″ will be $= MG = MB. \surd 2$.

Now produce the axis AM, and draw G″A″ parallel to BA; MA″ will be half the axis of the more acute pyramid, its horizontal projection being equal to FGF′G′. In the similar triangles BAM, G″A″M, the following proportion takes place:

$$BM : MA = G''M : MA'',$$

or

$$BM : MA = MB. \surd 2 : MA'',$$

therefore

$$MA'' = MA. \surd 2.$$

§. 101. SERIES OF ISOSCELES FOUR-SIDED PYRAMIDS.

Every derived pyramid may again be considered as a fundamental form, and the derivation may be continued. This will produce a series of icosceles four-sided pyramids, whose axes increase and decrease like the powers of the square root of 2; the horizontal projections of these forms being always supposed equal.

As in §. 90. the fundamental pyramid is designated by P, the more obtuse members in their succession by P — 1, P — 2, P — 3, &c. the more acute members by P + 1, P + 2, P + 3, &c. This designation not only rests upon the same principles as that in §. 90., but is in fact exactly the same. Since it is necessary to know before hand whether P is an isosceles or a scalene four-sided pyramid, if the forms derived from it are to be taken into consideration; this identity of the designation can neither in this nor in any other case, admit of, or give rise to, any ambiguity. Suppose the axis of $P = a$; the series of pyramids, and that of their axes, will appear as follows:

$$\ldots P-3,\ P-2,\ P-1,\ P,\ P+1,\ P+2,\ P+3 \ldots$$

$$\ldots \frac{a}{2\sqrt{2}},\ \frac{a}{2},\ \frac{a}{\sqrt{2}},\ a,\ \sqrt{2}.a,\ 2.a,\ 2\sqrt{2}.a \ldots$$

The ratio of the axes is

$$\ldots \frac{1}{2\sqrt{2}} : \frac{1}{2} : \frac{1}{\sqrt{2}} : 1 : \sqrt{2} : 2 : 2\sqrt{2} \ldots$$

that is to say

$$\ldots \sqrt{2^{-3}} : \sqrt{2^{-2}} : \sqrt{2^{-1}} : \sqrt{2^{0}} : \sqrt{2^{1}} : \sqrt{2^{2}} : \sqrt{2^{3}} \ldots$$

The axis of an undetermined n^{th} member, or of $P + n$, is $= \sqrt{2^n}.\ a = 2^{\frac{n}{2}}.\ a$; and this expression is the *Law of Progression* of the series, whose fundamental number is $\sqrt{2} = 2^{\frac{1}{2}}$.

It is evident, that subsequent members of the series are in a diagonal position, alternating members in a parallel position; and since the position of P may be taken for normal, all members of an even exponent will be in a parallel position, those of an odd exponent in a diagonal position.

The algebraic expressions in §. 52. refer to the edges of P. Those of $P + n$ are obtained by substituting $2^n.a^2$ in the place of a^2.

§. 102. LIMITS OF THE SERIES.

The limits of the series §. 101. are, on one side a plane perpendicular to the axis, on the other two rectangular four-sided prisms, one of which is in a parallel, the other in a diagonal position with the fundamental form.

The origin of these prisms is sufficiently evident from §. 91. If the diagonals of the base of the pyramid be supposed equal, a rectangular four-sided prism is obtained, instead of an oblique-angular one. It appears likewise, from the calculations in §. 101., that an isosceles four-sided pyramid of an infinite axis is transformed into a rectangular four-sided prism.

The series of scalene four-sided pyramids is limited on one side by a plane perpendicular to the axis, on the other by a single prism, because there exists no difference in the position of its members. But there is a difference of that kind, in the series of isosceles four-sided pyramids, in which the position of two subsequent members changes from the parallel to the diagonal, and from the diagonal again to the parallel position; and since the last member, or the limit of the series, may be considered in the one, as well as in the other of these positions, it becomes necessary to assume two rectangular four-sided prisms of infinite axes, as limits of this series, one for the parallel, the other for the diagonal position of two prisms limiting the series. This supposition is exactly conformable to experience.

The limits on the opposite side are squares equal to the horizontal projection, because the isosceles four-sided pyramid, if its axis becomes infinitely small, is transformed into a square plane figure. Here the difference of the position can no longer be considered, because this pyramid being nothing but a plane figure, cannot appear by itself in nature, and receives its boundaries from the intersection with the planes of other pyramids and prisms.

The designation of the limits is, agreeably to what has already been stated, $P - \infty$ and $P + \infty$. Since in the latter case it is impossible to argue from the exponent upon the position of the form, it is necessary to express this difference of the two four-sided prisms, by some other contrivance in the designation. The prism in the parallel position receives the sign as it has been employed above, the sign of that in the diagonal position is inclosed in crotchets, and the designation of the series of isosceles four-sided pyramids between its limits is therefore as follows :

$$P - \infty \ldots P + n \ldots \left\{ \begin{matrix} P + \infty \\ [P + \infty] \end{matrix} \right\}.$$

Five members of this series have been observed in pyramidal Garnet, four of them being consecutive, in the same species also the limits on both sides of the series. Four consecutive members are known in pyramidal Copper-pyrites, three in pyramidal Lead-baryte, pyramidal Tin-ore, pyramidal Titanium-ore. The limits of the series occur still more frequently, even in those species, which exhibit fewer members of finite dimensions.

§. 103. DERIVATION OF SCALENE EIGHT-SIDED PYRAMIDS.

From every member of the series §. 101., several scalene eight-sided pyramids may be derived.

This derivation is effected according to the second method (§. 81.), and it is applied after a preparation of the form, as described in §. 92.

The faces of the isosceles four-sided pyramids being isosceles triangles, the figures $BAC'A'$, $C'AB'A''$, &c., Fig. 40., will be rhombs, and the points A', A'', &c., X', X'', &c. will be situated in the angles of two squares, whose planes, like those of the rectangular figures, §. 92., are perpendicular to the axis in the terminal points A and X. These squares $A'A''A'''A''''$ and $X'X''X'''X''''$ are equal and parallel to the square circumscribed about the base $BCB'C'$.

Draw now from the points A', A'', &c. towards the upper, from the points X', X'', &c. towards the lower terminal point of the produced axis, the straight lines A'𝔄, A''𝔄, &c., X'𝔛, X''𝔛, &c. These lines, and accordingly their intersections S, S', &c. will be situated in planes, which are perpendicular to the faces of the isosceles four-sided pyramids; and the lines BS, C'S, &c. are therefore equal to each other. Thus likewise the triangles B𝔄S, S𝔄C', &c. are equal and similar to each other, and the form obtained by the derivation is a simple one, namely, the scalene eight-sided pyramid 𝔄'BSC'S'B'S'''CS''𝔛.

The designation of the scalene eight-sided pyramids is $(P+n)^m$, in agreement with §. 92. It comprehends as it were at once the two pyramids $(\breve{P}+n)^m$ and $(\bar{P}+n)^m$ of the mentioned paragraph, which forms, in the present case, would be equal and similar. The axis of the scalene eight-sided pyramid is $= 2^{\frac{n}{2}}.\ m.\ a$; where $2^{\frac{n}{2}}.\ a$ is the axis of $P + n$.

The relative length of the axis of the eight-sided and the four-sided pyramids, is expressed by the number m, as in §. 92. The only values of this number yet ascertained by observation relative to the pyramids, are 3, 4, and 5; and although it cannot be determined in general, yet it must always be rational, positive, and greater than $1 + \sqrt{2}$ (§. 56. 5.). This supposition is necessary for making it possible to determine the position of scalene eight-sided pyramids among themselves, and towards isosceles four-sided pyramids. If m is equal to $1 + \sqrt{2}$, the eight-sided pyramid is isosceles; if it is less than $1 + \sqrt{2}$, the acute terminal edges are transformed into the obtuse ones, and *vice versa.* And since every scalene four-sided pyramid, derivable from $P + n$, according to a certain m less than $1 + \sqrt{2}$, can likewise be derived according to another m greater than $1+\sqrt{2}$, from another more obtuse isosceles four-sided pyramid $P + n'$ connected with P; this supposition, by excluding the above mentioned values of m, produces at once simplicity and clearness in the consideration of these forms. By the supposition of m being greater than $1 + \sqrt{2}$, it is also possible to avoid a double designation of the same kind as that mentioned in §. 95. These considerations, however, yield a formula for changing any pyramid $(P + n)^m$, in which m is less than $1 + \sqrt{2}$ into another $(P + n')^{m'}$, in which m' is greater than $1 + \sqrt{2}$. It is

$$(P + n')^{m'} = (m - 1)\ (P + n - 1)^{\frac{m+1}{m-1}}.$$

Thus, for instance, 2 being less than $1 + \sqrt{2}$, the sign of a pyramid $(P + 1)^2$ may be transformed into another $(2 - 1)\ (P + 1 - 1)^{\frac{2+1}{2-1}} = (P)^3$, where the exponent 3 is greater than $1 + \sqrt{2}$.

The position in which the scalene eight-sided pyramid is obtained from the isosceles four-sided pyramid, supposing m to be greater than $1 + \sqrt{2}$, is the *parallel* position, that which differs from it for 45°, the *diagonal* position. In the parallel position, a plane through the axis and the acute terminal edge of the eight-sided pyramid, passes at the same time through the acute terminal edge of another eight-sided pyramid, or through the perpendicular line from the apex, upon the face of an isosceles four-sided one, whilst in the diagonal position, the same plane passes through the obtuse terminal edge of the other eight-sided, or through the terminal edge of the four-sided pyramid.

Pyramids expressed by the sign $(P + n)^3$ are frequently met with in nature, as in pyramidal Garnet, in pyramidal Zircon, &c.; those dependent upon other values of m occur more sparingly, as $(P + n)^4$ in pyramidal Garnet, and $(P + n)^5$ in pyramidal Tin-ore.

§. 104. THE BASES OF THE SCALENE FOUR-SIDED PYRAMIDS DEPEND UPON *m*.

For one and the same m, the bases of all forms contained under the sign of $(P + n)^m$, are equal and similar to each other.

The demonstration of this proposition follows from §. 93. The lines denoted in that paragraph by b and c, obtain here the determined value $= \sqrt{2}$; and hence we have

$$MS = \frac{2.\ m}{m + 1}.$$

The magnitude of the plane angles of the base, and therefore of all sections perpendicular to the principal axis of the scalene eight-sided pyramid, depend consequently solely upon m, and these bases are equal, whenever m is the same.

§. 105. SERIES OF SCALENE EIGHT-SIDED PYRAMIDS.

Every member of the series §. 101., gives for every determined m likewise a determined scalene eight-sided pyramid. The forms of this kind, derived from consecutive members of the series, according to one and the same m, produce a particular series among themselves, the axes of which increase and decrease, as the powers of $\sqrt{2}$, or as $2^{\frac{n}{2}}$.

These series arise like those in §. 95. The axes of their members are products of m, the number of derivation, with the axes of the members of the series of isosceles four-sided pyramids, $= 2^{\frac{n}{2}}.m.a$; and since $m.a$ is a factor common to them all, they will be among each other in the ratio of $2^{\frac{n}{2}}$.

If in the algebraic expressions §. 56., $2^{\frac{n}{2}}.a$ be substituted for a, the result will be expressions of the same kind for the cosines of the edges of $(P+n)^m$.

§. 106. LIMITS OF THE SERIES OF SCALENE EIGHT-SIDED PYRAMIDS.

The limits of the series of scalene eight-sided pyramids are, on one side a plane perpendicular to the axis, on the other unequiangular eight-sided prisms, whose transverse sections are equal and similar to those of the members of the series, and their axes infinite. The latter must be considered in two different positions.

That these limits are prisms of the kind above mentioned, follows from §. 95. But, as has been shewn in §. 102. regarding the rectangular four-sided prisms, the eight-sided prisms must, for the same reasons, be considered in either position, the parallel as well as the diagonal one; and, therefore, two limits of infinitely long axes must be assumed for the series of scalene eight-sided pyramids, one in a parallel, the other in a diagonal position to the fundamental form. The positions of these prisms are determined, like the positions of scalene eight-sided pyramids.

If n becomes $= \infty$, $(P+n)^m$ is transformed into $(P+\infty)^m$, $(P-n)^m$ into $(P-\infty)^m$; the latter, not being different from $P-\infty$, is denoted by that sign. In $(P+\infty)^m$ the position, which cannot follow from $n = \infty$, must be indicated by the designation, and this is effected as in four-sided prisms, by giving to the parallel prism the sign $(P+\infty)^m$, to the diagonal prism the sign $[(P+\infty)^m]$. The designation of the whole series between its limits, is therefore:

$$P - \infty \ldots (P+n)^m \ldots \left\{ \begin{matrix} (P+\infty)^m \\ [(P+\infty)^m] \end{matrix} \right\}.$$

The above-mentioned algebraic expressions give for $n = +\infty$, the cosines of the angles in the transverse sections of the unequiangular eight-sided prisms. Thus,

$$\cos. y = -\frac{2m}{m^2+1};$$

$$\cos. x = -\frac{m^2-1}{m^2+1}.$$

The following result is obtained, for the above-mentioned determined values of m (§. 103.).

PRISMS.	cos. y.	cos. x.	y.	x.
$(P+\infty)^3$	$-\frac{3}{5}$	$-\frac{4}{5}$	126° 52′ 12″	143° 7′ 48″
$(P+\infty)^4$	$-\frac{8}{17}$	$-\frac{15}{17}$	118° 4′ 10″	151° 55′ 50″
$(P+\infty)^5$	$-\frac{5}{13}$	$-\frac{12}{13}$	112° 37′ 12″	157° 22′ 48″

Examples of $(P + \infty)^3$ we have in pyramidal Garnet, in both positions; also in pyramidal Copper-pyrites, pyramidal Tin-ore, &c.; of $(P + \infty)^5$ in pyramidal Lead-baryte, and pyramidal Tin-ore.

§. 107. SUBORDINATE SERIES.

There are several *Subordinate Series* of isosceles four-sided pyramids, belonging to that in §. 103., which, in reference to these, is termed the *Principal Series.*

The derivation of the members of these subordinate series is exactly the same as that employed for the scalene four-sided pyramids, §. 96.; only being here applied to members of the series of isosceles four-sided pyramids, the result will be the required subordinate series of isosceles pyramids. The co-efficient thus obtained is likewise $= \frac{m+1}{2}$; and the subordinate series themselves proceed according to the law of the principal one, and are bounded by the same limits.

The same members of the subordinate series may also be obtained by laying tangent planes on the homologous terminal edges of the scalene eight-sided pyramids, &c. The latter process would be the same as that employed in §. 116., for the derivation of subordinate series of rhombohedrons from the principal one. The results of this and of the preceding process are identical. For if the tangent plane be laid on the acute edges of the scalene eight-sided pyramid, the co-efficient obtained will be $\frac{m+1}{2}$; if it be laid on its obtuse edges, the co-efficient will be $\frac{m}{\sqrt{2}}$. By substituting several values instead of m, for instance, those which are most commonly found in nature, we obtain members belonging to the same subordinate series. It is therefore sufficient to assume one of these terms as the general algebraic expression of the co-efficient. If the co-efficients become powers of the number 2, the members belong to the principal series. The members of the subordinate series in particular are designated by $\frac{m+1}{2}P+n$, agreeably to the rules given in §. 96.

The position of the members of the subordinate series in respect to those of the principal series, follows easily from their derivation; and the expressions of the cosines of their edges are found by substituting $\frac{m+1}{2}.a$ instead of a in the formulæ §. 56.

The values of the co-efficients hitherto ascertained by observation are, $\frac{3}{2\sqrt{2}}$, $\frac{5}{4}$, $\frac{2\sqrt{2}}{3}$, and $\frac{4}{5}$. Members of the series $\frac{3}{2\sqrt{2}}P+n$ occur in pyramidal *Zircon* and pyramidal Copper-pyrites; of the series $\frac{5}{4}P+n$ in pyramidal Tin-ore; of the series $\frac{2\sqrt{2}}{3}P+n$ in pyramidal Lead-baryte, pyramidal Kouphone-spar, of the series $\frac{4}{5}P+n$ in pyramidal Kouphone-spar and pyramidal Titanium-ore. It may be remarked here, that the two series, $\frac{3}{2\sqrt{2}}P+n$ and $\frac{5}{4}P+n$ are obtained together with the principal series, if, according to the first method of derivation, tangent planes are applied to the terminal edges of those eight-sided pyramids, which depend upon $m=3$, $m=4$, and $m=5$.

3. DERIVATIONS FROM THE RHOMBOHEDRON.

§. 108. DERIVATION OF HOMOGENEOUS FORMS.

From every rhombohedron, another form of the same kind, but more obtuse, may be derived. The derived rhombohedron is in a *transverse* position towards the fundamental form.

The first method, §. 81., is applied here without any further preparation; and it is evident that the form thus obtained will be a rhombohedron, which is more obtuse than the given one.

The inclined diagonals of this more obtuse rhombohedron, assume the situation of the terminal edges of the other; while the horizontal projections of both are parallel. One of these forms is said to be in a *transverse* position towards the other, because this position may be obtained, by turning a rhombohedron from its original position, round the principal axis, for an angle of 60° or 180°. If a rhombohedron is in the transverse position towards another, it may be brought into the parallel position, only by turning it again round the axis for the same quantity.

In the parallel position, a plane through the axis, and the inclined diagonal, or the terminal edge of one rhombohedron, passes at the same time through the inclined diagonal or the terminal edge of the other; in the transverse position, on the contrary, the plane through the axis and the inclined diagonal of the one at the same time passes through the terminal edge of the other rhombohedron.

In order to invert the process of derivation given above, draw the inclined diagonals upon the faces of the rhombohedron, and take away, by planes passing through every two adjacent diagonals, those parts of the form which lie on their outside. The remainder is the more acute rhombohedron, from which the given one in its due position may be derived according to the first method of derivation, as employed above.

§. 109. RATIO OF THE DERIVED RHOMBOHEDRONS.

The axis of the rhombohedron, whose faces touch the terminal edges of another, is to the axis of this rhombohedron $= \frac{1}{2} : 1$; and the axis of the rhombohedron whose terminal edges are touched by the planes of another, is to the axis of this in the ratio of $2 : 1$; the horizontal projections always being supposed equal.

Let AX, Fig. 45., be the axis, and ACXB the principal section (§. 37.) of the given rhombohedron; AC will be one of its terminal edges.

According to the method of derivation applied, the terminal point B′ of the inclined diagonal of the derived rhombohedron will be situated in the prolongation of the terminal edge AC of the fundamental form. But this point lies also in the prolongation of QS′, a line perpendicular to the axis in Q, AQ being $= \frac{2}{3}$ AX (§. 50. 11.). Therefore the terminal point B′ of the inclined diagonal of the derived rhombohedron coincides with the intersection of the two produced lines AC and QS′, and AB′ is this diagonal itself.

If in the same way, the lines XB and PS are produced till they intersect each other in C′; XC′ is the other inclined diagonal of the derived rhombohedron parallel to the former; and the lines joining the point X with B′, and A with C′, represent the terminal edges, AB′XC′ accordingly the principal section of the derived rhombohedron.

The line AX, or the axis, is common to both principal sections ACXB, and AB′XC′; but the side of the horizontal projection of the given rhombohedron is PC, while that of the derived rhombohedron is QB′.

In the similar triangles APC, AQB′ we have

PC : QB′ = AP : AQ $= \frac{1}{3}$ AX : $\frac{2}{3}$ AX = 1 : 2 (§. 50. 11.).

Hence the side of the horizontal projection of the derived rhombohedron is double the same line of the given rhombohedron, their axes being equal; and the inclined diagonal AB′ = XC′ of the derived rhombohedron, is double the terminal edge AC = XB of the given rhombohedron.

Draw now the line BA′ parallel to C′A, and B″A′ parallel to B′A; BA′B″X will be the principal section, and A′X the axis of the derived rhombohedron; provided the two rhombohedrons have the same horizontal projection, whose side is = BQ.

But in the similar triangles AC′X and A′BX the following proportion takes place:

$$AX : A'X = C'X : BX = 2 : 1.$$

Hence the axis of the derived rhombohedron is equal to half the axis of the fundamental one, if the horizontal projections are equal.

In order to find the ratio between the axis of the more acute rhombohedron and that of the fundamental form, let AB'XC' be the principal section of the latter, the side of the horizontal projection being C'P. In this case ACXB will be the principal section of the derived rhombohedron, the side of its horizontal projection = BQ, and its axis equal to that of the fundamental rhombohedron.

From the point C' draw the line C'B''' parallel to the axis, produce AB to B''', and complete the rhomboid AB'X'B'''. This is the principal section of the derived rhombohedron, the side of the horizontal projection being B'''Q' = C'P.

The similar triangles ACX and AB'X' give

$$AX : AX' = AC : AB' = 1 : 2.$$

The axis of the more acute rhombohedron is consequently double the axis of the fundamental form, their horizontal projections being equal.

§. 110. SERIES OF RHOMBOHEDRONS.

By continuing the derivation, a series of rhombohedrons is obtained, whose axes increase and decrease as the powers of the number 2; their horizontal projections being always supposed to be equal.

This series corresponds exactly to the series of scalene four-sided pyramids (§. 90.), in respect to the ratio between the axes of its members, in as much as it depends upon the same fundamental number.

Designate the fundamental form by R; R + n will be the general expression of an indeterminate member of the series.

Fig. 44. will assist us in giving a clearer idea of this se-

ries. Let ACXB be the principal section of R. Produce the axis AX, and from the points B and C draw lines parallel to it; every line drawn perpendicularly from these upon the axis, will be equal to the side of the horizontal projection of R.

The inclined diagonal of R becomes the terminal edge of $R + 1$. The inclined diagonal of $R + 1$ passes through the point S, the centre of the inclined diagonal XC of R. Hence draw the line AB′ through S, and produce this line till it arrives at the parallel from C; AB′ will represent the inclined diagonal, BAB′X′ the principal section, and $AX' = 2. AX$ consequently the axis of $R + 1$.

The inclined diagonal of $R + 1$ becomes the terminal edge of $R + 2$. The inclined diagonal of $R + 2$ passes through S′, &c. Draw the line AB″ till it arrives at the parallel from B; AB″ will be the inclined diagonal of $R + 2$, AB′X″B″ its principal section, and $AX'' = 2. AX' = 4. AX$ its axis.

Thus, considering AB″ as the terminal edge of $R + 3$, it will be found, that AB‴ is its inclined diagonal, AB″X‴B‴ the principal section, and $AX''' = 2. AX'' = 4. AX' = 8. AX$ the axis of $R + 3$; and in this manner we may continue the series, as long as we please.

The axis of R being $= a$, that of $R + 3$ is $= 2^3.a$, that of $R + n = 2^n.a$, that of $R + n + 1 = 2^{n+1}.a$. These values substituted in the expressions §. 50., give the cosines of the terminal edges for any required member of the series of rhombohedrons.

§. 111. LIMITS OF THE SERIES OF RHOMBOHEDRONS.

The limits of the series §. 110. are, on one side a plane perpendicular to the axis, on the other a regular six-sided prism of an infinite axis. The transverse section of that prism is equal and parallel to the transverse section of the fundamental form; the figure of the plane perpendicular to the axis is a regular hexagon, equal and similar to its horizontal projection.

Lay planes of intersection through the horizontal diagonals of a rhombohedron, whose axis is longer than the side of its horizontal projection. These planes will detach those parts of the rhombohedron which are contiguous to the terminal edges of this form. The remainder, contiguous to the lateral edges, is contained under two equilateral triangles in the direction of the axis, and under six isosceles triangles, being halves of the faces of the rhombohedrons. The equal sides of these triangles are the lateral edges of that form. This solid is the *Central Part* of the rhombohedron.

In the central part of a more acute rhombohedron, the angles at the bases of the isosceles triangles are greater, but the angles at the vertex are less; and the horizontal projection always being constant, the sum of the first approaches to two right ones, the latter to nothing, the more the axis of the rhombohedron is elongated. The equal sides in this case approach nearer and nearer to the parallelism with each other and with the axis, and to the equality with one-third of it, which is contained in the central part of the rhombohedron.

The limits to which these approximations lead, cannot be obtained, while the axis remains a finite quantity. But when the axis becomes infinitely long, these limits are obtained; the triangles are transformed into unlimited parallelograms, and contain a regular six-sided prism, which is still unlimited in the direction of its axis.

As to the transverse section of the prism, we may imagine, that in the proportion in which the axis of the rhombohedron increases, its faces turn round certain immoveable lines. These lines are the sides of the transverse section of the rhombohedron; and therefore they are likewise the sides of the transverse section of the prism.

Let HORZ, Fig. 46., be part of the horizontal projection, and the vertical lines C′D, EB′, &c. through the points

H, O, R, *Z*, the edges of that regular six-sided prism, in whose planes are situated the lateral edges CB, C′B′ of two rhombohedrons. These lateral edges, intersecting each other at the points M, N, turn as it were round these, and consequently the faces of the rhombohedron turn round the line MN, in a ratio, dependent upon the length of the axis. If the axis becomes infinitely long, the lateral edges of the rhombohedron assume the situation C″B″; the rhombohedron is transformed into a regular six-sided prism, upon the faces of which are drawn the horizontal lines MN, ML, &c.; and these lines, the sides of the transverse section of the rhombohedron, are therefore likewise the sides of the transverse section of the prism, whose position is thus determined in respect to the rhombohedrons of the series, and to their horizontal projection.

As to the opposite limit, it is evident, that if the axis becomes infinitely small, all the faces of the rhombohedron coincide in a single plane, and that this form is therefore changed into a regular hexagon, equal and similar to the horizontal projection of the fundamental form.

The crystallographic signs of the limits are $R + \infty$ and $R - \infty$, those of the series between its limits,

$$R - \infty \ldots R + n \ldots R + \infty.$$

Many examples are found in nature, illustrative of this series. Thus, rhombohedral Lime-haloide presents five consecutive members, and both the limits; in rhombohedral Tourmaline and rhombohedral Ruby-blende, four consecutive members and both limits have been observed; two or three consecutive members occur in many species; and in most of those, affecting forms which are in connexion with the rhombohedron, we likewise frequently meet with the limits on either side of the series.

§. 112. DERIVATION OF SCALENE SIX-SIDED PYRAMIDS.

From the members of the series in the preceding paragraph, several scalene six-sided pyramids may be derived, the lateral edges of which agree in their situation with the lateral edges of the rhombohedron.

We employ here the second method (§. 81.), without any farther preparation. Produce the axis of the rhombohedron on both sides, to an indefinite but equal length; or, what is the same thing, multiply the axis of the rhombohedron by the number of derivation m, which must be rational, positive, and greater than 1; draw from the angles of the rhombohedron, straight lines towards the terminal points of the axis produced, and lay planes on every adjacent pair of these lines. The result will be a scalene six-sided pyramid, whose lateral edges coincide with those of the rhombohedron.

Every determined prolongation of the axis of the rhombohedron, or every determined m, determines a scalene six-sided pyramid. A rhombohedron, and all the scalene six-sided pyramids derived from it, which therefore agree in the situation of their lateral edges, and also the pyramids among each other, are said to be forms *belonging together* or *co-ordinate*.

The position in which the scalene six-sided pyramid is placed by the derivation towards the rhombohedron, is termed the *parallel* position. The pyramid is in a transverse position towards rhombohedrons, which immediately precede or follow that from which it is derived, because the rhombohedrons themselves are in a transverse position towards each other. In general the pyramids partake of the position of the rhombohedron from which they are derived, and pyramids belonging together are in a parallel position.

In general, two scalene six-sided pyramids, or one pyramid and a rhombohedron, are said to be in a parallel position, when a plane through the obtuse terminal edge and the axis of the pyramid, intersects the face of the rhombohedron in its inclined diagonal, or in the other pyramid likewise passes through its obtuse terminal edge, and in both forms at the same time also through the axis. The same forms are in a transverse position when this plane passes through the terminal edge of the rhombohedron, or through the acute terminal edge of the other pyramid.

The crystallographic sign of a scalene six-sided pyramid derived from $P + n$ according to m is $(P + n)^m$; its axis is $= 2^n . m . a$; where $2^n . a$ is the axis of $R + n$, and a the axis of R.

The value of m is frequently found in nature to be $= 2$, $= 3$, or $= 5$, all of which occur in rhombohedral Lime-haloide. $(P + n)^3$ is also found in rhombohedral Tourmaline, in rhombohedral Ruby-blende, $(P + n)^5$ in rhombohedral Iron-ore. Besides these, we have $m = 7$ in rhombohedral Lime-haloide, $m = \frac{5}{3}$ in rhombohedral Fluor-haloide, rhombohedral Quartz, rhombohedral Tourmaline, $m = \frac{7}{3}$ in the two first of these species, $m = \frac{11}{3}$ and $= \frac{3}{2}$ in rhombohedral Quartz.

§. 113. THE TRANSVERSE SECTIONS OF THE SCALENE SIX-SIDED PYRAMIDS DEPEND ON *m*.

For one and the same m, the transverse sections of all forms contained under the sign of $(P + n)^m$ are similar to each other.

Let ABXC, Fig. 47., represent the principal section of the rhombohedron from which the scalene six-sided pyramid has been derived; $A\mathfrak{A}$, $X\mathfrak{X}$ the prolongations of the axis, and consequently $\mathfrak{A}B$, $\mathfrak{X}C$ the obtuse terminal edges, $\mathfrak{A}C$, $\mathfrak{X}B$ the acute terminal edges of this pyramid, and $\mathfrak{A}B\mathfrak{X}C$ its principal section.

Draw from M, the centre of the axis, in the plane of the transverse section, the line MG parallel to QB. This line will be intersected in F by the obtuse terminal edge $\mathfrak{A}B$; and F will therefore be that point in the transverse section, or MF that line situated in its plane, upon which depends the magnitude of the angle of the transverse section at the place of the obtuse terminal edge of the pyramid.

Draw now the line BG, parallel to the axis; we have $MG = QB = 1$; and $FG = 1 - MF$.

In the similar triangles FBG and F𝔄M, we have

$$GB : GF = M\mathfrak{A} : MF,$$

or,

$$\frac{a}{6} : 1 - MF = \frac{m.\,a}{2} : MF,$$

and consequently

$$MF = \frac{3\,m}{3\,m + 1};$$

from which it appears that the angles of the transverse section are dependent solely upon m, and that consequently they are the same in all pyramids derived according to the same m, whatever may be the fundamental rhombohedron.

If the section does not intersect the lateral edges of the pyramid, its figure is an unequiangular, but equilateral hexagon. The angle at the obtuse edge is as above; but the angle at the acute edge is likewise dependent upon m. For let CPF′ be in the plane of the section; the lines CP and PF′ will determine its figure. But $CP = 1$; and $PF = \frac{3\,m - 1}{3\,m + 1}$, as it follows from the similarity of the triangles Q𝔄B and P𝔄F′. Now

$$CP : PF = 3\,m + 1 : 3\,m - 1,$$

a ratio solely dependent upon m. Hence all scalene six-sided pyramids, derived according to the same m, have their sections through the terminal edges similar to each other.

§. 114. SERIES OF SCALENE SIX-SIDED PYRAMIDS.

From every member of the series §. 110., several scalene six-sided pyramids may be derived. The axes of those which are derived according to one and the same m, and consequently the pyramids themselves, produce a series which proceeds agreeably to the law of the series of rhombohedrons (§. 110.).

The axes of the members of this series may be considered as products of the axes of $R + n$ by m, that is to say, as being equal to $2^n . m . a$; and since m and a are common factors to them all, the axes are to each other in the ratio of 2^n (§. 110.).

If these values, namely $2^n . a$ instead of a, are substituted in the expressions §. 55., the cosines of the edges are obtained for any particular member of the series.

§. 115. LIMITS OF THE SERIES OF SCALENE SIX-SIDED PYRAMIDS.

The limits of the series of scalene six-sided pyramids are, on the one side unequiangular twelve-sided prisms of infinite axes, whose transverse sections are determined by m, and on the other side plane figures equal and similar to the horizontal projection.

The axis of a scalene six-sided pyramid, which belongs to a rhombohedron, whose axis is in a finite ratio to the side of its horizontal projection, can never become infinite, unless m itself should be infinite, a supposition, however, which is excluded from the relations to be taken here into consideration, in as much as such an m would give no series, or rather a series, all the members of which are equal to each other. Therefore the limits of the series of these pyramids must arise from rhombohedrons, whose axes are themselves infinite, or from the regular six-sided prism (§. 111.).

Lay planes of intersection through the terminal points of the lateral edges of a scalene six-sided pyramid derived from a rhombohedron, whose axis is longer than the side of its horizontal projection; and thus detach the apices of the pyramids, so that only the central part analogous to that in §. 111. remains, to which the lateral edges are adjacent.

This part contains one-third of the axis of the rhombohedron, from which the pyramid is derived. It is limited perpendicularly to the axis by equilateral hexagons, whose alternating angles are equal (§. 113.), and from the sides by twelve scalene triangles, whose bases are the sides of those hexagons, the longer sides being the entire lateral edges, and the shorter sides parts of the obtuse terminal edges of the scalene six-sided pyramid.

The central part of a pyramid derived according to the same m from a more acute rhombohedron, although contained under faces of the same kind, will yet differ from the preceding by the sum of the angles at the base of the triangles being greater, the angle at the vertex smaller, and the sides approaching nearer to parallelism and equality with each other, and to one-third of the axis of the rhombohedron.

The limits to which the pyramids approach, when thus derived according to the same m, from more and more acute rhombohedrons, are : the sum of the angles at the base of the triangles must be $= 180°$, the angle at the vertex $= 0$; and the sides equal and parallel among themselves, and to one-third of the axis of the rhombohedron. These limits are obtained when the central part of the pyramid, containing one-third of the axis of the rhombohedron, from which the pyramid is derived, and therefore the axis of the rhombohedron itself becomes infinite, or when the rhombohedron is changed into the regular six-sided prism. If therefore $(P + n)^m$ is changed into a prism, the expression of its axis $2^n . m. a$, will be changed into $2^{\infty} . m. a$, because n and not m is $= \infty$.

While the scalene triangles, the lateral faces of the central part, by the continual increase of the axis, are changed into unlimited parallelograms, the central part itself is changed into a twelve-sided prism, unlimited in the direction of the axis. Through all these changes, however, m remains unaltered; and since the angles of the transverse section are solely dependent upon m, the unequiangular twelve-sided prism will have the same transverse section as any other member of that series, whose limit it represents.

As to the opposite limit, it is evident, that if the height of the central part, $= \frac{1}{3}$ of the axis of the rhombohedron, disappears, the axis of this rhombohedron itself must also be $= 0$; and consequently that the rhombohedron must be a plane figure, equal and similar to the horizontal projection. Now the axis of the pyramid sought is $m . 0 = 0$; and the pyramid therefore likewise a plane figure, equal and similar to the horizontal projection.

The designation of the series between its limits is

$$R - \infty \; \ldots \; (P + n)^m \; \ldots \; (P + \infty)^m.$$

If an unequiangular twelve-sided prism, from its original position, is transferred into another different from it for 60° or 180°, its faces and edges resume exactly the situation they had before. Hence there is only one position existing for these forms: in which property it agrees with the regular six-sided prisms, the limit of the series of rhombohedrons, from which it derives its origin.*

If in the algebraic expressions mentioned in the last paragraph, n is made $= \infty$; the angles are obtained for the transverse section of the unequiangular twelve-sided prism, being the limit of the respective series of the scalene six-sided pyramid. Thus we find

$$\cos. y = -\left(\frac{3m^2 + 6m - 1}{2(3m^2 + 1)}\right);$$

$$\cos. z = -\left(\frac{3m^2 - 1}{3m^2 + 1}\right).$$

* Another regular six-sided prism, which in every respect, but the position, agrees with the former, will be considered in §. 118. This form, however, is in no immediate connexion with the scalene six-sided pyramids; and consequently no unequiangular twelve-sided prism can be considered in, or referred to, a position analogous to that regular six-sided one, although the angles of their transverse sections should seem to indicate a similar position.

The values of the cosines, and the angles of the transverse sections of the limits, relative to the above-mentioned series occurring in nature, are the following:

PRISMS.	cos. y.	cos. z.	y.	z.
$(P+\infty)^{\frac{3}{2}}$	$-\frac{59}{62}$	$-\frac{46}{62}$	162° 6′ 12″	137° 53′ 48″
$(P+\infty)^{\frac{5}{3}}$	$-\frac{13}{14}$	$-\frac{11}{14}$	158° 12′ 48″	141° 47′ 12″
$(P+\infty)^{2}$	$-\frac{23}{26}$	$-\frac{22}{26}$	152° 12′ 15″	147° 47′ 45″
$(P+\infty)^{\frac{7}{3}}$	$-\frac{22}{26}$	$-\frac{23}{26}$	147° 47′ 45″	152° 12′ 15″
$(P+\infty)^{3}$	$-\frac{11}{14}$	$-\frac{13}{14}$	141° 47′ 12″	158° 12′ 48″
$(P+\infty)^{\frac{11}{3}}$	$-\frac{46}{62}$	$-\frac{59}{62}$	137° 53′ 48″	162° 6′ 12″
$(P+\infty)^{5}$	$-\frac{26}{38}$	$-\frac{37}{38}$	133° 10′ 25″	166° 49′ 35″
$(P+\infty)^{7}$	$-\frac{47}{74}$	$-\frac{73}{74}$	129° 25′ 48″	170° 34′ 12″

Few of these prisms have as yet been observed in nature. Those which have been observed are $(P+\infty)^{\frac{5}{3}}$ and $(P+\infty)^{3}$ in rhombohedral Fluor-haloide, $(P+\infty)^{3}$ in rhombohedral Lime-haloide and in rhombohedral Tourmaline, $(P+\infty)^{\frac{3}{2}}$ in rhombohedral Quartz, $(P+\infty)^{5}$ in rhombohedral Corundum. The angles of other prisms have not been exactly ascertained.

The angles of the first and sixth of the preceding prisms, those of the second and fifth, and those of the third and fourth, are inversely equal to each other. In general, the number of derivation which produces the one, may be found from that which produces the other, by the formula:

$$m = \frac{3\ m' + 1}{3\ (m' - 1)}.$$

§. 116. SUBORDINATE SERIES.

To the series of rhombohedrons, §. 110., belong several subordinate series, in reference to which the former is termed the principal series.

For the derivation of these subordinate series, the first process is applied to the scalene six-sided pyramids.

Let ABXC, Fig. 47., represent the principal section of a rhombohedron, 𝔄B𝔛C that of a scalene six-sided pyramid derived from it according to a certain m.

If tangent planes are laid on the terminal edges 𝔄B, &c. of this pyramid; these edges become the inclined diagonals of the resulting rhombohedron. Suppose its axis = a′; that part of it which corresponds to the inclined diagonal 𝔄B will be

$$\mathfrak{A}Q = \tfrac{2}{3}.\ a' = M\mathfrak{A} + MQ = \frac{3\ m + 1}{6}.\ a,$$

and

$$a' = \tfrac{3}{2}.\ \frac{3\ m + 1}{6}.\ a = \frac{3\ m + 1}{4}.\ a,$$

the side of the horizontal projection BQ being = 1.

If on the other side tangent planes be laid on the acute terminal edges 𝔄C, &c.; these terminal edges again will become the inclined diagonals of the derived rhombohedron. The axis of this rhombohedron being = a″; that part of it which corresponds to the inclined diagonal 𝔄C will b

$$\mathfrak{A}P = \tfrac{2}{3}.\ a'' = M\mathfrak{A} - MP = \frac{3\ m - 1}{6}.\ a$$

and

$$a'' = \frac{3\ m - 1}{4}.\ a$$

for the same horizontal projection.

Hence $\frac{3\ m + 1}{4}$ and $\frac{3\ m - 1}{4}$ are the co-efficients, with which a, the axis of R, or 2^n. a, the axis of R + n must be multiplied for obtaining members of the subordinate series. When these co-efficients become powers of the number 2, the rhombohedron produced is a member of the principal series; when they are not powers of the number 2, members are produced belonging to a subordinate series, which is determined by m.

Designate the subordinate series by $\frac{3\ m + 1}{4}$ R + n. The quantity $\frac{3\ m + 1}{4}.\ 2^n.$ a substituted for 2^n. a in the above mentioned algebraic expressions, gives the cosines of the edges for the members of the subordinate series.

The position of the members of the subordinate series towards each other, and towards those of the principal series, follows from their derivation. The limits are common to the principal, and to all subordinate series.

§. 117. DERIVATION OF THE ISOSCELES SIX-SIDED PYRAMIDS.

From every rhombohedron an isosceles six-sided pyramid may be derived, whose axis is to the axis of the rhombohedron in the ratio of 2 : 3, the horizontal projections of the two forms being supposed equal.

The third method of derivation (§. 82.) is applicable to the present case.

Let ABXC, Fig. 48., represent the principal section of the given rhombohedron, and suppose a horizontal plane to pass through M, the centre of its axis. In this plane is situated the base of the six-sided pyramid, which is to be derived.

The terminal edge AC of the rhombohedron, being produced to Z, will be changed in AZ the terminal edge of the pyramid. In the same way XB is changed into XH, so that, if we draw AH and XZ, AZXH will be the principal section of the derived isosceles six-sided pyramid, its axis being equal to that of the rhombohedron, the side of its horizontal projection MZ = MH.

Draw the lines BG and CG′ perpendicularly to HZ; MG′ will be = MG = PC, equal to the side of the horizontal projection of the given rhombohedron; and if now the lines GA′, G′A′, GX′, G′X′, be drawn parallel to the sides of the principal section of the pyramid, A′X′ will represent the axis of this form for the side of its horizontal projection MG′ = PC, that is to say, equal to the horizontal projection of the given rhombohedron.

The two triangles APC, A′MG′ are equal and similar to each other. Therefore A′M = AP; which if the axis of the pyramid be called a′, may be expressed thus:

$$\tfrac{1}{2}.\ a' = \tfrac{1}{3}.\ a,$$

and consequently

$$a' = \tfrac{2}{3}.\ a.$$

The above-mentioned constant co-efficient (§. 54. 4.) accordingly is $= \tfrac{2}{3}$.

§. 118. SERIES OF ISOSCELES SIX-SIDED PYRAMIDS.

There is a series of isosceles six-sided pyramids in connexion with the principal series of rhombohedrons, with which it proceeds after the same law, and is limited by infinite six-sided prisms.

The axes of the members producing this series, are equal to the axes of the rhombohedrons, multiplied by $\tfrac{2}{3}$, the horizontal projections being always supposed equal; so that, if P + n denotes an undetermined n^{th} member of the series, its axis will be $= \tfrac{2}{3}\, 2^n.\ a$. In the expression for the axis of any member, the common factor $\tfrac{2}{3}.\ a$ is contained; and by dividing with it, we find that the axes of the series increase and decrease like the powers of the number 2; and 2^n consequently expresses also in this series the law of progression.

The limits of this series are isosceles six-sided pyramids belonging to rhombohedrons, whose axes are on one side infinitely long, on the other infinitely short. It is evident that an isosceles six-sided pyramid of an infinitely long axis can be nothing else but a regular six-sided prism, whose transverse section is equal and similar to the horizontal projection of the pyramid; and that an isosceles six-sided pyramid of an infinitely short axis can be nothing else but a plane figure perpendicular to the axis, and equal and similar to the same horizontal projection.

The regular six-sided prism, which limits the series of isosceles six-sided pyramids, can be distinguished by its position from the other regular six-sided prism, which limits the series of rhombohedrons. The faces of the two prisms differ in their situation for 30° and 150°: and the prisms therefore themselves are two different well defined forms, which must not be confounded with each other.

If, in the algebraic expressions in §. 54., m is made $= \frac{2}{3}$, and $2^{2n}. a^2$ is substituted for a^2; the expressions produced refer to the cosines of the edges for $P + n$. They are:

$$\cos. x = -\left(\frac{2. 2^{2n}. a^2 + 27}{4. 2^{2n}. a^2 + 27}\right);$$

$$\cos. z = -\left(\frac{4. 2^{2n}. a^2 - 27}{4. 2^{2n}. a^2 + 27}\right).$$

The series of isosceles six-sided pyramids between its limits, receives the following designation:

$$R - \infty \ldots P + n \ldots P + \infty.$$

Several members of this series, together with its limits, occur in rhombohedral Fluor-haloide, rhombohedral Quartz, rhombohedral Corundum, &c. There exist also series appertaining to rhombohedrons of certain subordinate series, and which, on that account, receive a co-efficient in their representative sign, like the rhombohedrons from which they are derived. Examples of the series $\frac{3}{4}$ P + n, $\frac{5}{3}$ P + n, and $\frac{7}{3}$ P + n, have been found in rhombohedral Corundum; of the two first also in rhombohedral Quartz, of the first in rhombohedral Iron-ore, &c.

4. DERIVATIONS FROM THE HEXAHEDRON.

§. 119. DIFFERENT POSITIONS OF A MOVEABLE PLANE.

A plane, moveable round the terminal point of a rhombohedral axis of the hexahedron, is liable to assume *four* different classes of positions. One of these is exactly determined, and admits of only *one* case; the others allow of a farther distinction in *two* cases.

Let ACBC′B″C″B′X, Fig. 36., represent a hexahedron, which is brought into an upright position by supposing one of its rhombohedral axes AX vertical. AC, AC′, &c. are therefore the terminal edges, AB, &c. the inclined diagonals of this hexahedron, if considered as a rhombohedron of 90°.

Direct now the planes MNOO′, PQRR′ and TUVV′ through the axis AX, so as to make them pass through the inclined diagonals AB, AB′, and through the terminal edges AC″, AC′ which are opposite to these edges. The planes will intersect each other at angles of 60° and 120°.

The part MNSS′ of the plane MNOO′ turned towards the observer, may be termed the *Section of the Face*, the part PQSS of the plane PQRR′, similarly situated, the *Section of the Edge*, in so far as they refer to the upper apex A; because the former passes through the inclined diagonal AB, and bisects the face, while the latter passes through the terminal edge AC of the hexahedron, and bisects the angle at which the faces meet.

The sections of the face divide every face of the hexahedron into two equal and similar triangles, as ABC, ABC′, &c. and thus the solid angle A may be conceived to consist of six faces, which, for the sake of derivation, are considered moveable, and their situation is ascertained in respect to both the sections, to that of the face, and that of the edge. Whatever results are found for one of these faces, likewise applies to the other, because the hexahedron is a solid of several axes, and it will therefore be sufficient to consider the situation of *one* of these six faces, because the rest must assume an analogous position. This refers evidently not only to those contiguous to A, but also to those belonging to the other solid angles B, C, &c. of the hexahedron.

The moveable plane may assume the following situations:

1. *Perpendicular to both sections.*

Upon this supposition, ABC will be perpendicular to the intersection of MNSS′ and PQSS′, which is the line SS′, or the rhombohedral axis of the hexahedron. All the six planes contiguous to that point, will necessarily coincide in that plane.

2. *Perpendicular to the section of the edge; but inclined to the section of the face.*

Here every two faces situated like ABC and AB′C, &c. coincide in a single plane, which, though always perpendicular upon PQSS′, yet may be differently inclined to AX.

3. *Perpendicular to the section of the face, inclined to the section of the edge.*

In this case again, pairs of faces like ABC and ABC′ coincide in a single plane perpendicular to MNSS′, inclined to AX.

4. *Perpendicular to none of the sections.*

No two planes contiguous to the same solid angle of the hexahedron coincide, but every two meeting in the same section, as ABC and AB′C in PQSS′, or ABC and ABC′ in MNSS′, are inclined to that section at the same angle.

In the first of the above mentioned cases, the situation of the moveable plane is perfectly determined.

In the second case, the plane must either

a) touch AC, the edge of the hexahedron, or

b) the line of its intersection with PQSS′ must include an angle with AX, which is greater then CAX.*

Supposing the first to take place, two faces of the solid angle C, coincide in one single plane, with two faces of the solid angle A, for instance CC′A with ABC, and CC″A with AB′C. This does not take place upon the latter supposition.

In the third case, the moveable plane may either

a) pass through the diagonal AB, and consequently coincide with the face of the hexahedron itself, or

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* Should this angle be less than CAX, it would be necessary to refer the whole derivation, from the solid angle A, to the solid angle C, where the case is confined within the one above mentioned.

b) its intersection with MNSS′ may include an angle with AX, greater than BAX.

In the first case, two planes of the solid angle B coincide with two planes of the solid angle A, as for instance BAC with ABC, and BAC′ with ABC′, and consequently two from C likewise with two from C′, being altogether eight planes coinciding in a single one, which is not the case in the second.

The differences which may occur in the fourth case are the following:

a) the intersection of every two faces, as ABC and AB′C, with the plane PQSS′, may coincide with the edge of the hexahedron, or

b) it includes an angle with AX greater than CAX.

Suppose the former to take place; two faces from the solid angle A will coincide with two faces from the solid angle C, viz: ABC with CC′A, and ACB′ with CC″A; which is not the case upon the latter supposition.

The different situations, which a plane moveable round the point A may assume, are therefore the following:

1. Perpendicular to both the sections;
2. Perpendicular to the section of the edge, touching the edge of the hexahedron;
3. Perpendicular to the section of the edge, intersecting the edge of the hexahedron;
4. Perpendicular to the section of the face, in the face of the hexahedron;
5. Perpendicular to the section of the face, not in the face of the hexahedron;
6. Inclined to both the sections, touching the edge of the hexahedron;
7. Inclined to both the sections, intersecting the edge of the hexahedron.

§. 120. PRODUCTION OF THE FORMS OF SEVERAL AXES.

Whatever situation of those mentioned in the preceding paragraph the moveable plane may assume, it corresponds to the face of a form of several axes.

We obtain or derive the forms of several axes from the hexahedron, by considering the space limited by all those faces which are homologous to the one whose situation has been ascertained.

Hence there will exist as many different kinds of forms of several axes, as there are possible situations of the moveable plane, and no more; and we obtain, therefore, the *complete* number of these forms, whilst at the same time every form is excluded which does not belong to this assemblage.

In the preceding paragraphs 57—77, we have met with more than seven forms of several axes. Those which are not immediately produced according to the present consideration, are nevertheless contained in its results, the mode of which will be explained in the paragraphs 128—134.

§. 121. THE OCTAHEDRON.

In the first situation the moveable plane is the face of *the Octahedron* (§. 59.).

Of the forty-eight faces which are moveable round the eight solid angles of the hexahedron, every six contiguous to one of these solid angles coincide in one and the same plane, perpendicular to a rhombohedral axis of the form (§. 59. 2.).

§. 122. THE DODECAHEDRON.

In the second situation the moveable plane is the face of *the Dodecahedron* (§. 63.).

A pair of faces from every solid angle of the hexahedron coincides with another pair of faces contiguous to an adjacent solid angle in a plane which touches the edge of the hexahedron, and joins these two solid angles. Hence, of the forty-eight faces, four and four will coincide, and the solid obtained will be limited by twelve faces. The prismatic axes pass through the centres of the edges of the hexahedron, and consequently also through the centres of the faces now obtained. Thus the faces of the derived form become perpendicular to the prismatic axes, and are themselves the faces of the monogrammic Tetragonal-dodecahedron (§. 63. 3.).

§. 123. THE OCTAHEDRAL TRIGONAL-ICOSITETRAHEDRON.

In the third position the moveable plane is the face of an *octahedral Trigonal-icositetrahedron* (§. 72.).

In this case there are no pairs of faces from one solid angle, coinciding with pairs from another; but of the six faces contiguous to one and the same solid angle, two and two faces will coincide. Hence the number of faces of this form is twenty-four. Each of these faces is intersected by the two other faces contiguous to the same, and by one contiguous to the adjacent solid angle; with the last of these faces it produces an edge in the direction of the greater diagonal of the dodecahedron, or in the direction of the edge of the octahedron. Its faces therefore assume the figure of isosceles triangles; the rhombohedral solid angles of the form consist of three faces, and they are monogrammic; the pyramidal solid angles are formed by eight faces, and they are digrammic; the form itself is an octahedral Trigonal-icositetrahedron (§. 72. 1. 2.).

The different varieties of octahedral trigonal-icositetrahedrons may be considered as forms intermediate between the dodecahedron and the octahedron. If the angle measuring the inclination of the moveable plane to the axis AX, Fig. 36., becomes greater than CAX, the face of the monogrammic tetragonal-dodecahedron is divided into two isosceles triangles, whose common base is the longer diagonal of the rhomb. The triangles retain their isosceles figure, though the angles may vary, till the moveable plane intersects the axis of the form at an angle of 90°. In this case, all the faces contiguous to the same solid angle coincide in a single plane, which is the face of the octahedron. All possible varieties of octahedral Trigonal-icositetrahedrons are therefore contained between the two forms just mentioned, and the dimensions of their varieties depend upon the magnitude of the above mentioned angle.

§. 124. THE HEXAHEDRON.

In the fourth situation, the moveable plane is the face of *the Hexahedron* (§. 58.).

In this situation pairs of faces from all the four solid angles A, B, C, C′ coincide in a plane perpendicular to the pyramidal axis (§. 58. 3.).

§. 125. THE DIGRAMMIC TETRAGONAL-ICOSITETRAHEDRON.

In the fifth situation, the moveable plane is the face of a *digrammic Tetragonal-icositetrahedron* (§. 74.).

The pairs of faces from the angles A and B, and those from the angles C and C′, do not coincide, but they intersect each other at equal angles in a determined point of the lengthened pyramidal axis of the hexahedron. A solid angle of three faces is produced at the point A. The edges which produce these two kinds of solid angles unite with each other in the prismatic axes prolonged, and thus produce solid angles, which contain likewise four faces, but two different kinds of edges.

Each face is intersected by four other faces, two of which are contiguous to the same solid angle, the other two to the adjacent ones. The faces are four-sided; and on account of the two kinds of edges which the form contains, it is a digrammic Tetragonal-icositetrahedron (§. 74. 2.).

The digrammic Tetragonal-icositetrahedrons may be considered as forms intermediate between the hexahedron and the octahedron. For if the angle measuring the inclination of the moveable plane to the axis AX becomes greater than BAX, a digrammic Tetragonal-icositetrahedron will be produced, and the varieties of this form will succeed each other, till the angle just mentioned becomes = 90°, when the derived form is changed into the octahedron. The dimensions of the different varieties, are dependent upon the value of that angle.

§. 126. THE HEXAHEDRAL TRIGONAL-ICOSITETRAHEDRON.

In the sixth situation the moveable plane is the face of a *hexahedral Trigonal-icositetrahedron* (§. 71.).

This icositetrahedron is produced by the coincidence of two faces contiguous to adjacent solid angles. From every edge of the hexahedron faces rise towards the prolongation of the pyramidal axes, at which they form a solid angle of four faces, intersecting each other at equal angles, while the general aspect of the hexahedron is retained in the derived form. The rhombohedral solid angles are equiangular, but they consist of two kinds of edges.

Each of the faces of this form is intersected by three other faces, of which one is contiguous to the same, and one to an adjacent solid angle of the hexahedron, the third face being common to both these solid angles. The faces of this form are consequently triangular, and intersect each other at equal angles in its pyramidal solid angles. The form, therefore, will be a hexahedral Trigonal-icositetrahedron.

The hexahedral Trigonal-icositetrahedrons are forms intermediate between the hexahedron and the dodecahedron. The moveable plane passes through the edge of the hexahedron; the limits to which its inclination approaches will therefore be on one side a plane perpendicular to the section of the edge, on the other a plane perpendicular to the section of the face, of which the first produces the dodecahedron (§. 122.); and the other the hexahedron (§. 124.).

The varieties are determined according to the mutual inclination of the faces at the place of the edge of the hexahedron. Every different inclination, greater than 90° and less than 180°, yields a particular hexahedral Trigonal-icositetrahedron.

§. 127. THE TETRACONTAOCTAHEDRON. DESIGNATION OF THE TESSULAR FORMS.

In the seventh situation, the moveable plane is the face of a *Tetracontaoctahedron* (§. 77.).

No two faces coincide in a single plane. The form therefore is contained under forty-eight faces, which are scalene triangles, on account of their intersection with two faces from the same, and with one face from the adjacent solid angle of the hexahedron. The rhombohedral solid angles are formed by six faces, the pyramidal solid angles by eight, and the prismatic solid angles by four. All of them are equiangular and digrammic. Hence the form will be a Tetracontaoctahedron.

It may here be observed, that the seven forms thus connected with the hexahedron, the hexahedron itself being one of the number, perfectly agree with each other in respect to the kind, number, and situation of their axes, which is an immediate consequence from the method of derivation employed. The same property does not extend to the rest of the forms of several axes; and thus the number of the different kinds of axes in their peculiar situation, becomes the character by which it is possible to ascertain whether or not a form of several axes belongs to the compass of those which may be derived *immediately* from the hexahedron. All these forms are contained under the first degree of regularity (§. 47.).

If, instead of a rhombohedron, we substitute the hexahedron, or, instead of an isosceles four-sided pyramid the octahedron, and apply to them the modes of derivation described above (§. 80. 81. 82.): certain combinations between the derived forms will likewise represent the whole compass of those obtained by the process of the moveable plane; some of them even with their determined dimensions, which may be considered as an advantage of this method. It requires, however, some knowledge of compound forms (§. 34.), upon the resolution or developement of which it depends.

The designation of the forms of several axes has been framed upon a principle different from that followed in the designation of the rest of simple forms. The great number of distinct kinds of forms, and the few varieties of each, known or occurring in the compass of derived forms, have been the reason why it was impossible to trace in the crystallographic signs, all those relations of the forms to each other which distinguish the designation of the forms derived from the four-sided pyramids and from the rhombohedrons. The following method, although it does not possess the advantage of expressing these relations, yet is recommended by its brevity and distinctness.

Designate the hexahedron by the letter H, the octahedron by O, the dodecahedron by D, the tetracontaoctahedron by T; these being the initial letters of their respective names. The initial letters of the icositetrahedrons cannot be employed in the same way, because there are three different kinds of such forms among those immediately derived from the hexahedron;—namely, the hexahedral and the octahedral trigonal-icositetrahedrons, and the digrammic tetragonal-icositetrahedron. Designate the first of these by A, the second by B, the third by C; and add to them, as well as to the sign of the tetracontaoctahedrons, the number which denotes the variety of these forms, as contained in the preceding paragraphs 71–77.

§. 128. RESOLUTION OF FORMS BELONGING TO THE FIRST DEGREE OF REGULARITY.

To *resolve* a form of several axes, means to produce from it two or more equal and similar forms of several axes, the faces of which agree in number and situation with one-half or one-fourth of the faces of the original form. These forms reproduce the original one, if combined in the required position.

A form of several axes, produced by the resolution of another, if it contains half the number of its faces, is termed a *Half;* if it contains only one-fourth of the faces, it is termed a *Fourth* of the resolved or original form.

Those halves must not be taken for half forms, or such as might be obtained by cutting in two, one of the original forms, as would be a simple pyramid. Nor are the fourths real quarters of original forms, because they have not been obtained by cutting in two, one of the preceding halves.

The method by which the resolution is effected, is the following:

Place the given form in an upright position, so as to make one of its rhombohedral axes vertical.

Call the upper terminal point of this axis a *Principal Point*, the lower one a *Subordinate Point*, and transfer those names to all the terminal points of rhombohedral axes, distant from the vertical one, for 109° 28′ 16″. In the hexahedron, as represented Fig. 36., the principal points are A, B, B′, B″, and the subordinate points, X, C, C′, C″.

Enlarge now,

1. *all the faces contiguous to the principal points*, till those contiguous to the subordinate points disappear; or,
2. *the alternating faces from the principal points, and those from*

the subordinate ones which are parallel to them, again, till the rest disappear: or,

3. *the alternating faces from the principal points, and those from the subordinate ones, which are not parallel to them*, till the rest disappear.

The enlarged faces, if they can geometrically include a space by themselves, will produce a form of many axes, which is a Half, if only one of the three methods has been applied; a Fourth, if two at the same time, or subsequently, have been employed in resolving the original form.

If, in the first mode of resolution, instead of enlarging the faces contiguous to the principal points, we enlarge those from the subordinate ones, the result from the same original form will be another half, equal and similar to the first, but in a different situation from the other form. The two halves can be brought into a parallel position, by inverting the perpendicular axis of one of them. The position now mentioned is called the *inverse*, in reference to the other or *normal* position; and one half of this kind is said to be the *Inverse* of the other, which has been obtained in the normal position.

A similar result takes place, if, in applying the second mode of resolution, those faces are made to disappear, which produce the half in the normal position, while the others are enlarged. Both these kinds of halves are remarkable for the parallelism of their faces, which, however, is a consequence of the method of resolution applied.

The third method of resolution, if treated in the same manner, enlarging those faces which had been made to disappear before, and *vice versa*, does not yield exactly the same result. In respect to position, there is no difference among the two halves; but there is a difference according to *Right* and *Left*, as mentioned in §. §. 67. and 76. The same relation exists in the Fourths, which, like the halves of the first and third method of resolution, have no parallel faces.

Some of the forms derived from the hexahedron, do not allow of any resolution at all; either because half the number of their faces would not be sufficient to include a space from all sides; or because none of the methods mentioned is applicable to them. The first is the reason why the hexahedron, the other why the dodecahedron, have no halves. Besides, it is not every form that can be resolved by every one of the above-mentioned methods; but certain properties of the form are required to render one of these methods applicable.

The *first* method supposes the faces of the form which is to be resolved not to touch the terminal points of two rhombohedral axes; or, which is the same, not to touch a principal point and a subordinate one at the same time. For as it is required to effect the reverse on one of those points, from what has been done on the other, it would thus be requisite, that one and the same face should at the same time be made to enlarge itself and disappear. For this reason, the hexahedral trigonal-icositetrahedron cannot be resolved according to the first method.

The *second* and *third* method supposes the number of faces at the rhombohedral solid angles to be such as will render it possible to enlarge the alternating ones. This cannot take place, if the solid angles are formed of three faces. In this case, the resolution too is impossible; and therefore, the two methods require the rhombohedral solid angles to consist of six faces. The third method requires moreover the condition of the first method to take place, else it would be necessary to enlarge all the faces; and consequently no resolution at all could take place. By this last condition, the hexahedral trigonal-icositetrahedron is excluded, and the method becomes applicable only to the tetracontaoctahedron, which, however, can be resolved according to both the other methods.

The axes undergo very remarkable changes by the resolution. The rhombohedral ones remain unaltered; the prismatic axes disappear entirely in all the halves; the changes in the pyramidal axes are various. If the third method has been applied, they remain constant like the rhombohedral axes; they are changed into prismatic axes in the first and second processes; and then there are no more than three axes of this kind to be found in the solid. In the resolution of halves in fourths, or in the application of two of the methods at once, they disappear entirely. The peculiar character of the halves is, therefore, that they have *six* axes less; of the fourths, that they have *nine* axes less than the original forms.

The halves arising from the first method of resolution, and the fourths, into the formation of which this method likewise enters, assume the general aspect of the tetrahedron.

The crystallographic signs of these forms are obtained by indicating a division by the numbers 2 and 4, the references as to position by the signs + and —, and those to Right and Left, by the letters r and l prefixed to the crystallographic sign of the original form.

§. 129. THE TETRAHEDRON.

The half of the octahedron is *the Tetrahedron* (§. 57.).

The octahedron allows of the application of the first process. The number of faces of its half is four, and these faces are perpendicular to the rhombohedral axes.

The crystallographic signs of the two tetrahedrons, of which one is in the normal, the other in the inverse position, Figs. 13. 14., as halves of the octahedron, are $\frac{O}{2}$ (o) and $-\frac{O}{2}$ (o').

§. 130. THE HEXAHEDRAL PENTAGONAL-DODECAHEDRON.

The half of the hexahedral trigonal-icositetrahedron is the *hexahedral Pentagonal-dodecahedron* (§. 66.).

Here the second process must be applied. If the alternating faces contiguous to the principal points disappear, and at the same time those which are parallel to the former at the subordinate points; every one of the remaining enlarged faces is intersected by five others, and thus assumes a pentagonal figure. The number of faces is evidently twelve; the form produced will therefore be a pentagonal-dodecahedron, which is a hexahedral one because the second mode of resolution does not change the general aspect of the form. The latter property also might be derived from the equality and similarity of the eight solid angles of three faces, which correspond to those of the hexahedral trigonal-icositetrahedron, which are formed by six faces (§. 66. 1.).

The crystallographic signs of the hexahedral pentagonal-dodecahedrons, one of them being in the normal, the other in the inverse position, Figs. 19. 20., are $\frac{An}{2}$ (a) and $-\frac{An}{2}$ (a'), where n denotes the variety which is to be expressed.

§. 131. THE DIGRAMMIC TETRAGONAL-DODECAHEDRON.

The half of the octahedral trigonal-icositetrahedron is the *digrammic Tetragonal-dodecahedron* (§. 64.).

The resolution is effected after the first method.

Each of the enlarged faces is intersected by four others, of which two belong to the same, and two to other principal points. Thus they become four-sided, and their number is twelve. Hence the form is a tetragonal-dodecahedron; and since it assumes the general aspect of a tetrahedron, the first mode of resolution having been applied, it will be that described in §. 64. 1., or the digrammic tetragonal-dodecahedron.

The crystallographic signs of these forms in the normal and in the inverse position, Figs. 17. 18., are $\frac{Bn}{2}$ (*b*) and $-\frac{Bn}{2}$ (*b'*).

§. 132. THE TRIGONAL-DODECAHEDRON.

The half of the digrammic tetragonal-icositetrahedron is the *Trigonal-dodecahedron* (§. 61.)

This resolution is likewise effected after the first method.

Each of the enlarged faces is intersected by two faces of the same, and one of another principal point. These faces, isosceles triangles, are twelve in number; and the half therefore is a trigonal-dodecahedron.

The crystallographic signs of these forms in the normal and in the inverse position, Figs. 15. 16., are $\frac{Cn}{2}$ (*c*) and $-\frac{Cn}{2}$ (*c'*).

§. 133. THE TETRAHEDRAL TRIGONAL-ICOSITETRAHEDRON, THE TRIGRAMMIC TETRAGONAL-ICOSITETRAHEDRON, AND THE PENTAGONAL-ICOSITETRAHEDRON.

The halves of the tetracontaoctahedron are,

1. The *tetrahedral Trigonal-icositetrahedron* (§. 70.);

2. The *trigrammic Tetragonal-icositetrahedron* (§. 75.); and

3. The *Pentagonal-icositetrahedron* (§. 76.).

Any one of the three methods of resolution may be applied to the tetracontaoctahedron; and this form consequently has three kinds of halves, which at first sight seem to agree with each other only in their being icositetrahedrons.

The first mode yields the tetrahedral trigonal-icositetrahedron. The solid angles of six faces of the original form are likewise to be found in the half, because according to this method all the faces contiguous to the principal points are enlarged. The faces are intersected by two other faces of the same principal point, and by one face contiguous to another, exactly as in the preceding case; the faces remain triangles, which are likewise scalene, but not similar to those of the original form. The half therefore is a trigonal-icositetrahedron, which owes its tetrahedral aspect to the application of the first process.

In the designation of these forms, it is necessary to indicate the mode of resolution upon which they depend. The sign of the tetrahedral trigonal-icositetrahedrons in both, the normal and the inverse positions, Figs. 25. 26., will therefore be $\frac{\mathrm{Tn}}{2\,\mathrm{i}}$ and $-\frac{\mathrm{Tn}}{2\,\mathrm{i}}$. The trigrammic tetragonal-icositetrahedron is the result of the second mode of resolution. The intersection of the enlarged faces takes place with two faces of the same principal points, and with two faces contiguous to adjacent subordinate points. In comparing the pyramidal solid angle of the tetracontaoctahedron with the corresponding solid angle in its half, we find, that of the eight faces which constitute the first, alternating *Pairs* of faces are enlarged, and not the alternating single faces. From this it becomes evident that the angle formed by four faces above the face of the hexahedron cannot consist of equal edges, and that consequently the faces of the tetragonal-icositetrahedron thus formed cannot by a straight line be divided in two isosceles triangles. Besides the two kinds of edges meeting in the solid angle of four faces, the form contains still another kind of edges, which meet in the rhombohedral solid angle formed by three planes; it is therefore a trigrammic tetragonal-icositetrahedron (§. 75. 2.).

The crystallographic signs of the trigrammic tetragonal icositetrahedrons in both, the normal and the inverse positions, Figs. 27. 28., are $\frac{\mathrm{Tn}}{2\,\mathrm{ii}}$ and $-\frac{\mathrm{Tn}}{2\,\mathrm{ii}}$.

If we apply the third method, the result will be a pentagonal-icositetrahedron. Here all the alternating faces of the original form are enlarged, while the rest disappear, so that each face is intersected by five others, of which two belong to the same principal point, the rest to the adjoining subordinate points, and the faces assume therefore a pentagonal figure. Of the pyramidal solid angle of the tetracontaoctahedron likewise the alternating faces are enlarged; this solid angle therefore remains a pyramidal one, and the axis which passes through it, a pyramidal axis. These properties will suffice for determining the form to be a pentagonal icositetrahedron.

The crystallographic signs of the pentagonal-icositetrahedrons, in reference to their being as it were twisted to the right or to the left, Figs. 29. 30., are $r\frac{Tn}{2}$ and $l\frac{Tn}{2}$. It would be superfluous to add here the number iii for indicating the mode of resolution, except in comparing these forms with other halves of the tetracontaoctahedron.

§. 134. THE TETRAHEDRAL PENTAGONAL-DODECAHEDRONS.

The three icositetrahedrons of the preceding paragraph, which are halves of the tetracontaoctahedron, allow of a farther resolution, and then yield the fourths of that form. The fourths of the tetracontaoctahedron, are the tetrahedral *Pentagonal-dodecahedrons* (§. 67.).

The resolution of the tetrahedral trigonal-icositetrahedron, is effected by enlarging its alternating faces, till they limit the space by themselves. Each of the enlarged faces is intersected by five others; and the resultant fourth is therefore a pentagonal-dodecahedron, whose general aspect is that of the tetrahedron, on account of the application of the first process. The fourth itself is the tetrahedral pentagonal-dodecahedron.

This method of resolution produces the same result as if the first and the second, or the first and the third, had been applied to the tetracontaoctahedron. The first requires the faces of the subordinate points to disappear; the others require in this case only the enlargement of the alternating faces of the remainder.

If in the icositetrahedron considered above, we enlarge those faces which have disappeared, and *vice versa*, let those disappear which have been enlarged before; the result in respect to that obtained first, will be a *Left* tetrahedral pentagonal-dodecahedron. But the icositetrahedron may be resolved both in the normal and in the inverse position. Hence both the differences, as to Right and Left, and as to Normal and Inverse, come into consideration in the tetrahedral pentagonal-dodecahedron.

The trigrammic tetragonal-icositetrahedron may be resolved after the first process, by enlarging all the faces contiguous to its principal points, &c. Each of these faces is intersected by five others, two of which belong to the same, the other three to adjacent principal points. For the rest, every thing is as above; and the trigrammic tetragonal-icositetrahedron yields exactly the same fourths.

The pentagonal-icositetrahedron is resolved according to the first method, by enlarging all the faces contiguous to the principal points, &c. Each of these faces again is intersected by five others, and the result of the resolution is likewise a tetrahedral pentagonal-dodecahedron.

These four pentagonal-dodecahedrons, different on one side as to right and left, on the other as to their normal or inverse position, reproduce in binary combinations the icositetrahedrons, and in a quadruple combination the tetracontaoctahedron itself, from the resolution of which they have been obtained.

The first of these differences is expressed by the letters r and l, the second by the signs + and —, prefixed to $\frac{Tn}{4}$ the general notation of one-fourth of the tetracontaoctahedron. The four dodecahedrons will therefore be:

$$+ r\frac{Tn}{4}\ (t)\ \text{Fig. 21.,}\quad - r\frac{Tn}{4}\ (i)\ \text{Fig. 22.,}$$

$$+ l\frac{Tn}{4}\ (t'')\ \text{Fig. 23.,}\quad - l\frac{Tn}{4}\ (t''')\ \text{Fig. 24.}$$

These four solids yield six binary aggregates:

1. $+ r\frac{Tn}{4}. - r\frac{Tn}{4}$, which is $= l\frac{Tn}{2\,iii}$ Fig. 30.;
2. $+ r\frac{Tn}{4}. + l\frac{Tn}{4}$, which is $= +\frac{Tn}{2\,i}$ Fig. 25.;
3. $+ r\frac{Tn}{4}. - l\frac{Tn}{4}$, which is $= -\frac{Tn}{2\,ii}$ Fig. 28.;
4. $- r\frac{Tn}{4}. + l\frac{Tn}{4}$, which is $= +\frac{Tn}{2\,ii}$ Fig. 27.;
5. $- r\frac{Tn}{4}. - l\frac{Tn}{4}$, which is $= -\frac{Tn}{2\,i}$ Fig. 26.;
6. $+ l\frac{Tn}{4}. - l\frac{Tn}{4}$, which is $= r\frac{Tn}{2\,iii}$ Fig. 29.

Of these, 1 and 6 are pentagonal-icositetrahedrons, 1 is the left, and 6 the right one; 2 and 5 are tetrahedral trigonal-icositetrahedrons, 2 is in the normal, and 5 in the inverse position; and 3 and 4 are trigrammic tetragonal-icositetrahedrons, of which 4 is in the normal, and 3 in the inverse position. Every two homogeneous forms of these six reproduce by combination the tetracontaoctahedron itself.

The halves and fourths belong to the second degree of regularity.

The preceding methods of resolving the original forms of several axes yield all those forms which have been described above (§. 57.—77.), and which could not be obtained by immediate derivation. Thus, resolution completes what by derivation would have remained imperfect; and we are entitled to consider as complete the number of simple forms of several axes.

The method of resolving simple forms, is not confined to those which possess several axes, in as much as it may also be applied to pyramids of every description, and even to rhombohedrons. This requires, however, certain restrictions, which will be mentioned along with the results of that process, in some of the paragraphs referring to the *Character of Combinations.*

GENERAL IDEAS OF SIMPLE FORMS.

§. 135. SYSTEM OF CRYSTALLISATION.

The assemblage of simple forms derivable from one fundamental form (§. 87.), independent of all consideration of its dimensions, is termed a *System of Crystallisation*, and denominated after the fundamental form, from which it is derived.

The term System of Crystallisation has often been made use of in a sense different from that of the present definition.

A System of Crystallisation is not a mere aggregation of forms, according to their different kinds, or according to certain properties peculiar to them; but it is the *Assemblage of those Relations* which take place among certain simple forms, in as far as they are derived from one fundamental form.

From the above mentioned four fundamental forms, there arise four different Systems of Crystallisation; and no more systems are possible, if there exist only four forms of this kind. We have no reason to assume a new fundamental form in Crystallography, unless we have discovered or observed a form, which cannot be derived from any one of those which are known. As this is the case with the scalene four-sided pyramids with an inclined axis (§. 98.), the number of systems of crystallisation will be increased to *six*, or perhaps to *seven*. In none of these systems can there be any objection against considering all those forms which geometrically must enter into it, as really belonging to them, although nature should not as yet have produced them as simple forms.

The System of Crystallisation derived from the rhombohedron is termed the *Rhombohedral System.* The system whose fundamental form is the isosceles four-sided pyramid, is called the quadrato-pyramidal, or, in shorter terms, the *Pyramidal System;* that from the scalene four-sided pyramid is the rhombeo-pyramidal system, which, on account of the great variety of prisms which it contains (§. 91. 95. 98.), receives the denomination of the *Prismatic System;* and that from the hexahedron is called the *Tessular System*, not the hexahedral one, in order to intimate, that experience has not as yet given any reason for assuming another system of tessular forms, although geometrically we may conceive a system of forms of several axes, which stands in the same relation to the regular dodecahedron of Geometry, in which the tessular system is to the hexahedron. The other systems, comprising the Hemiprismatic (§. 153.) and the Tetartoprismatic (§. 154.) forms, have not as yet been provided with particular denominations.

§. 136. SERIES OF CRYSTALLISATION.

The fundamental form being supposed to possess determined dimensions, the assemblage of derived forms becomes a *Series of Crystallisation.*

The system of crystallisation is an idea of very great extent, but it is liable to certain restrictions in particular determinate cases. These restrictions consist in ascertaining or fixing the dimensions of the fundamental forms in the different systems: for the relations of the derived forms among each other and to the fundamental one are general, and must remain unchanged in all series of crystallisation, which belong to the same system. If, therefore, the fundamental form is supposed to possess determined dimensions, the derivation will yield a System of Crystallisation, which is determined for a particular case; or, which is the same thing, it will produce a Series of Crystallisation.

From these considerations it follows that the tessular system, being the only one in which the simple forms possess invariable dimensions, will comprehend only one Series of Crystallisation; while all the other systems, which possess variable dimensions, may comprehend an unlimited number of such series, that is to say, as many as there may be differences in the dimensions of their fundamental forms.

Since these series represent the systems of crystallisation themselves, though determined for particular dimensions; it is plain that they must not be confounded with the series of homogeneous simple forms (§. 85.) considered above, as, for instance, with the series of rhombohedrons, or of the different pyramids. It is also evident that, if different members of the same series, for instance R, R + 3, R — 1, &c., be considered as fundamental forms, they will not yield different series of crystallisation, because upon this supposition, the results obtained by derivation will be identical.

§. 137. THE SYSTEM OF CRYSTALLISATION DETERMINED FROM A SINGLE FORM.

From the observation of any single form, except the right rectangular prism, the System of Crystallisation to which this form belongs may be inferred. This extends to the Series of Crystallisation, if the dimensions of the form be given or known.

The facility of ascertaining the System of Crystallisation, by observing one single form, is obvious, and is founded upon the difference among those simple forms, which constitute the different systems. Thus the rhombohedral system is composed of rhombohedrons, of six-sided pyramids, and of six-sided and twelve-sided prisms; the pyramidal system, of isosceles four-sided pyramids, and of scalene eight-sided pyramids, besides rectangular four-sided prisms, and eight-sided prisms of alternately equal angles, &c. It is therefore not very difficult from one form being known, to trace or find out the system to which it belongs. The only exception occurring here, is the right rectangular four-sided prism, which may belong, as a simple form, or as a compound one, to three, or at least to two different systems, if we abstract from the plane perpendicular to the axis. As a simple form, it is the hexahedron, and belongs to the tessular system; as a compound form, which consists of two simple ones ($P - \infty$ and $P + \infty$), it is a right rectangular prism, and belongs to the pyramidal system; and as a compound form, consisting of three simple ones ($P - \infty$. $\breve{P}r + \infty$. $\bar{P}r + \infty$.), it is likewise a right rectangular prism, but belongs to the prismatic system. The abstract geometrical consideration of these forms, yields no characters by which they could be distinguished from one another, though the means will be afterwards (§. 159.) pointed out, by which this uncertainty may be removed, and which principally depend upon the connexion of certain forms with each other, and upon several peculiarities occurring along with them in natural bodies.

If the given form possesses finite dimensions, these are either known, or may be found by immediate measurement; in both cases, therefore, it is possible to obtain those of the fundamental form, and consequently also of the series to which it belongs.

III. OF COMBINATIONS.

OF COMBINATIONS IN GENERAL.

§. 138. DEFINITION.

A compound form is termed a *Combination* (§. 34.).

A mineral occurring in a compound form is nevertheless a simple mineral (§. 21.), and may be an individual; for that composition, which is the subject of our present consideration, regards only the external form of the mineral. Yet an individual, affecting a compound form, or more than one simple form at a time, may be imagined to *represent* two, three, or more individuals, if we suppose that all the natural historical properties of the individual, with the exception of the geometrical ones, are connected with every one of the simple forms contained in the combination.

A combination may, in some instances, assume the aspect of a simple form, being contained under faces which are equal and similar to each other (§. 35.). This takes place when two equal and similar forms combine in different positions, which positions, nevertheless, are always peculiar to the system in which these forms are found. Combinations of this kind may be ascertained to be such, and distinguished from really simple forms, either by the number of their faces, which is greater than that produced by derivation, or by their relations and the position, which exclude these apparently simple forms from the series to which, as simple forms, they would necessarily belong.

The form of a combination is the space contained at the same time within all the simple forms constituting it. Hence, none of the angles of incidence of a combination can be greater than 180°, or re-entering angles. Such angles are produced, though not always, if two or more individuals, of the same or of different forms, are connected in different positions; these compositions will be properly considered in §. 178. &c.

The number of forms entering into a combination, is undetermined. There may be only two, but there also may be a great number of them. A combination containing two simple forms is also termed a *binary combination;* one containing three simple forms, a *triple combination*, &c. The exact knowledge of binary combinations is the most interesting department of Crystallography, in as far as it refers to compound forms. The knowledge of binary combinations may be considered as the elements of the knowledge of multiple combinations; and a triple combination may be resolved into two, a quadruple combination into six, one consisting of n simple forms, into $\frac{n(n-1)}{1 \cdot 2}$ binary combinations.

§. 139. FIRST LAW OF COMBINATION.

The first Law of Combination is: *That the combined simple forms must belong not only to the same System* (§. 135.), *but also to the same Series of Crystallisation* (§. 136.); *they must be derived from one and the same Fundamental Form.*

If one of the simple forms contained in a combination, belong to a certain System of Crystallisation, the rest of them also must belong to the same system. Let, for instance, this form be a regular six-sided prism; the rest of the forms combined with it will belong to the rhombohedral system; and thus one single form recognised in a combination, though it be a limiting one, will be sufficient to determine the System of Crystallisation (§. 137.). But the form recognised may be a finite one, and moreover it may be, according to its dimensions, a member of the Series of Crystallisation peculiar to rhombohedral Lime-haloide; the rest of the forms will belong to the same series (though from this only it does not necessarily follow that the individual is rhombohedral Lime-haloide); and their dimensions may be calculated, as soon as we know in what relation they stand to that which has been determined. Hence also, in this case, the observation of one single form suffices for determining the whole Series of Crystallisation to which all the simple forms of the combination belong (§. 137.). Nature confirms this law in all combinations, without any exception.

The only combinations to be considered in Crystallography are therefore such as are produced by forms belonging to the same Series of Crystallisation.

§. 140. SECOND LAW OF COMBINATION.

The second Law of Combination is: *That the simple forms contained in the combination, must be in such Positions towards each other as are peculiar to them in the systems to which they belong.*

According to the preceding derivations, the simple forms of every system are obtained in certain determined positions. In these positions, and only in these, they join in combinations. Thus, in the rhombohedral system we have the parallel position and the transverse position; in the pyramidal system the parallel position and the diagonal position, &c. In most cases, the combined forms assume those positions, in which they have been derived. Thus, in the rhombohedral system, the subsequent rhombohedrons R and R + 1 are in a transverse position, the alternating rhombohedrons R and R + 2 in a parallel position, in regard to each other. Yet there are some exceptions in this respect. The position of R, the fundamental form of the rhombohedral system, is considered as the *normal* one, to which the position of all other forms is referred; yet this rhombohedron sometimes appears in a transverse position, whilst other forms, though according to the derivation obtained in the transverse position, nevertheless affect the parallel one in the combinations. If thus one and the same simple form appears in both positions at once in a combination, a remarkable result will be obtained, after the necessary enlargement of their faces, all other faces having been made to disappear. They produce a form contained under equal and similar faces, which assumes the aspect of a simple one (§. 138.), though it is really compound, as results from the process by which it has been obtained.

Nature confirms this second law relative to the position of the forms as the preceding one (§. 139.), in every combination.

§. 141. SYMMETRY OF COMBINATIONS.

The *Symmetry* of combinations is founded upon the two laws in §. 139. and §. 140.

The Symmetry of combinations consists in the sameness of disposition of the faces, edges, and angles of each of those simple forms which they contain, in respect to the homologous parts of the other; or it consists in the sameness of situation of the different edges and angles produced by the combination of these simple forms. Symmetry refers only to combinations, Regularity only to simple forms (§. 45.).

All combinations produced by nature are symmetrical, and experience thus confirms the truth of the two abovementioned laws; since the symmetry of combinations depends upon the relative dimensions and the position of simple forms. These Laws of Combination, and not the Symmetry of the latter, are fundamental laws in Crystallography, because the latter is a mere and necessary consequence of the former.

Sometimes there occur in nature apparent exceptions to this Symmetry. Yet they are merely accidental, and arise from an unequal and disproportionate enlargement of certain faces of Crystallisation; and this sometimes goes so far, as to cause some of these faces entirely to disappear. Sometimes, however, certain faces are enlarged, and others diminished, according to constant laws, and then the symmetry is not destroyed altogether, though it assumes a peculiar character, different from what it has been before. The differences thus produced, are called the *Character of Combinations*, and will be considered afterwards in greater detail.

In considering the combinations, we must abstract from all casual deviations from symmetry, as in like manner has been done with certain irregularities of simple forms (§. 45.), and reduce the combinations themselves to their peculiar symmetry.

§. 142. EDGES OF COMBINATION.

The edges, in which the faces of two different forms contained in a combination, meet or intersect each other, are termed *Edges of Combination*.

Compound forms contain sometimes a great many edges of combination: in this case they are determined and distinguished from each other by attending to the simple forms, between the faces of which they are situated, or whose intersections they represent. What has been said in §. 29. 32. of edges in general, applies likewise to edges of combination.

It sometimes happens that the edges of the simple forms disappear entirely in a compound form, so that every edge to be met with is an edge of combination.

§. 143. DEVELOPEMENT OF COMBINATIONS.

The developement of a combination consists in determining, 1. the kind of all the simple forms contained in it; 2. their peculiar position; and 3. their relations to each other.

It is not difficult to recognise the kind of simple forms contained in a combination. For this purpose, enlarge one set of homologous faces which it contains after the other, till the rest disappear; and the kind of the form will become evident, from their number and disposition. Each of these forms is obtained at the same time in their peculiar position.

The determination of the mutual relations of the forms is in many cases somewhat more circumstantial. They might be derived immediately from their dimensions, comparing them with each other, as obtained from the observation of the angles of incidence at the edges of these simple forms themselves, or at the edges of combination: this process, however, being deficient in geometrical precision, cannot lead to any generality, and is therefore not an appropriate foundation for a scientific method of Crystallography. Besides, it would suppose a great number of measurements, which, in establishing general laws, we must avoid as much as possible, because the crystals themselves are very seldom found in such perfection, and under such circumstances, as to allow of any observations of this kind, upon the correctness of which we might rely.

The method of Crystallography intended for the use of the Natural History of the Mineral Kingdom, and employed in the present work, is founded solely upon the situation of those edges, in which the faces of several simple forms intersect each other; that is to say, of the edges of combination; and this method is therefore independent of all measurement. The situation of those edges is a consequence of the relations among the simple forms, and it is changed as soon as any change takes place in these. It will be possible, therefore, to determine the simple forms contained in the combination, provided the situation of the edges affords sufficient data. Only if these be wanting, it will be necessary to resort to immediate measurement.

The number of data required for this purpose, depends entirely upon the quality of the form itself. Thus a rhombohedron depends upon a single datum; for in order to determine it exactly, nothing is required but to know the relative length of its axis, or what place it occupies in one of the series developed above. At the same time, it appears whether it be a member of the principal series or of a subordinate one. For a scalene six-sided pyramid, two data are required, in order to determine, first, the rhombohedron from which it is derived, and then with what number the axis of this rhombohedron must be multiplied in the derivation. In general, since the situation of any plane is perfectly determined by three points given in it, there is no possible case, in which more than two data are required, each of these data consisting in the situation of a straight line upon one of the faces of that form.

This method of determining the relations of simple forms contained in a combination, is evidently founded upon the knowledge of the series produced by these forms, which have been explained above: by these it acquires a perfect generality, because in every stage the same relations exist among the members.

The developement itself may be effected either *analytically* or *synthetically*. The synthetical method speaks more plainly to the eye, and is therefore particularly recommended to beginners. The analytical method is more easy, elegant, and general. Several examples of the synthetical method are contained in the course of this work; and since it would far exceed its limits to treat of these methods at large, I shall only subjoin a short sketch of the process of the analytical method.

Let ABC, A′B′C′, Fig. 49., represent the faces of two forms of the rhombohedral system, for instance, of two scalene six-sided pyramids, whose horizontal projection is the same, and which are placed in a parallel position. The lines CB, C′B′ will intersect each other in the point G, and six points situated like G will be common to both the forms. The points G, G &c. are situated in a horizontal plane, perpendicular to the axis AX in M, the centre of the form; they are constant in all forms of the rhombohedral system; for though the situation of the points C, C′ and B, B′ may vary, yet this never can have any influence upon their intersection in G. In the other systems of crystallisation, the situation of the points G, G &c. is not invariable; but it may easily be shewn, that this situation depends upon the diagonals of the bases of the forms combined.

The acute terminal edges AC, A′C′, intersect each other in the points G′, G′, &c., which points, therefore, are likewise common to both the forms. The situation of these points is variable, and depends upon the relations of the axes, belonging to the combined forms. If A, or A′, or both at the same time change their places, G′, G′, &c. necessarily must change theirs likewise.

The straight line GG′, joining the points G′ and G, is the intersection of the faces of the two forms, and represents, therefore, their edge of combination (§. 142.); it lies, on that account, both in the plane ABGG′, or in the face ABC of the one, and in the plane A′B′GG′, or in the face A′B′C′ of the other pyramid. Hence it appears that the situation of the line GG′ depends upon the relations of the bases, or of the horizontal projections, and upon those of the axes; or in general, upon the dimensions of the combined forms themselves.

If we now produce the obtuse terminal edges AB and A′B′ of the combined pyramids, till they intersect each other in F; F will again be a point common to both the forms. Hence it follows, that F, G and G′ must be situated in one straight line; and that if the one of the variable points F and G′ moves, the other likewise must be affected by this alteration. This demonstrates the immediate dependence of the situation of F upon the dimensions of the combined forms.

The horizontal plane HZ intersects the obtuse terminal edges of the pyramids in E and E′. The situation of these points, or their distance from the centre M, is likewise variable; but it depends upon the dimensions of the combined forms, exactly as the rest of the variable points, and becomes determined for determined forms.

Thus the length of the lines EF or E′F will be perfectly determined, being a function of the above-mentioned relations.

The line EF or EF′ is termed the *Line of Combination*. Its length can be measured by comparing it with the terminal edge, or with the diagonal, of which it is a part, or in which it lies, if produced to a sufficient length. A single equation is sufficient for expressing this line in the rhombohedral system. Two expressions are required in the pyramidal system, on account of the differences arising from the parallel and the diagonal position of the forms themselves, and of their horizontal projection, which differences cannot be comprehended in a single formula. A single equation again is sufficient in the prismatic system, where there exists no such difference.

These equations contain every possible case in respect to the kind and the position of forms within one and the same system, and to certain differences in the edges of combination, in as much as these may be produced by faces contiguous either to the same, or to different apices; and which again belong either to that side which may be conceived to be turned towards the observer, or to the opposite side of the forms under consideration. These differences are expressed in the equations by the addition of the signs + and —.* The possibility of thus comprehending every binary combination of a system in one, or, at the utmost, in two expressions, is at the same time the most convincing proof of the simplicity and generality of the method.

I shall now shortly explain the use of the Line of Combination, in the developement of compound forms, which is here reduced to the determination of the relations among the simple forms contained in the compound one, that are already known as to their kind and position.

Let ABC, A′B′C′, Fig. 50., represent the faces of the same forms as in Fig. 49.; and the points G, G′, F be identical with those in the same figure which are marked by the same letters. Combine now a third form with these, whose face is A″B″C″, and whose dimensions are such as to have the points G and G′ common to all these three forms. The points G must always keep the same place, if we suppose the horizontal projection to be equal. The edge of combination thus produced between the new form, and any one of the others, will evidently coincide with that

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* Vide *Gilbert's Annalen der Physik.* 1821. 8., where these formulae have been published, along with examples of their application.

in which the faces of the two first forms meet. Hence the algebraic expression of the Line of Combination must be exactly the same for the third form and the first, or the third and the second, which is found when calculating the Line of Combination for the first and the second form among themselves.

After these preparations, the length of the Line of Combination EF or EF′ must be calculated from the known or given dimensions of the two first forms. An expression will be obtained for it, finite or infinite, positive or negative, in which every single quantity is determined.

The same line EF or EF′ must also be calculated from the dimensions, either of the first and the third, or of the second and the third form. An expression similar to that mentioned above will be found, in which every thing is known, except what refers to, or is dependent upon, the dimensions of the third form, to ascertain whose relations is the object of the inquiry. The two values of EF, or EF′ being equal to each other, they may be joined in an equation to be resolved for one of the unknown quantities.

If this equation contains only one unknown quantity, the resolution yields its value at once, and by this the perfect determination of the third form is obtained. But if there are two unknown quantities, the process must be repeated, by constructing another equation, for a case in which the unknown form which is to be determined combines in the same way with other known forms; the second equation is procured in the same way as the first.

In the application of this process, we may take advantage of many circumstances, which may very often render the second equation superfluous; this, however, is so obvious in every particular case, that it will not be necessary here to enter upon this subject more at large.

§. 144. MAGNITUDE OF THE EDGES OF COMBINATION.

To a complete knowledge of a combination is also required the knowledge of the angles at which the faces of different forms intersect each other, or of the edges of combination.

In every particular case, the magnitude of the edges of combination may easily be calculated from the dimensions of the simple forms. There is, however, also a general solution of this problem, effected in every system by the same number and kind of equations as those for the Line of Combination.* Those parts by which the simple forms are determined, enter into these equations as variable quantities, whose real value is found by the developement (§. 143.). These values being substituted instead of the variable quantities, we obtain trigonometrical functions for the Edge of Combination.

The application of these equations pre-supposes the dimensions of one of the forms to be known. These dimensions must be found by immediate measurement, whenever the form belongs to one of the systems whose dimensions are variable. If in a species one of the forms, for instance, the fundamental one, is known in respect to its dimensions, no new measurement is required for the combinations of this species, provided the situation of the edges contain sufficient data for their developement. Hence it appears that we must endeavour to ascertain the dimensions of the fundamental form, with the utmost accuracy, but at the same time also that the measurement of different forms of the same series may be useful in correcting each other. This subject, however, will be treated of more at large in the *Elementary Treatise on Crystallography*.

The designation of compound forms must be founded upon the relations of the simple forms among each other. It will therefore be sufficient to indicate, by their peculiar signs, the simple forms, in order to express the combinations

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* *Gilbert's Annalen*, l. c.

which they produce. The signs of the simple forms follow each other in the designation of the compound one in a certain order, so as to let those precede which refer to forms whose faces are more inclined to the axis, and those follow whose faces are less inclined or parallel to the same line. The signs of two different forms are separated by a full stop. The sign of a combination of R and R + 1 will therefore be

$$R. \quad R + 1.$$

An example of the designation of a compound form containing a greater number of simple ones will be found in §. 148.

From the preceding observations it is evident, that the designation contains every thing required for calculating the combinations.

OF THE COMBINATIONS OF THE DIFFERENT SYSTEMS OF CRYSTALLISATION.

§. 145. RHOMBOHEDRAL COMBINATIONS.

A combination of the rhombohedral system is more particularly said to possess a *Rhombohedral Character*, if each of the simple forms contained in it appears only in one position, but with the whole number of their faces.

Hence a combination of rhombohedrons, six-sided pyramids, and prisms, exhibits a rhombohedral character, if these forms are found in only one position, and if those which cannot assume any other position, as the isosceles six-sided pyramid, and the six-sided and the twelve-sided prisms, appear with the number of faces peculiar to them. In all other cases the combinations receive a particular denomination, provided the changes which take place in respect to the enlargement of some of their faces, are not owing to an accidental irregularity (§. 31.). In general, and supposing, for the sake of brevity, only one series of rhombohedrons, the binary combinations of this system are

i. R + n. R + n′,
ii. R + n. $(P + n')^{m'}$,
iii. R + n. P + n′,
iv. $(P + n)^{m}$. $(P + n')^{m'}$,
v. $(P + n)^{m}$. P + n′,
vi. P + n. P + n′.

Some of the most common and remarkable of these combinations may be shortly noticed.

i. R + n. R + n′.

1. Let n′ be = n $\pm$ 1. The two forms are consecutive members of the series §. 110., and as such in a transverse position to each other. The edges of combination which they produce are parallel among themselves, and at the same time also to the terminal edges of the more acute rhombohedron, and to the inclined diagonals of the more obtuse one. *Example*, R — 1 (*n*) and R (*P*), or R (*P*) and R + 1 (*r*) in rhombohedral Kouphone-spar. Vol. II. Fig. 120. We may also argue inversely: when two rhombohedrons join in a transverse position, and produce edges of combination of the above mentioned kind, that is to say, parallel to each other, parallel to the terminal edges of the more acute rhombohedron, and parallel also to the inclined diagonals of the more obtuse rhombohedron, the relations between the two forms must be the same as those between two consecutive members of the series §. 110. This is a direct consequence from the derivation of those forms (§. 108.).

2. Let n′ be = n $\pm$ 2 r, where r may be any whole number. In this case an odd number of members of the series §. 110. is wanting between the two combined forms, the forms therefore are in a parallel position, and the edges of combination produced are horizontal. *Example*, R (*P*) and R + 2 (*m*) in rhombohedral Lime-haloide. Vol. II. Fig. 115. A similar result is obtained from rhombohedrons

belonging to different series (§. 116), provided they are in a parallel position. Any two rhombohedrons in a parallel position intersect each other in horizontal edges, and their dimensions therefore cannot be determined solely from a combination of this kind.

The reason why the edges of combination here are horizontal is evident, because the horizontal projections being supposed equal, those faces of two rhombohedrons in a parallel position, which are contiguous to the same apex,* must intersect each other in the line GG (§. 143.); and this line itself is horizontally situated in the upright combination.

3. Let n′ be $= -\infty$, R + n′ therefore $=$ R $- \infty$. This form appears as a face perpendicular to the axis of the rhombohedron R + n (§. 111.), and the resulting edges of combination must evidently be horizontal, whatever value n may affect. *Example*, R $- \infty$ (*o*) and R (*P*) in rhombohedral Alum-haloide. Vol. II. Fig. 111.

4. Let n′ be $= + \infty$. Upon this supposition, R + n′ is $=$ R $+ \infty$, or a regular six-sided prism, the limit of the series of rhombohedrons (§. 111.). The edges of combination produced by its alternating faces with those of the upper apex of R + n are horizontal; so are the similarly situated ones between the rest of its faces and the faces of the rhombohedron contiguous to the lower apex; and their situation is independent of the dimensions of R + n (2.). There are also inclined edges of combination to be found here, which depend upon R + n. If we consider the regular six-sided prism as a rhombohedron of an infinite axis (§. 111.), they are produced by the intersection of those of its faces which are contiguous to the lower apex, and those faces of R+ n which are contiguous to the upper apex

* It would lead us too far to consider more at large the intersections of faces contiguous to opposite apices, and the edges of combination which they produce. A few examples contained in the subsequent part of the work, will shew their application.

of the combination, and *vice versa*. *Ex.* R (*P*) and R + ∞ (*c*) in rhombohedral Lime-haloide. Vol. II. Fig. 114. The cosine of the angle at the horizontal edge, is equal to double the cosine of the angle at the inclined edge of combination.

From the horizontal edges of combination between a rhombohedron and a regular six-sided prism, we may infer that the latter is R + ∞ and not P + ∞ (§. 118.). The demonstration of this depends upon §. 111.

ii. $R + n. \quad (P + n')^{m'}$.

1. Let n′ be = n. Upon this supposition the forms become co-ordinate (§. 112.). The edges of combination which they produce are parallel to the edges of the rhombohedron, and to the lateral edges of the pyramid, whatever be the value of m′. The figure of the faces of the rhombohedron remains a rhomb, and they appear contiguous to, or in the place of the apices of the pyramid. *Ex.* R (*P*) and $(P)^3$ (*r*), or R (*P*) and $(P)^5$ (*y*) in rhombohedral Lime-haloide. Vol. II. Fig. 116. From the rhombic figure of the faces, which is a consequence of the situation of the edges of combination, follow the relations of the combined forms, as is immediately evident from the derivation (§. 112.).

2. Let n or n′ be = — ∞. One of the forms becomes = R — ∞, and the edges of combination are horizontal (i. 3.).

3. Let n be = + ∞; R + n therefore = R + ∞. The figure which the faces of this prism assume in the combination with a pyramid, is that of an irregular tetragon, which may be divided by a horizontal line into two isosceles triangles. The relative heights of these triangles are to each other in the ratio of m′ — 1 : m′ + 1. The more obtuse triangle is produced by the intersection of the faces of R + ∞ with the upper faces of $(P + n')^{m'}$, while the more acute one results from the intersection of the same faces of R + ∞ with the lower ones of $(P + n')^{m'}$. *Ex.* R + ∞ (*c*) and $(P)^5$ (*y*) in rhombohedral Lime-haloide. Vol. II. Fig. 116. The figure of the faces of the regular six-sided prism *c* at once indicates it to be R + ∞, and not P + ∞.

4. Let n' be $= n - 1$, and $m' = 3$. The combination is $= R + n. (P + n - 1)^3$. Under these circumstances, the forms are in a transverse position, because $R + n - 1$, the rhombohedron from which the pyramid is derived, is itself in a transverse position towards $R + n$. The faces of $R + n$ have the situation of the more acute terminal edges of the pyramid; and the edges of combination are parallel to each other, to the above mentioned acute terminal edges, and to the inclined diagonals of the rhombohedron *Ex.* $R - 1$ (*z*) and $(P - 2)^3$ (*t*) in rhombohedral Ruby-blende. Vol. II. Fig. 126. Inversely from the situation of the edges, in which the faces of the two forms meet in the given position, we may infer the above mentioned relation to exist between the two combined forms.

In order to demonstrate this, let ABXC, Fig. 47., be the principal section of the rhombohedron, from which the pyramid is derived, AX its axis, and M𝔄 half the axis of the pyramid: 𝔄C becomes its acute terminal edge, and at the same time the inclined diagonal of the rhombohedron, whose plane touches the pyramid in this terminal edge, if the horizontal projections of the two forms are equal. Let now $a = AX$, be the axis of that rhombohedron, from which the pyramid is derived; and a' the axis of the rhombohedron sought; it follows in respect to the pyramid, that

$$\mathfrak{A}P = \frac{3\,m' - 1}{6}.\ a,$$

in respect to the rhombohedron, that

$$\mathfrak{A}P = \tfrac{2}{3}.\ a';$$

and on account of the equality of both expressions, that

$$\frac{3\,m' - 1}{6}.\ a = \tfrac{2}{3}.\ a',$$

$$\text{and } a = \frac{4}{3\,m' - 1}.\ a'.$$

If now we suppose $m' = 3$,

$$a \text{ becomes} = \tfrac{1}{2}.\ a', \text{ or } a' = 2.\ a.$$

and $n' = n - 1$.

But, let m' be $= 2$; the result, obtained in the same way, will be

$$a = \tfrac{4}{5}.\ a', \text{ or } a' = \tfrac{5}{4}.\ a = \tfrac{5}{4}.\ 2^0.\ a\ ;$$

and $R + n = \frac{5}{4} R + n'$, or that member of the first subordinate series, which belongs to $R + n'$.

Let now m' be $= 5$; we find that

$$a = \tfrac{2}{7}.\ a', \text{ or } a' = \tfrac{7}{2}.\ a = \tfrac{7}{4}.\ 2^1.\ a,$$

and $R + n = \frac{7}{4} R + n' + 1$, or that member of the second subordinate series, which belongs to $R + n' + 1$.

Each of the two co-efficients, $\frac{5}{4}$ and that immediately following $\frac{7}{4}$, determines a particular subordinate series, which may be distinguished by the name of *the first* and *the second.* In itself it is quite arbitrary which of their members are considered to be in the nearest relation to members of the principal series. But it is very useful to fix upon a certain member, and this has been done here by supposing, that, when the axis of $R + n$ of the principal series is $= 2^n.\ a$, that of $R + n$ of the first subordinate series is $\frac{5}{4}.\ 2^n.\ a$, and that of $R + n$ of the second subordinate series $\frac{7}{4}.\ 2^n.\ a$. Members of these dimensions are said to belong together, or to be co-ordinate (§. 116.).

5. Let n' be $= n - 2$ and $m' = 5$. The sign of the combination will be $R + n.\ (P + n - 2)^5$. Under these circumstances, the forms are in a parallel position. The faces of the rhombohedron appear in the place of the more obtuse terminal edges of the pyramid. The edges of combination are parallel with each other, with the above mentioned terminal edges of the pyramid, and with the inclined diagonals of the rhombohedron. *Ex.* $R + 2$ (m) and $(P)^5$ (y) in rhombohedral Lime-haloide. Vol. II. Fig. 116. We may infer inversely from this situation of the edges, that the above mentioned relations really take place.

For, making again use of Fig. 47., $\mathfrak{A}B$ will represent the obtuse terminal edge of the pyramid, but at the same time also the inclined diagonal of that rhombohedron, whose faces touch the more obtuse terminal edges of the pyramid, their horizontal projections always being supposed equal. Hence, for the pyramid, we have

$$\mathfrak{A}Q = \frac{3\ m' + 1}{6}.\ a,$$

for the rhombohedron

$$\mathfrak{A}Q = \tfrac{2}{3}.\ a';$$

therefore

$$\frac{3\ m' + 1}{6}.\ a = \tfrac{2}{3}.\ a', \text{ and } a = \frac{4}{3\ m' + 1}.\ a'.$$

If now we suppose $m' = 5$,

$$a \text{ becomes } = \tfrac{1}{4}.\ a', \text{ or } a' = 4.\ a,$$

and $n' = n - 2$.

If here we suppose $m' = 3$, we find

$$a = \tfrac{2}{5}.\ a';\ a' = \tfrac{5}{2}.\ a = \tfrac{5}{4}.\ 2^1.\ a,$$

and $R + n = \frac{5}{4}\ R + n' + 1$, the member of the first subordinate series belonging to $R + n' + 1$.

For $m' = 2$, follows

$$a = \tfrac{4}{7}.\ a' \text{ or } a' = \tfrac{7}{4}.\ a = \tfrac{7}{4}.\ 2^0.\ a,$$

and $R + n = \frac{7}{4}\ R + n'$, that member of the second subordinate series, which belongs to $R + n'$.

6. Let n' be $= n - 2$, $m' = 3$. The combination is $R + n.\ (P + n - 2)^3$. The forms are in a parallel position; the more acute terminal edges of the pyramid coincide with the terminal edges of the rhombohedron, and the edges of combination are parallel as well among themselves as also with both the mentioned terminal edges. *Ex.* R (*P*) and $(P - 2)^3$ (*t*) in rhombohedral Ruby-blende. Vol. II. Fig. 126. From this situation and position we may inversely conclude, that the given relations really take place among the forms.

For let $\mathfrak{A}C$ be the more acute terminal edge of the pyramid, in which the terminal edge of the rhombohedron is situated. We shall have for the pyramid

$$\mathfrak{A}P = \frac{3\ m' - 1}{6}.\ a;$$

for the rhombohedron

$$\mathfrak{A}P = \tfrac{1}{3}.\ a',$$

and therefore $$\frac{3\ m' - 1}{6}.\ a = \tfrac{1}{3}.\ a'.$$

Now m' being $= 3$, we obtain

$$a = \tfrac{1}{4}.\ a', \text{ or } a' = 4.\ a = 2^2.\ a,$$

and $R + n = R + n' + 2$.

If m' is supposed $= 2$, we find

$$a = \tfrac{2}{5}.\ a', \text{ or } a' = \tfrac{5}{2}.\ a = \tfrac{5}{4}.\ 2^1.\ a$$

and $R + n = \frac{5}{4} R + n' + 1$.

The number 5 substituted for m′, gives

$$a = \tfrac{1}{7}.\ a', \text{ or } a' = 7.\ a = \tfrac{7}{4}.\ 2^2.\ a,$$

and $R + n = \frac{7}{4} R + n' + 2$.

7. Let n′ be $= n - 3$, $m' = 5$. The combination is $R + n.\ (P + n - 3)^5$. The forms are in a transverse position. The more obtuse terminal edges of the pyramid are parallel to the terminal edges of the rhombohedron, and at the same time also to the edges of combination arising from the intersection of their faces.

Suppose the horizontal projections of the forms to be the same; the terminal edge of the rhombohedron will lie in the more obtuse terminal edge of the pyramid. Hence for the pyramid we have

$$\mathfrak{A}Q = \frac{3\ m' + 1}{6}.\ a;$$

for the rhombohedron

$$\mathfrak{A}Q = \tfrac{1}{3}.\ a',$$

and consequently $\dfrac{3\ m' + 1}{6}.\ a = \tfrac{1}{3}.\ a'$.

For $m' = 5$ this expression gives

$$a = \tfrac{1}{8}.\ a', \text{ or } a' = 8.\ a = 2^3.\ a,$$

and $R + n = R + n' + 3$;

for $m' = 3$,

$$a = \tfrac{1}{5}.\ a', \text{ or } a' = 5.\ a = \tfrac{5}{4}.\ 2^2.\ a,$$

and $R + n = \frac{5}{4} R + n' + 2$;

for $m' = 2$,

$$a = \tfrac{2}{7}.\ a', \text{ or } a' = \tfrac{7}{2}.\ a = \tfrac{7}{4}.\ 2^1.\ a,$$

and $R + n = \frac{7}{4} R + n' + 1$.

iii. $R + n.\ P + n'$.

1. For $n' = n$, the combination is $R + n.\ P + n$. The forms are co-ordinate, and the faces of the pyramid appear in pairs in the place of the terminal edges of the rhombohedron. The edges of combination are parallel to these, to the alternating terminal edges of the pyramid, and among each other. In a combination, which, besides these

two forms, contains P + ∞ (4.), the faces of the rhombohedron retain their rhombic figure; and this may be considered as a proof of the existence of the above mentioned relations. This, however, follows immediately from the derivation (§. 117.). *Ex.* R (*s*) and P (*P*, *z*) in rhombohedral Quartz. Vol. II. Fig. 145.

2. Let n′ be = n + 1. The combination is R + n. P + n + 1. The faces of the rhombohedron appear instead of the alternating terminal edges of the pyramid, and the edges of combination are parallel to each other, to the alternating terminal edges of the pyramid, and to the inclined diagonals of the rhombohedron. *Ex.* R (*P*) and P + 1 (*r*) in rhombohedral Corundum. Vol. II. Fig. 121. From this parallelism again follows the equal inclination upon the axis of the alternating terminal edges in the pyramid, and of the diagonals in the rhombohedron, and from this the relations existing among the axes of the two forms.

3. n or n′ = — ∞ produces horizontal edges of combination (i. 3.).

4. Let n′ be = + ∞, the combination R + n. P + ∞. The regular six-sided prism P + ∞ is the limit of the series of isosceles six-sided pyramids (§. 118.), and its faces appear with parallel edges of combination in the place of the lateral edges of R + n, or the faces of the rhombohedron terminate the regular six-sided prism, their figure being that of a rhomb. *Ex.* R + 1 (*r*) and P + ∞ (*s*) in rhombohedral Emerald-malachite. Vol. II. Fig. 118. This rhombic figure of the faces, or the situation of the edges in general, affords the means of distinguishing a combination of R + n and P + ∞ from a combination of R + n and R + ∞ (i. 4.). The difference depends upon the position of P + ∞ in respect to R + n, and P + n (§. 118.).

5. Let n be = + ∞. The combination is R + n. P + n′. The faces of R + ∞ appear as rhombs in place of the lateral angles of P + n′. This figure by itself is a sufficient character for distinguishing the prism R + ∞ in this combination from P + ∞.

6. If n and n′ at the same time are = + ∞, the com-

bination therefore $R + \infty . P + \infty$; the result is an equiangular, and if the faces are of equal extent, it is a regular twelve-sided prism; because the faces of one of the six-sided prisms appear under equal inclinations in the place of the edges of the other prism, their edges of combination being parallel to each other and to the axis. *Ex.* $R + \infty$ (e) and $P + \infty$ (M) in rhombohedral Fluor-haloide. Vol. II. Fig. 149. Hence the twelve-sided prism is a compound form, and not a simple one. Also this combination depends upon the different situations of the two prisms.

iv. $(P + n)^m . (P + n')^{m'}$.

1. Let n' be $= n$. The combination is $(P + n)^m . (P + n)^{m'}$; the forms will be co-ordinate scalene six-sided pyramids. The faces of the more acute pyramid are situated in the place of the lateral edges of the more obtuse one. The edges of combination are parallel to each other and to the lateral edges of both the pyramids. *Ex.* $(P)^3$ (r) and $(P)^5$ (y) in rhombohedral Lime-haloide. Vol. II. Fig. 116. If, on the contrary, the edges assume the mentioned position, we may infer that n is $= n'$, which is exactly what follows from their derivation.

2. Let m' be $= m$, the combination will be $(P + n)^m . (P + n')^m$. Suppose at the same time the forms to be in a parallel position, or $n' = n \pm 2r$ (i. 2.). Under these circumstances, the edges of combination which they produce become horizontal. For the transverse section of one of these pyramids is similar to the transverse section of the other (§. 113.), and therefore a plane passing through those edges in which the faces of the two forms meet, will be perpendicular to their axis. *Ex.* $(P - 2)^3$ (t) and $(P)^3$ (r) in rhombohedral Lime-haloide. Vol. II. Fig. 129. This situation of the edges cannot take place, if the scalene six-sided pyramids are in a transverse position. From such horizontal edges, therefore, we infer not only the parallel position of the two forms, but also that they are derived according to the same m, or that m' is $= m$.

3. The horizontal situation of those edges is not altered,

if n or n′ should become infinite and negative, whatever may be the value of m and m′.

4. But let m′ be = m, and n′ or n = + ∞: the combination therefore $(P + \infty)^m . (P + n')^m$ or $(P + n)^m . (P + \infty)^m$. The edges of combination between those faces of the two forms, which are contiguous to the same apex, must here likewise be horizontal, because the unequiangular twelve-sided prism is the limit of the series of pyramids, and as such contains an identical transverse section (§. 115.). *Ex.* $(P)^{\frac{5}{3}}$ (*u*) and $(P + \infty)^{\frac{5}{3}}$ (*c*) in rhombohedral Fluor-haloide. Vol. II. Fig. 148. The intersections of the faces from two different apices, however, assume an inclined situation, dependent upon the dimensions of the simple forms (i. 4.). The inferences drawn in 2. extend also to the present case.

5. In the series of scalene six-sided pyramids,
$\dots (P + n)^5, (P + n + 1)^3, (P + n + 2)^2, (P + n + 1)^5 \dots$
the law of progression is evident. Instead of n any whole number, positive or negative, may be substituted, and the series arbitrarily continued on either side. If now from the above mentioned series we select a combination of any two subsequent members, as $(P + n)^5$ and $(P + n + 1)^3$, or $(P + n + 1)^3$ and $(P + n + 2)^2$, &c.; the obtuse terminal edges of every more obtuse pyramid appear in the place of the acute terminal edges of the more acute member, the edges of combination being parallel to each other, and to the mentioned terminal edges of the two pyramids. Examples occur in rhombohedral Lime-haloide, between $(P)^3$ (*r*) and $(P + 1)^2$ (*x*), &c.

The situation of the edges is a consequence of the transverse position of every two subsequent members of the above mentioned series, and of its peculiar property, that the more obtuse terminal edge of every lower member is inclined to the axis at the same angle as the more acute one of the higher member. We have in Fig. 47.

$$\sin. \mathfrak{A}BQ = \frac{(3\,m + 1)\,2^n.\,a}{\sqrt{[(3\,m + 1)^2.\;2^{2n}.\;a^2 + 36]}};$$

$$\sin. \mathfrak{A}CP = \frac{(3\,m' - 1)\,2^{n'}.\,a}{\sqrt{[(3\,m' - 1)^2.\;2^{2n'}.\;a^2 + 36]}}.$$

If in the hypothesis of $(P + n)^5. (P + n + 1)^3$, we substitute in sin. $\mathfrak{A}BQ$ the number 5 instead of m, and in sin. $\mathfrak{A}CP$ the expression $n + 1$ for n', and 3 for m'; we obtain

$$\text{sin. } \mathfrak{A}BQ = \frac{16.\ 2^n.\ a}{\sqrt{(16^2.\ 2^{2n}.\ a^2 + 36)}};$$

$$\text{sin. } \mathfrak{A}CP = \frac{8.\ 2.\ 2^n.\ a}{\sqrt{(8^2.\ 4.\ 2^{2n}.\ a^2 + 36)}};$$

two values which are equal.

If the above mentioned relations take place among the consecutive members of the series, the parallelism of the edges of combination always must follow. But these relations are not a necessary consequence of that parallelism, and many pyramids really exist and produce in their regular positions, parallel edges of this kind; and yet they are not derived from rhombohedrons of the same series at all, or according to other values of m, than those of 2, 3, and 5. This is the case in the combinations of $(P - 1)^3$ (*a*) and $(\frac{5}{4} P - 1)^3$ (*b*) in rhombohedral Ruby-blende. Vol. II. Fig. 126. In the above mentioned series, one datum has been assumed, upon which the rest is dependent. If this be changed, the consequences also must be altered, without, however, in the least affecting the parallelism of the edges of combination. Supposing the pyramids to be derived from the members of the principal series, we may argue with perfect security, from the parallelism of the edges to the values of m, and determine the one whenever we know the other. Restrictions of this kind frequently occur in general solutions of crystallographic problems.

v. $(P + n)^m.\quad P + n'$.

1. Let m be $= 5$, $n' = n + 3$. We have the combination $(P + n)^5.\ P + n + 3$. The faces of the scalene pyramid meeting in its obtuse terminal edges, appear in the place of the alternating terminal edges of the isosceles one. The edges of combination are parallel to each other, and to the above mentioned terminal edges of the two forms. For on these suppositions the inclination of the terminal edges of

the isosceles pyramid to the axis, is equal to the inclination of the more obtuse terminal edges of the scalene pyramid to that line. For the latter we have (iv. 5.)

$$\sin. \mathfrak{A}BQ = \frac{(3\,m + 1)\, 2^n.\, a}{\sqrt{[(3\,m + 1)^2.\, 2^{2n}.\, a^2 + 36]}};$$

and for the former

$$\sin. \mathfrak{A}BQ = \frac{m'.\, 2^{n'}.\, a}{\sqrt{(m'^2.\, 2^{2n}.\, a^2 + 4)}};$$

two values which become equal, if in that for the scalene pyramid we substitute 5 instead of m, and in that for the isosceles pyramid the expressions, $\frac{2}{3}$ instead of m, and n + 3 instead of n′. Examples occur in $(P - 2)^5$ and P + 1 (*n*) of rhombohedral Iron-ore.

2. Let m be = 3, n′ = n + 2; the combination will be $(P + n)^3.\ P + n + 3$. In this case, similar to the preceding, the faces of the isosceles pyramid appear in the place of the more acute terminal edges of the scalene one; the edges of combination being parallel among each other, and to the above mentioned terminal edges, from the same reason as in 1., because the inclinations of the terminal edges to the axis are equal in these two pyramids. This may be shewn by effecting the necessary substitutions in the values of sin. $\mathfrak{A}$CP, which here takes the place of sin. $\mathfrak{A}$BQ.

3. For n or n′ = − ∞ all the edges of combination become horizontal (i. 3.).

4. Let n′ be = + ∞. The combination is $(P + n)^m.\ P + \infty$. The edges of combination are parallel to the lateral edges of the pyramids. *Ex.* $(P)^3$ (*h*) and P + ∞ (*n*) in rhombohedral Ruby-blende. Vol. II. Fig. 126. The situation of the edges distinguishes P + ∞ from R + ∞ in ii. 3.

vi. P + n. P + n′.

1. The edges of combination are always horizontal, whatever may be the value of n or n′, even though this be + ∞ or − ∞. *Ex.* P + 1 (*r*) and P + 2 (*b*) in rhombohedral Corundum. Vol. II. Fig. 122.

§. 146. DI-RHOMBOHEDRAL COMBINATIONS.

A combination of the rhombohedral system is said to possess a *Di-rhombohedral Character*, if one or more of the simple forms contained in it appear at once in both their positions (§. 108.).

The rhombohedrons and the scalene six-sided pyramids are the only forms of this system which may assume two different positions; for if the position of the other forms is supposed to change, their faces resume the situation they had before, or, properly speaking, another face exactly takes the place of that which just has been removed from it.

1. Suppose, therefore, in §. 145. i., n′ to be = n, or the combination, R + n. — R + n. The sign — indicates, that the two rhombohedrons are in a transverse position to each other. The combination assumes the aspect of an isosceles six-sided pyramid (§. 35.), which is a simple form, and the edges of combination are parallel to those lines, which, in the faces of the single rhombohedrons, join the apices with the centres of the lateral edges. This form is now designated by the name of a *Di-rhombohedron*, and its crystallographic sign is either as above R + n. — R + n, or it is 2 (R + n). The difference of this form from the isosceles six-sided pyramids consists in the position, and in the relations existing between their axes and horizontal projections. From the di-rhombohedrons the combinations are denominated, in which these forms occur.

Let the terminal edge of the rhombohedron be = x, that of the di-rhombohedron = C, being the edge of combination; we obtain the formulæ

$$\cos. x = 3 \cos. C + 2,$$
$$\cos. C = \frac{\cos. x - 2}{3};$$

which are useful for finding the dimensions of any di-rhombohedron from those of its rhombohedron, and *vice versa.*

In §. 145. iii. 1., the faces of the isosceles six-sided pyramids are transformed into rhombs, if, instead of R + n, 2 (R + n) enters into the combination.

2. If in §. 145. iv., n′ is made = n, and m′ = m, the combination is $(P + n)^m . — (P + n)^m$; the forms are considered here in two different positions. The combination assumes the aspect of a *scalene twelve-sided Pyramid*, Fig. 51.; the edges of combination are parallel to lines which, in the simple pyramids, join the apices with the centres of the lateral edges. A form of this kind receives the name of a *Di-pyramid*, and is designated by $(P + n)^m . — (P + n)^m$ or by $2\,((P + n)^m)$. The di-pyramids form part of the di-rhombohedral forms, and combinations, which contain them, are likewise considered as di-rhombohedral.

Di-rhombohedral combinations occur in rhombohedral Emerald. Vol. II. Fig. 150.

§. 147. HEMI-RHOMBOHEDRAL AND HEMI-DI-RHOMBOHEDRAL COMBINATIONS.

A combination of the rhombohedral system is said to be *hemi-rhombohedral*, if only *half* the number of the faces appear of some of the simple forms which it contains. The combination is termed *hemi-di-rhombohedral*, if one or several of the *di-rhombohedral* forms constituting it, enter with only half the number of their faces into the combination.

It has already been observed (§. 141.), that such combinations are perfectly symmetrical: hence appearances of this kind are by no means in opposition to the symmetry of the combinations.

The rhombohedron itself cannot assume a hemi-rhombohedral appearance in the same way as other forms of this system, because three faces cannot be distributed symmetrically on two different apices.

If in a scalene six-sided pyramid we enlarge the alternating faces contiguous to one of the apices, the symmetry will require us also to enlarge three alternating faces on the opposite side; but the enlarged faces may be either those which are parallel to the former, or those which are not parallel to them.

The first process produces a form exactly similar to a rhombohedron, if considered as a geometrical solid, which nevertheless cannot be considered as a rhombohedron in Crystallography, because in a compound form its faces can never assume the position of the faces of a rhombohedron. A combination in which one or several such forms appear, is more particularly designated by the expression of a hemi-rhombohedral form *of parallel faces.* In a combination of this kind, particular attention must be given to the situation of the faces, in as much as two such forms similar to a rhombohedron arise from the resolution of a single pyramid, whose faces are situated either to the right or to the left of a face of the fundamental form, or of any other complete form contained in the combination.

The second process yields two forms, contained under irregular trapezoidal faces, which on that account are called *three-sided Trapezohedrons.* They are equal and similar, but distinguished from each other by the character of being twisted as it were, to the Right or to the Left (§. 67. 4.). Fig. 53. represents a Right Trapezohedron, which is produced by the enlargement of the faces, *a*, *a*, &c. Fig. 11, while Fig. 54. shews a Left one, which is contained under the faces *b*, *b*, &c. of the same pyramid. A combination partly or entirely consisting of such forms, is termed a hemi-rhombohedral one *of inclined faces.* The contorted aspect of the trapezohedrons extends likewise to these combinations.

The same process applied to the isosceles six-sided pyramid, gives in the first case, or by means of the enlargement of parallel faces, forms likewise similar to a rhombohedron, which yet, for the reasons mentioned above, cannot be considered as rhombohedrons. If, however, we enlarge those faces, which are not parallel, the result is an *isosceles three-sided Pyramid*, Fig. 52. The faces of those rhombohedron-like forms, as well as those of the three-sided pyramids, keep the situation of those of the isosceles six-sided pyramid, from which they are derived, and appear as such in the combinations; and this is the character by which they may be recognised. The combinations themselves receive the same denominations as above.

The regular six-sided prism $P + \infty$ may assume a hemi-rhombohedral aspect, like a finite isosceles six-sided pyramid. In this case its faces must be symmetrically distributed in the combination, like those of an equilateral three-sided prism in a position which is characteristic and peculiar to it.

If the unequiangular twelve-sided prism enters a hemi-rhombohedral combination, its faces likewise must be symmetrically distributed, which comprises two cases. By enlarging the alternating faces, it will appear either as a regular six-sided prism, different from $R + \infty$ and $P + \infty$ by its position; or by enlarging the alternating pairs of faces, it assumes the aspect of a six-sided prism, whose alternating angles only are equal.

The rhombohedron itself may produce a hemi-di-rhombohedral combination, but only one of inclined faces, because if the parallel faces of the di-rhombohedron are enlarged, the rhombohedron will be reproduced, and the combination itself will be simply rhombohedral (§. 145.). The result is an isosceles three-sided pyramid, differing both in position and dimensions, from that which may be obtained from the isosceles six-sided pyramid. In the same way the di-pyramid (§. 146.) also enters into a hemi-di-rhombohedral combination, although its halves and fourths may appear both as forms of inclined faces, and as forms of parallel faces. For if we enlarge the alternate faces contiguous to the upper apex of the di-pyramid, and those contiguous to the lower apex, which are parallel to the former, two forms will arise, which, though exactly similar to isosceles six-sided pyramids, are yet different from these forms in respect to the situation of their faces. Combinations of this kind are said to be hemi-di-rhombohedral of parallel faces. The characteristic form of the hemi-di-rhombohedral combinations of inclined faces is a *six-sided Trapezohedron*, a solid contained under twelve equal and similar trapezoidal faces. Pairs of them are obtained by enlarging the alternating faces contiguous to one of the apices, and those of the other which are not parallel to them; they shew, like some of the forms considered above, the differences of Right and Left. A Right Trapezohedron of this kind is represented in Fig. 57., a Left one in Fig. 58. The first is obtained by enlarging the faces, a, a; and a', a', &c.; the second, by enlarging the faces marked b, b; and b', b', &c.

The three-sided trapezohedrons, Figs. 53. and 54., are obtained by enlarging all those faces of the six-sided ones which are marked a, a, &c. or b, b, &c. in Fig. 51.; but if we enlarge the faces a, a, &c. or b, b, &c. contiguous to the upper apex, and the faces a', a', &c. or b', b', &c. contiguous to the lower apex, forms will result, like Fig. 55. and Fig. 56., and which are right and left trapezohedrons like the former, but which contain faces of the scalene six-sided pyramids in both positions. The situation of these faces determines their existence in the combinations.

In order to re-obtain the simple forms from a di-pyramid, it is necessary to enlarge the alternating pairs of faces, a and b, or those which meet in the obtuse terminal edges of the six-sided pyramid from the upper apex, and the alternating ones, a and b from the lower apex. If, on the contrary, we enlarge those on the lower apex which produce with the former horizontal edges of combination like a' and b', the result will be scalene six-sided pyramids, whose bases are hexagons of alternately equal angles, similar to the figure produced by a section perpendicular to the axis which does not intersect the lateral edges. Simple minerals may assume this form, although they have as yet not been found in nature; but they are very common in compound minerals, and as such they will be the subject of farther investigations.

Hemi-rhombohedral and hemi-di-rhombohedral combinations occur in rhombohedral Fluor-haloide, and in rhombohedral Quartz.

The mode in which the rhombohedrons themselves, and the limits of their series, produce hemi-rhombohedral combinations, is particularly remarkable, and very distinct from what we have seen till now. Here *all* the faces belonging to one apex enter into the combination, whilst *all* those contiguous to the opposite apex disappear. The results are combinations of a dissimilar configuration in their opposite terminations. The crystallisations of rhombohedral Tourmaline give the most generally known examples of this peculiarity, which, however, is likewise frequently met with in rhombohedral Ruby-blende. It is evident that the six-sided prism $R + \infty$, if subject to this modification, will only shew half the number of its faces; and that the three-sided prism occurring in these two species, can only be explained upon the supposition, that it is a rhombohedron of an infinite axis, three faces of which belong to one of the apices of the combination, and are enlarged, while those belonging to the other apex disappear.

The designation of hemi-rhombohedral and hemi-di-rhombohedral forms, depends upon the same principle as that of the halves in the tessular system (§. 129.—133.). The number 2 is added to the sign of the entire, simple form, as a divisor; and by the signs + and —, or r and l, is indicated the parallel and the transverse position, or the difference between right and left.

The following signs refer to the figures 51 — 58, comprehending the different forms of a di-rhombohedral, hemi-rhombohedral, or hemi-di-rhombohedral character:

$$2\,((P+n)^m) \text{ Fig. 51.;}$$

$$\frac{r}{r}\,\frac{P+n}{2}, \text{ or } \frac{l}{l}\,\frac{P+n}{2}, \text{ or } \frac{+}{-}\,\frac{R+n}{2} \quad \text{Fig. 52.;}$$

$$\frac{r}{r}\,\frac{(P+n)^m}{2} \quad \text{Fig. 53.;} \quad \frac{l}{l}\,\frac{(P+n)^m}{2} \quad \text{Fig. 54.;}$$

$$\frac{+r}{-r}\,\frac{(P+n)^m}{2} \quad \text{Fig. 55.;} \quad \frac{+l}{-l}\,\frac{(P+n)^m}{2} \quad \text{Fig. 56.;}$$

$$\frac{r}{r}\,\frac{2\,((P+n)^m)}{2} \text{ Fig. 57.;} \quad \frac{l}{l}\,\frac{2\,((P+n)^m)}{2} \text{ Fig. 58.}$$

§. 148. DEVELOPEMENT OF RHOMBOHEDRAL AND DI-RHOMBOHEDRAL COMBINATIONS.

The developement of the combinations containing three or more simple forms, is founded upon the knowledge of binary combinations.

A few examples will be the best means of instructing the beginner how to proceed in similar cases. The example chosen for rhombohedral combinations, shews at once the sufficiency of the few particular cases mentioned in §. 145.—147. of binary combinations for the developement of such as consist of a greater number of simple forms.

The 59th figure represents a rhombohedral combination (§. 145.), consisting of four rhombohedrons, two scalene six-sided pyramids, and a regular six-sided prism. Its indeterminate designation is

$$\underset{a}{R + n}.\ \underset{c}{R + n^{I}}.\ \underset{e}{R + n^{II}}.\ \underset{d}{R + n^{III}}.\ \underset{b}{(P + n^{IV})^{m}}.$$

$$\underset{f}{(P + n^{V})^{m'}}.\ \underset{g}{R + \infty}.$$

The only form immediately determined in this combination, is $R + \infty$. The edges of combination between the faces of this form and those of the rhombohedrons $R + n^{III}$, $R + n^{II}$ and $R + n$ are horizontal (§. 145. i. 4.), whereas $P + \infty$, if it were contained in the combination, would produce edges of combination parallel to the lateral or terminal edges of these rhombohedrons (§. 145. iii. 4.).

Among the rhombohedrons, one must be selected and fixed upon as the fundamental form, and the letter n in its sign therefore, must be made = 0. The figure represents a crystal of rhombohedral Lime-haloide, which mineral is cleavable (§. 162.), parallel to the faces of the rhombohedron here designated by $R + n^{I} = 105° 5'$. According to this we determine the rhombohedron $R + n^{I}$ to be the fundamental form, and consequently n' to be $= 0$, and $R + n' = R$. The position of R is considered as the normal position.

The rhombohedron $R + n$ is in a transverse position towards R; and since the edges of combination between R, $(P + n^{IV})^m$ and $R + n$ are parallel to the terminal edges of R and to the inclined diagonals of $R + n$, those between R and $R + n$ will assume the same situation, if the faces of $(P + n^{IV})^m$ disappear. Hence the two forms are in the relation of $R + n : R + n - 1$ (§. 145. i. 1.); or, for $n = 0$, in that of $R : R - 1$. Consequently n is $= -1$ and $R + n = R - 1$.

Suppose the faces of R and those of $R + n^{IV}$ to be enlarged till they intersect each other.* $R + n^{IV}$ is in the same position towards R as $R - 1$; the two forms will produce edges of combination parallel to the inclined diagonals of R, and consequently to the terminal edges of $R + n^{IV}$. The forms $R + n^{IV}$ and R are again in the ratio of $R + n$ and $R + n - 1$ (§. 145. i. 1.). And since for R, the expression $n - 1$ is $= 0$, n^{IV} for $R + n^{IV}$ will be $= 1$, and $R + n^{IV} = R + 1$.

The three rhombohedrons $R - 1$, R and $R + 1$ are consecutive members of one and the same series.

The edges of combination between $(P + n^{V})^{m'}$ and R are parallel to the terminal or to the lateral edges of R, and to the lateral edges of $(P + n^{V})^{m'}$; the scalene six-sided pyramid belongs to the rhombohedron (§. 145. ii. 1.). In $(P + n^{V})^{m'}$ therefore n^{V} is $= 0$, and the pyramid itself $= (P)^{m'}$.

The rhombohedron $R + 1$ is in a transverse position towards this pyramid, which is itself parallel to R, and the faces of $R + 1$ take away the more acute terminal edges of $(P)^{m'}$ with parallel edges of combination. The relation of the forms is therefore as in §. 145. ii. 4.; and we have

$$\frac{3\,m' - 1}{6} . \, a = \tfrac{2}{3} . \, 2 . \, a,$$

---

* The faces of R may easily be enlarged by cleavage, (§. 162.).

and hence $m' = 3$. The pyramid accordingly will be perfectly designated by the crystallographic sign $(P)^3$.

The more obtuse pyramid produces with $(P)^3$ horizontal edges of combination, m is therefore $= m' = 3$ (§. 145. iv. 2.); and $(P + n^{IV})^m = (P + n^{IV})^3$.

But the faces of $R - 1$ appear with parallel edges of combination, in the place of the more acute terminal edges of this pyramid. If, therefore, a′ be the axis of the rhombohedron to which the pyramid belongs, whilst a is the axis of R, we have from §. 145. ii. 4.,

$$\frac{3.3 - 1}{6} \cdot a' = \frac{2}{3} \cdot \frac{a}{2};$$

from which follows

$$a' = \tfrac{1}{4} \cdot a = 2^{-2} \cdot a;$$

n^{IV} is therefore $= -2$, and $(P + n^{IV})^3 = (P - 2)^3$.

The edges of combination between the pyramid just now determined and the rhombohedron $R + n^{II}$ are parallel to the more obtuse terminal edges of the former, and to the terminal or lateral edges of the latter. The two forms rank under the head of §. 145. ii. 7.

Let a′ be the axis of the rhombohedron $R + n^{II}$. We have

$$\frac{3.3 + 1}{6} \cdot \tfrac{1}{4} \cdot a = \tfrac{1}{3} \cdot a';$$

and accordingly,

$$a' = \tfrac{5}{4} \cdot a = \tfrac{5}{4} \cdot 2^0 \cdot a$$

n^{II} is therefore $= 0$; but the rhombohedron belongs to the first subordinate series, and $R + n^{II}$ is $= \frac{5}{4}$ R.

If now we write down the signs of the forms as we have found them in the developement in their regular order; we obtain the crystallographic designation of the combination, disposed according to the different angles which perpendicular lines drawn upon the faces produce with the axis,

$R - 1.$	$(P - 2)^3.$	$R.$	$\frac{5}{4}R.$	$R + 1.$	$(P)^3.$	$R + \infty.$
a	*b*	*c*	*d*	*e*	*f*	*g*

The 60th figure represents a di-rhombohedral combination of rhombohedral Emerald, designated indeterminately thus:

$$\underset{a}{R - \infty}. \quad \underset{b}{P + n}. \quad \underset{d}{P + n^{I}}. \quad \underset{c}{2\,(R + n^{II})}. \quad \underset{e}{P + \infty}.$$

Here $R - \infty$ (*a*) and $P + \infty$ (*e*) the limits of the series of isosceles six-sided pyramids are immediately determined. The only rhombohedron which it contains is $R + n^{II}$ (*c*). It is present in both positions; the combination therefore assumes a di-rhombohedral character (§. 146.). If this rhombohedron be considered as the fundamental form, the value of n^{II} will be $= 0$, and $2\,(R + n^{II})$ therefore $= 2\,(R)$.

The faces of $P + n$ (*b*) if duly enlarged, appear as rhombs in the place of the apices of the di-rhombohedron; the faces of the latter likewise would be rhombs, were they not intersected by the faces of other forms. Hence n is also $= 0$, or the pyramid $P + n$, and the di-rhombohedron $2\,(R)$ are co-ordinate forms (§. 146. 1.; and §. 145. iii. 1.). $P + n$ therefore is $= P$.

The faces of the di-rhombohedron appear with parallel edges of combination in the place of the terminal edges of $P + n^{I}$ (*d*). The relations of the pyramid and the rhombohedron will therefore be those considered in §. 145. iii. 2. From these it follows that n^{I} is $= n^{II} + 1 = 0 + 1 = 1$. The pyramid will be $= P + 1$.

The determined designation of the developed di-rhombohedral combination is therefore:

$$\underset{a}{R - \infty}. \quad \underset{b}{P}. \quad \underset{c}{2\,(R)}. \quad \underset{d}{P + 1}. \quad \underset{e}{P + \infty}.$$

These developements, as represented by the signs, contain every thing required for calculations referring to the compound forms; since the designation contains all those determined definite values of m and n, which must be substituted in the general equations referred to in §. 144.

The developement of the combinations is peculiarly applicable, if it is not limited to the forms of a single individual, but if it refers to all the crystalline varieties known in a natural-historical species. It will be treated of more at large than the proposed limits of this work would allow, in a particular work on Crystallography. In general, those simple forms which are known from one developed combination, become the foundation of every farther developement; and this method of proceeding is also used in compound forms of a single individual, if some of its forms can be identified with others, which have been developed in other crystalline varieties of the same species.

§. 149. PYRAMIDAL COMBINATIONS.

A combination of the pyramidal system is more particularly said to possess a *Pyramidal character*, if the simple forms contained in it appear with the full number of their faces in their peculiar position.

The binary combinations of this system, under the same restrictions as to subordinate series as in §. 145., are in general

i. $P + n. \quad P + n'$,
ii. $P + n. \quad (P + n')^m$,
iii. $(P + n)^m. \ (P + n')^{m'}$.

i. $P + n. \quad P + n'$.

1. Let n be $= n \pm 1$. Under these circumstances, the forms will be consecutive members of the series §. 101. As such, they are in a diagonal position, and the edges of combination which they produce, must be parallel among each other, but at the same time they must be parallel also to the terminal edges of the more acute pyramid, and to those lines in the faces of the more obtuse one, which may be drawn perpendicularly from the apices to the lateral edges. *Ex.* $P - 1$ (t) and P (P) in pyramidal Zircon. Vol. II. Fig. 99. Inversely from the described situation of the edges follow the above mentioned relations of the two

forms among each other; which is immediately evident from the derivation.

2. Let n′ be = n ± 2 r. The forms are in a parallel position. The edges of combination therefore will be horizontal. *Ex.* P (*c*) and P + 2 (*b*) in pyramidal Garnet. Vol. II. Fig. 96. They retain this situation, even though the combined forms belong to different series, and their relative dimensions therefore in such a combination remain undetermined. The reason why the faces of all parallel pyramids intersect each other in horizontal edges, consists in the parallel situation of their respective lateral edges, or of the sides of their horizontal projections.

3. Let n or n′ be = — ∞. One of the forms under these circumstances appears as a face perpendicular to the axis of the other; and the edges of combination, independently of the dimensions of the forms, must be horizontal, as is evident from 2. *Ex.* P — ∞ (*a*) and $\frac{2\sqrt{2}}{3}$ P — 3 (*b*) in pyramidal Lead-baryte. Vol. II. Fig. 92.

4. Let n or n′ be = + ∞; one of the forms becomes a regular four-sided prism. In a parallel position, the edges in which the faces of the two forms meet, must be parallel to the lateral edges of the finite form, and therefore horizontal (2). *Ex.* P (*P*) and P + ∞ (*l*), or P + 1 (*s*) and [P + ∞] (*g*) in pyramidal Tin-ore. Vol. II. Fig. 102. In a diagonal position they are parallel to the rhombic principal section (§. 53. 2.) of the finite member, and their situation depends on its dimensions. *Ex.* P + 1 (*s*) and P + ∞ (*l*) in pyramidal Tin-ore. Vol. II. Fig. 102. If the faces of the prism do not intersect each other, they appear as rhombs. The combinations P — ∞. P + ∞, and P — ∞. [P + ∞], are right rectangular four-sided prisms.

5. If both n and n′ are = + ∞, and the forms in a diagonal position, the faces of one of the prisms appear in the place of the edges of the other; their combination produces an equiangular, and if the faces are of equal extent, a regular eight-sided prism. *Ex.* P + ∞ (*l*) and [P + ∞] (*g*) in pyramidal Tin-ore. Vol. II. Fig. 102.

ii. $P + n. \quad (P + n')^{m'}.$

1. Let n' be $= n$. The forms in this case are co-ordinate, and as such, in a parallel position. The faces of the four-sided pyramid appear in the place of the apices of the eight-sided one as rhombs, their edges of combination being parallel to the terminal edges of the four-sided pyramid. *Ex.* P (*c*) and $(P)^3$ (*s*), or P (*c*) and $(P)^4$ (*x*) in pyramidal Garnet. Vol. II. Fig. 96. The situation of the edges produced between the two forms, is independent of m′, but in the parallel position already mentioned, it equally requires the above relation of the forms, whatever may be the value of m.

If the forms are in a diagonal position, and therefore not co-ordinate ones, there exists for every sca ne eight-sided pyramid a particular four-sided pyramid, whose faces appear as rhombs in the place of the apices of the former. *Ex.* P — 1 (*o*) and $(P - 2)^3$ (*a*), or P (*P*) and $(P - 1)^3$ (*z*) in pyramidal Garnet. Vol. II. Fig. 96. In these combinations the rhombic figure of the faces of the isosceles four-sided pyramids depends on a certain relation of m′, n′ and n, which is expressed in the equation:

$$m' = 2^{\frac{n - n' + 1}{2}} + 1.$$

This equation is very useful, from two of these quantities being known, to find the value of the third.

2. Let n or n′ be $= -\infty$. One of the forms becomes $= P - \infty$, and as a face perpendicular to the axis, it produces horizontal edges of combination (i. 3.).

3. Let n be $= +\infty$. The pyramid P + n in this case appears as a rectangular four-sided prism. In either position its faces assume a rhombic figure, by their intersection with those faces which form the lateral solid angles of the pyramid. In the parallel position the angles of those rhombs, and therefore the situation of the edges of combination are altogether dependent on m′, and different in different pyramids. In the diagonal position, however, these angles and edges of combination do not depend on m′, since the angles are equal to the angles of the rhombic

principal section, and the edges of combination consequently parallel to the alternating terminal edges of that isosceles four-sided pyramid, from which the scalene eight-sided pyramid is derived, or to which it belongs. Examples of both cases are contained in pyramidal Zircon, Vol. II. Fig. 99. The scalene eight-sided pyramids $(P)^3$ (*x*), $(P)^4$ (*y*) and $(P)^5$ (*z*) are in a parallel position with $P + \infty$ (*l*), but they are in a diagonal position with $[P + \infty]$ (*s*).

If, therefore, the edges of combination produced by a scalene eight-sided pyramid and an isosceles four-sided one, in a parallel position, are found to be parallel to those edges which are produced by the same eight-sided pyramid, and a rectangular four-sided prism in a diagonal position; it follows that the isosceles four-sided pyramid and the eight-sided pyramid must be co-ordinate forms.

4. Let $n' = n - 2$, and $m' = 3$. The combination will be $P + n. \ (P + n - 2)^3$, and the forms in a parallel position. The faces of the four-sided pyramid appear in the place of the more acute terminal edges of the eight-sided pyramid; the edges of combination being parallel among themselves, to the above-mentioned terminal edges of the eight-sided pyramid, and to those lines in the four-sided pyramid, which, from its apices, can be drawn perpendicularly to its lateral edges. Examples occur in pyramidal Zircon, of $(P)^3$ and $P + 2$.

Suppose A′M, Fig. 69., to be half the axis, A′C the more acute terminal edge of the eight-sided pyramid. If CM be made equal to half the side of the horizontal projection, MA′ becomes half the axis, A′C the above-mentioned perpendicular line upon the face of the four-sided pyramid; and consequently, supposing MD, half the side of the horizontal projection $= \frac{1}{2}$, we have

$$MA = 2^{\frac{n}{2}}. a = \frac{m'+1}{2}. 2^{\frac{n'}{2}}. a.$$

If, according to the supposition, m′ be $= 3$, the values of n and n′ will follow thus:

$$n = n' + 2, \text{ and } n' = n - 2.$$

But if we substitute 4, instead of m′, we obtain

$$2^{\frac{n}{2}} = \tfrac{5}{4}.\, 2^{\frac{n'}{2}} = \tfrac{5}{4}.\, 2^1.\, 2^{\frac{n'}{2}} = \tfrac{5}{4}.\, 2^{\frac{n'+2}{2}},$$

and $P + n = \frac{5}{4} P + n' + 2$, or that member of the second subordinate series (§. 107.), which belongs to $P + n' + 2$.

For $m' = 5$, the result is

$$2^{\frac{n}{2}} = 3.\, 2^{\frac{n'}{2}} = \frac{3}{2\sqrt{2}}.\, 2^{\frac{3}{2}}.\, 2^{\frac{n'}{2}} = \frac{3}{2\sqrt{2}}.\, 2^{\frac{n'+3}{2}};$$

and $P + n = \frac{3}{2\sqrt{2}} P + n' + 3$, that member of the first subordinate series (§. 107.), which belongs to $P + n' + 3$.

5. Let n' be $= n - 3$, $m' = 4$. The combination is $P + n. (P + n - 3)^4$; under these circumstances, the forms will be in a diagonal position. The faces of the four-sided pyramids appear in the more obtuse terminal edges of the eight-sided ones. The edges of combination are parallel to these, to the perpendicular lines upon the faces of the four-sided pyramid, and among each other.

Suppose, Fig. 70., MA′ to be half the axis of the eight-sided pyramid, and A′B its more obtuse terminal edge. If now BM is half the side of the horizontal projection, we have in MA′ half the axis, and in A′B the perpendicular line upon the face of the four-sided pyramid; and consequently, if half the side of the horizontal projection MD is supposed $= \frac{1}{2}$, MA will be

$$= 2^{\frac{n}{2}}.\, a = m'.\, 2^{\frac{n'-1}{2}}.\, a.$$

But we have $m' = 4$; therefore

$$n = n' + 3, \text{ and } n' = n - 3.$$

If m' be $= 3$, it will follow that

$$2^{\frac{n}{2}} = 3.\, 2^{\frac{n'-1}{2}} = \frac{3}{2\sqrt{2}}.\, 2^{\frac{3}{2}}.\, 2^{\frac{n'-1}{2}} = \frac{3}{2\sqrt{2}}.\, 2^{\frac{n'+2}{2}};$$

and thus $P + n$ becomes $\frac{3}{2\sqrt{2}} P + n' + 2$, or that member of the first subordinate series, which belongs to $P + n' + 2$.

$m' = 5$ makes

$$2^{\frac{n}{2}} = 5.\, 2^{\frac{n'-1}{2}} = \tfrac{5}{4}.\, 2^2.\, 2^{\frac{n'-1}{2}} = \tfrac{5}{4}.\, 2^{\frac{n'+3}{2}}.$$

$P + n$ therefore becomes $= \frac{5}{4} P + n' + 3$, which is that member of the second subordinate series which belongs to $P + n' + 3$.

6. Let n' be $= n - 3$, $m' = 3$, the combination therefore $P + n. (P + n - 3)^3$. The forms again are in a diagonal position, and the more acute terminal edges of the eight-sided pyramid, therefore, fall into the same vertical plane, which passes through the terminal edges of the four-sided pyramid. The edges of combination, arising between the faces of the two forms, become parallel among themselves, and to the above-mentioned terminal edges of the pyramids. *Ex.* $P + 2$ (*b*) and $(P - 1)^3$ (*z*) or $P + 4$ (*r*) and $(P + 1)^3$ (*e*) in pyramidal Garnet. Vol. II. Fig. 96.

For, the rest being as in the other example, let A′C, Fig. 69., represent the terminal edge of the four-sided pyramid: it will follow that

$$MA = 2^{\frac{n}{2}}. a = \frac{m'+1}{2} \cdot 2^{\frac{n'+1}{2}}. a.$$

If now, according to the supposition, m' be $= 3$; we have

$$n = n' + 3, \text{ and } n' = n - 3.$$

But if m' is $= 4$; $P + n$ becomes $= \frac{5}{4} P + n' + 3$, or that member of the second subordinate series, which belongs to $P + n' + 3$; if m' is $= 5$, the pyramid becomes $\frac{3}{2\sqrt{2}} P + n' + 4$, or that member of the first subordinate series, which belongs to $P + n' + 4$.

7. Let n' be $= n - 4$, $m' = 4$, or the combination $P + n. \ (P + n - 4)^4$. The forms are in a parallel position; the more obtuse terminal edges of the eight-sided pyramid coincide with the terminal edges of the four-sided pyramid. In this situation of the faces, the edges of combination between the two forms are parallel to each other, and to both the mentioned terminal edges. *Ex.* $P + 4$ (*r*) and $(P)^4$ (*x*) in pyramidal Garnet. Vol. II. Fig. 96.

For we have

$$MA = 2^{\frac{n}{2}}. a = m'. 2^{\frac{n'}{2}}. a,$$

from which, 4 being substituted for m', we obtain

$$n = n' + 4, \text{ or } n' = n - 4;$$

$m' = 3$ gives $P + n = \frac{3}{2\sqrt{2}} P + n' + 3$, the member of the first subordinate series belonging to $P + n' + 3$; $m = 5$ gives $P + n = \frac{5}{4} P + n' + 4$, or that member of the second subordinate series which belongs to $P + n' + 4$.

iii. $(P + n)^m$. $(P + n')^{m'}$.

1. Let n' be $= n$. The combination is $(P + n)^m$. $(P + n)^{m'}$; the forms are co-ordinate pyramids, and as such in a parallel position. The situation of the edges in which the faces of the two pyramids intersect each other, do not depend upon m or m′ (ii. 1.). The situation of the edges produced between any one of those pyramids, and the four-sided one from which they are derived, is such that the faces of the latter become rhombs, or the edges of combination are parallel to their terminal edges. The edges of combination between two such co-ordinate scalene eight-sided pyramids, will therefore likewise be parallel to the edges of that isosceles four-sided pyramid to which they belong; and if more than two appear in one and the same compound form, the above mentioned parallelism will be observable in the edges of combination between them all. *Ex.* The pyramids *x*, *y*, and *z*, already mentioned in pyramidal Zircon. Vol. II. Fig. 99. From the observed parallelism, we may decide inversely whether the pyramids are co-ordinate forms or not, and whether or not they are derived from that isosceles four-sided pyramid, with which they enter into combination.

2. Let m be $= m'$. The combination will be $(P + n)^m$. $(P + n')^m$. If, moreover, the forms are in a parallel position, the edges produced by the intersection of their faces become horizontal. *Ex.* $(P - 1)^3$ (*z*) and $(P + 1)^3$ (*e*) in pyramidal Garnet. Vol. II. Fig. 96. The transverse sections of the two forms are similar to each other, since these forms are members of one and the same series (§. 105.). The observations in respect to scalene six-sided pyramids in §. 145. iv. 2. extend likewise to this case.

3. If one of the combined forms, by n becoming $= \pm \infty$

is changed either into an unequiangular eight-sided prism, or into a plane perpendicular to the axis, the situation of the edges in 2. remains nevertheless unchanged, provided in the case of the prism, the position still remains the parallel one and $m = m'$. *Ex.* of the latter, $(P + 1)^3$ (*e*) and $[(P + \infty)^3]$ (*f*) in pyramidal Garnet. Vol. II. Fig. 96.

4. There exists in the pyramidal system, a series of scalene eight-sided pyramids analogous to that of the scalene six-sided pyramids in the rhombohedral system (§. 145. iv. 4.). Its members succeed each other in the following order: $\ldots (P + n)^5, (P + n + 1)^4, (P + n + 2)^3, (P + n + 1)^5 \ldots$ in which the consecutive members assume a diagonal position towards each other. *Ex.* $(P)^4$ (*x*) and $(P + 1)^3$ (*e*) in pyramidal Garnet. Vol. II. Fig. 96. The edges of combination between the faces of every two subsequent pyramids are parallel to each other, to the more obtuse terminal edges of the lower, and to the more acute terminal edges of the higher member in the series. The demonstration of this property depends upon the same suppositions as in the six-sided pyramids.

Let A′B, Fig. 70., represent the more obtuse terminal edges of $(P + n)^m$; we find the algebraic expression of

$$\sin A'BM = \frac{m \cdot 2^{\frac{n}{2}} \cdot a}{\sqrt{(m^2 \cdot 2^n \cdot a^2 + 2)}}.$$

If in the same way we suppose A′C, Fig. 69., to be the more acute terminal edge of $(P + n')^{m'}$; a similar algebraic expression will give

$$\sin A'CM = \frac{(m' + 1)\, 2^{\frac{n'}{2}} \cdot a}{\sqrt{[(m' + 1)^2 \cdot 2^{n'} \cdot a^2 + 4]}}.$$

These two expressions become equal, if in the first we substitute 5 for m, and in the second $n + 1$ for n', and 4 for m'. They again become equal if in the first we suppose $m = 4$; in the second $n' = n + 1$ and $m' = 3$: and again for $m = 3$ in the first, and $n' = n - 1$, and $m' = 5$ in the second expression. For the rest, the remarks of §. 145. iv. 5. find here equally their full application.

§. 150. HEMI-PYRAMIDAL COMBINATIONS.

A combination of the pyramidal system possesses a *Hemi-pyramidal Character*, if one or more of the simple forms contained in it, appear with only half the number of their faces.

The isosceles four-sided pyramid may be resolved into halves, like the octahedron (§. 129.). The result is a pair of forms contained under four equal and similar isosceles triangles, Figs. 61. 62., the first being produced from the isosceles four-sided pyramid, Fig. 8., by the enlargement of the faces *a* and *a*, the second by the enlargement of *b* and *b*. None of their faces are parallel to each other. This form is analogous to the tetrahedron. A combination, containing one or several forms of this kind, is termed a hemi-pyramidal combination *of inclined faces*. If we enlarge parallel faces, no finite forms can be obtained; and this seems to be the reason why hemi-pyramidal forms thus possessing parallel faces have not yet been found in nature.

The scalene eight-sided pyramid, if resolved by enlarging its alternate faces, gives forms contained under eight irregular trapezoidal faces, Figs. 63. 64., which on that account receive the denomination of *four-sided Trapezohedrons.* These forms agree exactly with the six-sided trapezohedrons obtained by the same mode of resolution from the di-pyramid, §. 146., except in the number of their faces. The two forms thus produced from the eight-sided pyramid, are also distinguished from each other by the difference of Right and Left, as the forms of the rhombohedral system already mentioned. Fig. 63. represents a right four-sided Trapezohedron, produced from the eight-sided pyramid, Fig. 12., by the enlargement of the faces *a*, *a*, while Fig. 64. is the left one which arises from the enlargement of *b* and *b*.

By enlarging parallel faces we obtain two forms absolutely similar to isosceles four-sided pyramids, except in their position, since the faces of such hemi-pyramidal forms are always situated like the faces of that eight-sided pyramid, whose halves they are. Combinations, into which forms of this kind enter, are termed hemi-pyramidal combinations *of parallel faces.*

But a scalene eight-sided pyramid may also be resolved according to the process, applied in §. 146. to the di-pyramid, for reproducing the simple forms which it contains. This is effected by enlarging the alternate pairs of faces which meet in the acute terminal edges of the one, and those of the other apex which are not contiguous to them. The resulting forms are contained under eight equal and similar scalene triangles, all their faces being inclined to one another, Figs. 65. 66.; hence they likewise change those pyramidal combinations in which they are contained, into hemi-pyramidal ones of inclined faces. These forms are in respect to the eight-sided pyramids, what the forms analogous to the tetrahedron considered above are to the four-sided pyramids, and occur along with them in the same combinations.

Hemi-pyramidal combinations of parallel faces occur in pyramidal Scheelium-baryte, and hemi-pyramidal combinations of inclined faces in pyramidal Copper-pyrites. No example has yet been found in nature of di-pyramidal combinations, or such as would contain one and the same form of the pyramidal system in two different positions; nor have we any reason to suspect their existence, because no rational number of derivation can produce from any pyramidal form, another diagonally situated, and equal and similar to that pyramidal form.

Hemi-pyramidal forms, like the hemi-rhombohedral ones, are designated by adding the divisor 2 to the crystallographic signs of the entire forms. The situation of those faces which occur in the combination, are moreover indicated by the signs + and —, or r and l.

The following signs refer to the figures 61 — 66:

$$+\frac{P+n}{2} \text{ Fig. 61.}; \quad -\frac{P+n}{2} \text{ Fig. 62.};$$

$$\frac{r}{r}\,\frac{(P+n)^m}{2} \text{ Fig. 63.}; \quad \frac{l}{l}\,\frac{(P+n)^m}{2} \text{ Fig. 64.};$$

$$+\frac{(P+n)^m}{2} \text{ Fig. 65.}; \quad -\frac{(P+n)^m}{2} \text{ Fig. 66.}$$

§. 151. DEVELOPEMENT OF PYRAMIDAL COMBINATIONS.

The following developements will shew how the knowledge of binary combinations relative to the pyramidal system in §. 149., is to be applied in particular cases.

Fig. 67. represents a pyramidal combination, whose undetermined designation is

$$\underset{a}{P+n}. \quad \underset{b}{P+n^{I}}. \quad \underset{c}{(P+n^{II})^{m}}. \quad \underset{d}{P+\infty}. \quad \underset{e}{[P+\infty]}.$$

First of all, the more obtuse of the two four-sided pyramids, or $P + n$ (*a*), is supposed to be the fundamental form. The value of n will therefore be $= 0$, and $P + n = P$. This determination must precede that of the vertical prisms, which, however, is now very easily effected; the prism *d*, or that whose intersections with P are horizontal, being $P + \infty$, whilst *e*, the other prism, is $[P + \infty]$, and produces intersections with the faces of P, which are parallel to the terminal edges of this form (§. 149. i. 4.).

The edges of combination between P and $P + n^{I}$ (*b*), are parallel among themselves, but at the same time also to the perpendicular lines drawn upon the faces of the former, and to the terminal edges of the latter pyramid. The forms are in a diagonal position to each other; and they are therefore in the relation of $P + n$ and $P + n + 1$; and since $n = 0$, $P + n^{I}$ will be $= P + 1$, as follows immediately from the derivation.

The scalene eight-sided pyramid *c* belongs to P; for it is in a parallel position with it, and the faces of the four-sided pyramid appear as rhombs in the place of the apices of the eight-sided pyramid (§. 149. ii. 1.). For $n^{II} = 0$, $(P + n^{II})^{m}$ becomes $= (P)^{m}$.

$P + 1$ is in a diagonal position with $(P)^{m}$; its faces, however, likewise appear as rhombs in the combination, for the edges between the two forms are parallel to the terminal edges of P + 1. From the equation given in §. 149., we have

$$m = 2^{\frac{1-0+1}{2}} + 1 = 2 + 1 = 3, \text{ and } (P)^m = (P)^3.$$

The perfectly determined designation of the developed compound form, is therefore

$$\underset{a}{P.}\quad \underset{b}{P+1.}\quad \underset{c}{(P)^3.}\quad \underset{d}{P+\infty.}\quad \underset{e}{[P+\infty].}$$

The indeterminate designation of the compound form, represented in Fig. 68., is

$$\underset{a}{P+n.}\quad \underset{f}{P+n^{I}.}\quad \underset{c}{(P+n^{II})^m.}\quad \underset{d}{P+\infty.}\quad \underset{e}{[P+\infty].}$$

If we compare the present combination with the preceding one, we find a perfect identity between the forms P + n and P, and between the forms $(P + n^{II})^m$ and $(P)^3$, the rectangular prisms likewise being common to both; and accordingly we may consider these forms as already known, so that in

$$\underset{a}{P.}\quad \underset{f}{P+n^{I}.}\quad \underset{c}{(P)^3.}\quad \underset{d}{P+\infty.}\quad \underset{e}{[P+\infty].}$$

the only form still to be determined is the four-sided pyramid $P + n^{I}$ (*f*).

The faces of the scalene eight-sided pyramid $(P)^3$, meeting in their more obtuse terminal edges, are found in the combination to appear in the place of the terminal edges of $P + n^{I}$. The two forms are therefore in a parallel position, and the relation existing among them, if compared with those considered above, is comprised under the case §. 149. ii. 7.

If the number 3 be substituted for m, the axis of the isosceles four-sided pyramid will be $= 3.\ 2.^{\frac{n}{2}}.\ a$, or $= 3.\ a$ if $n = 0$. Hence the pyramid is that member of the first subordinate series which belongs to P + 3 (§. 149. ii. 4.), that is to say, it is $= \frac{3}{2\sqrt{2}} P + 3$.

The complete designation of the compound form follows according to this developement, thus:

$$\underset{a}{P.}\quad \underset{f}{\tfrac{3}{2\sqrt{2}} P + 3.}\quad \underset{c}{(P)^3.}\quad \underset{d}{P + \infty.}\quad \underset{e}{[P + \infty].}$$

§. 152. PRISMATIC COMBINATIONS.

A combination of the prismatic system is said more particularly to possess a *Prismatic Character*, if the forms contained in it appear with the whole number of their faces.

The number of binary combinations in the prismatic system is so great, on account of the great variety of different relations among its forms, that it becomes impossible in the present place to consider them all, even though this should be done in the most general manner. We can therefore notice only as many as will be sufficient for explaining the greater part of the cases commonly occurring in nature, and which, at the same time, shew how to proceed in the application of the methods of developement mentioned above (§. 143.). These binary combinations are:

i. $P + n.\quad P + n'$,
ii. $P + n.\quad (\breve{P} + n')^{m'}$,
iii. $P + n.\quad (\bar{P} + n')^{m'}$,
iv. $P + n.\quad (\breve{P}r + n')^{m'}$,
v. $P + n.\quad (\bar{P}r + n')^{m'}$,
vi. $P + n.\quad \breve{P}r + n'$,
vii. $P + n.\quad \bar{P}r + n'$,
viii. $\breve{P}r + n.\quad \bar{P}r + n'$.

i. $P + n. P + n'$.

The bases of the forms contained in this combination are similar to each other, because these forms are members of one and the same series, and no different position can have any influence upon them, whatever members of the series may be combined. The edges of combination, therefore, become horizontal for every value of n and n′, even though this be $= + \infty$ or $= - \infty$. The same reasoning applies to all such combinations as are produced by simple forms of similar bases, and inversely, horizontal edges of combination may be considered as a certain character of this property of forms, as is evident from the derivation of the series itself, and of its limits. *Ex.* $P - \infty$ (*P*), $\frac{4}{3}P - 1$ (*s*), P (*o*) and $P + \infty$ (*M*) in prismatic Topaz. Vol. II. Fig. 34.

ii. $P + n. (\breve{P} + n')^{m'}$.

Let n′ be = n; and the forms accordingly co-ordinate ones. The faces of $P + n$, meeting in their more obtuse terminal edges, appear in the place of those terminal edges of $(\breve{P} + n)^{m'}$, which are situated contiguous to the prolonged diagonal, and the edges of combination will be parallel among themselves, and to the above mentioned edges of the two pyramids. This parallelism follows from the simultaneous increment of the axis and of the variable diagonal of $(\breve{P} + n)^{m'}$ (§. 94.).

iii. $P + n. (\bar{P} + n')^{m'}$.

Let n′ be = n. Every thing is as in ii.; except that the faces of $P + n$ meeting in the more acute terminal edges, appear in the place of the similarly situated terminal edges of $(\bar{P} + n)^{m'}$. This likewise follows from §. 94. *Ex.* P (*P*) and $(\bar{P})^3$ (*a*) in prismatic Melane-glance. Vol. II. Fig. 34.

iv. $P + n. (\breve{P}r + n')^{m'}$.

Let n′ be = n; m = 3. The combination will be $P + n. (\breve{P}r + n)^3$. The faces of $P + n$ meeting in its more obtuse terminal edges, are situated in the place of the corresponding terminal edges of $(\breve{P}r + n)^3$. The edges of combination are parallel to each other, and to the above mentioned edges of the pyramids. For, if in the general co-efficients of $(\breve{P}r + n)^m$ (§. 95.), m is supposed $= 3$, the ratio $\frac{m+1}{2} 2^n. a \; : \; \frac{m+1}{m-1}. c$ becomes equal to $2^n. a \; : \; c$, which is identical with the ratio of the analogous lines in $P + n$. *Ex.* P (*P*) and $(\breve{P}r)^3$ (*n*) in Serpentine. Vol. II. Fig. 33.

If $m' = 5$, and $n' = n - 1$, the same situation of the edges takes place. A similar result is obtained by substituting 4 instead of m'. In this case, however, the pyramid, from which $(P + n')^{m'}$ is derived, would not be one belonging to the same series as $P + n$, but it would be a pyramid belonging to that subordinate series of which $\frac{3}{4}$ is the co-efficient. This becomes evident from a comparison of the general co-efficients. The observed parallelism of the edges in one direction alone is therefore insufficient for the determination of the form, if there is not another datum supplying this want from another side.

v. $P + n. (\breve{P}r + n')^{m'}$.

Let n' be $= n$; $m' = 3$, or the combination $P + n. (\breve{P}r + n)^3$. As the preceding case (iv.) refers to the more obtuse terminal edges, so the present one applies to the more acute ones; which is evident from the comparison of the general co-efficients of the forms concerned. The situation of the edges being as described here, if m be supposed to assume such values as do not make $m' + 1$ equal to a power of the number 2; the pyramids $P + n$ and $P + n'$ will not belong to one and the same series.

vi. $P + n. \breve{P}r + n'$.

The pyramids and horizontal prisms considered here are supposed to belong to one and the same series.

Let n' be $= n$. The faces of the horizontal prism appear in the place of the acute terminal edges of the pyramid, and the edges of combination are parallel among each other, and to the above mentioned terminal edges of the pyramid, which is evident from the derivation. The same takes place for $(\breve{P}r + n)^m. \bar{P}r + n'$, and if m be $= 3$, also for $(\breve{P}r + n)^m. \bar{P}r + n'$; and it is therefore a very useful and evident datum for the determination of forms. *Ex.* P (*P*) and $\breve{P}r$ (*o*) in diprismatic Olive-malachite. Vol. II. Fig. 5.

vii. $P + n. \bar{P}r + n'$.

1. What has been said in vi. of the acute terminal edges of $P + n$, applies here on the same supposition to the obtuse ones, in the combination $P + n. \bar{P}r + n$, as well as in those of $(\bar{P} + n)^m. \breve{P}r + n$, and, in the case of $m = 3$, also to $(\bar{P}r + n)^m. \breve{P}r + n$. *Ex.* P (*o*) and $\bar{P}r$ (*P*) in diprismatic Iron-ore. Vol. II. Fig. 4.

2. Suppose now a triple combination, whose crystallographic sign is

$$P + n. \breve{P}r + n'. \bar{P}r + n'',$$

and in this $n' = n$, $n'' = n - 1$; the faces of $\bar{P}r + n''$ will assume a rhombic figure in the combination.

Let AM, Fig. 71., represent part of the axis, MB one of the diagonals, and BG part of that terminal edge of the pyramid $P + n$, which is contiguous to BG; FGHI, FGH'I' will be the faces of the horizontal prism $\breve{P}r + n$.

The rhomb AQB'P is a face of the horizontal prism $\bar{P}r + n''$; AN is $=$ NB', the triangle NB'N' therefore similar to the triangle AB'M', and equal and similar to the triangle NGA. Hence N'N $=$ NG $= \frac{1}{2}$ N'G.

The line N'B' is at the same time the diagonal of the pyramid $P + n$, and of the pyramid $P + n''$, to which the horizontal prism $\bar{P}r + n''$ belongs. N'G therefore represents the axis of the first, N'N that of the second pyramid, and from the ratio of these $= 2 : 1$, we infer that n'' is $= n - 1$.

3. If instead of $P + n$, the triple combination contains the vertical prism $P + \infty$, and n' is $= n''$, the faces of all the three forms become rhombs. The rhombic figure of the faces of a horizontal prism, if produced by the intersection

with such forms as are known, will always suffice for its perfect determination.

viii. $\breve{P}r + n. \bar{P}r + n'$.

1. Let n' be $= n$. The combination $\breve{P}r + n. \bar{P}r + n$ will be the intermediate form belonging to $P + n$ (§. 97.).

2. If the two forms do not belong to one and the same series, due attention must be given to the co-efficients of the different series. If n or n′ become infinite, the edges of combination are parallel to the terminal edges of that scalene four-sided pyramid, to which the finite prism belongs, whatever may be its co-efficient.

3. If both n and n′ are $= + \infty$, the combination $P - \infty. \breve{P}r + \infty. \bar{P}r + \infty$ is transformed into a right rectangular prism, whose transverse section is an oblong. This prism must not be confounded with the right rectangular four-sided prism of the pyramidal system, whose transverse section is a square, and whose crystallographic sign may be either $P - \infty. P + \infty$ or $P - \infty. [P + \infty]$.

§. 153. HEMI-PRISMATIC COMBINATIONS.

A combination possesses a *Hemi-prismatic Character*, if one or several of the forms contained in it, and bearing to each other the general relations of those in the prismatic system, appear only with half the number of their faces, or in which these faces shew differences in their angles referring to an axis which is inclined in a plane perpendicular upon the base, and passing through one of its diagonals.

The hemi-prismatic combinations depend upon the fundamental form, §. 98. Fig. 41., whose axis is inclined in a plane perpendicular to the base, and passing through one of its diagonals.

The hemi-prismatic forms are designated like the prismatic ones, with that difference only, that the signs of those forms, of which only half the number of faces appear, receive the additional divisor 2; and that those faces which are turned towards the observer, contiguous to the upper apex, are provided with the sign +, while those contiguous to the same apex, but on the opposite side, are distinguished by the sign —.

Only a few observations shall be made in the present place on these combinations, in order to explain their general appearance.

If the inclination of the axis be supposed = 0, the combination of $P - \infty$ with $P + \infty$, or with $(\breve{P} + \infty)^m$, $(\bar{P}r + \infty)^3$, &c. will be a right oblique-angular four-sided prism, which is a compound form, bearing altogether the character of prismatic combinations, considered above.

But if the axis be inclined in the plane of the greater diagonal, or if the inclined face of the horizontal prism $\frac{\breve{P}r+n}{2}$ terminate the prism $P + \infty$; then the combination will assume the appearance of an oblique-angular four-sided prism, the basis of which is inclined to its acute lateral edges. A similar prism is produced by a combination of $P - \infty$ or of $\frac{\bar{P}r+n}{2}$ with an oblique-angular four-sided prism, only that the oblique terminal face is inclined towards the obtuse lateral edges of the prism. If in $\frac{\breve{P}r+n}{2}$ or $\frac{\bar{P}r+n}{2}$ n becomes $= + \infty$; the horizontal prism is transformed into a pair of planes parallel to the axis of the four-sided prism, and these faces appear with parallel edges of combination, in the case of $\frac{\breve{P}r+n}{2}$ instead of its acute lateral edges, in the case of $\frac{\bar{P}r+n}{2}$ instead of its obtuse ones, the prism itself remaining unlimited in the direction of its axis. Combinations of this kind cannot be distinguished from the prismatic ones $P + \infty . \breve{P}r + \infty$, and $P + \infty . \bar{P}r + \infty$, unless some other faces be present, which, by their inclination upon the axis, shew the hemi-prismatic character of the combination.

Supposing again the inclination of the axis to be $= 0$, the triple combination $P - \infty . \breve{P}r + \infty . \bar{P}r + \infty$ will be a right rectangular four-sided prism, its base an oblong rectangular face, and the combination itself a prismatic one, as it appears from what has been stated above, §. 152. The same triple combination $P - \infty . \breve{P}r + \infty . \bar{P}r + \infty$, upon the supposition of the axis being inclined in the plane of the longer, or the shorter diagonal, or the triple combinations $\frac{\breve{P}r + n}{2} . \breve{P}r + \infty . \bar{P}r + \infty$ or $\frac{\bar{P}r + n}{2} . \breve{P}r + \infty . \bar{P}r + \infty$, appear as oblique rectangular four-sided prisms, two faces of which are perpendicular to the rectangular base, while the two others produce with it horizontal edges of combination, supplemental to each other.

The axis is not always inclined to the base at the same angle, but varies according to the different species in which it occurs.

Prismatoidal Gypsum-haloide, prismatic Azure-malachite, paratomous, hemi-prismatic and prismatoidal Augite-spar, may be quoted as examples of hemi-prismatic combinations.

§. 154. TETARTO-PRISMATIC COMBINATIONS.

The *Tetarto-prismatic Character* of a combination requires, that of the forms which constitute it, the scalene four-sided pyramids shew only one-fourth, and the prisms, both horizontal and vertical, only one-half the number of their faces, or that these faces are distinguished from each other by their angles, which refer to an axis inclined in a plane perpendicular upon the base, and passing through neither of its diagonals.

Tetarto-prismatic combinations consist, like the prismatic and the hemi-prismatic combinations, of forms, whose general relations to each other are those developed for the prismatic system.

The tetarto-prismatic combinations depend upon the fundamental form, §. 98., Fig. 42., whose axis is inclined in a plane which is perpendicular to the base, and passes through neither of its diagonals.

The designation of tetarto-prismatic forms must distinguish all the faces of the pyramid contiguous to one and the same apex. Thus the face BAC being turned towards the observer on his right hand, is noted $r\frac{P}{4}$, BAC′ to his left is noted $l\frac{P}{4}$; on the opposite side the face B′AC turned to his right hand is distinguished by $-r\frac{P}{4}$, the face B′AC′ turned to the left by $-l\frac{P}{4}$. In the same manner also the front and back faces of the horizontal prisms belonging to the diagonal BB′, and the right and left faces of the horizontal prisms belonging to the diagonal CC′ are indicated by $\pm\frac{\breve{P}r+n}{2}$, and $\begin{matrix}r\\l\end{matrix}\frac{\bar{P}r+n}{2}$, the latter of which extends also to those prisms whose axis is parallel to the principal axis AA′ of P.

The tetarto-prismatic combination $\frac{P+n}{4}$. $P+\infty$ gives an oblique oblique-angular four-sided prism, in which the alternating edges mutually are supplemental of each other. Two of them assume a horizontal position, if the inclination of the axis is $=0$. But if the angle of inclination is an appreciable magnitude, or if these forms do not belong to one and the same series, none of the edges of combination can become horizontal.

The tetarto-prismatic combination $\frac{P+n}{4}$. $\breve{P}r+\infty$. $\bar{P}r+\infty$ is an oblique four-sided prism, in which none of the edges of combination are horizontal. This prism is rectangular if the base of the fundamental form is a rhomb, but its transverse section will differ more or less from a rectangular figure, if this base be a rhomboid.

Examples of tetarto-prismatic combinations occur in tetarto-prismatic Vitriol-salt, in several species of the genus Feld-spar, in prismatic Axinite, and other species.

§. 155. DEVELOPEMENT OF PRISMATIC COMBINATIONS.

The following developement will fully explain the application of what has been stated above in respect to binary combinations.

Fig. 72. represents a prismatic combination, indeterminately designated by

$$\underset{b}{P + n}.\ \underset{e}{P + n^{I}}.\ \underset{d}{\left\{\begin{matrix}(\breve{P} + n^{II})^{m} \\ (\breve{P}r + n^{II})^{m}\end{matrix}\right\}}{}^{*}\ \underset{c}{\breve{P} + n^{III}}.$$

$$\underset{a}{\bar{P}r + n^{IV}}.\ \underset{h}{\breve{P}r + \infty}.\ \underset{f}{P + \infty}.\ \underset{g}{\left\{\begin{matrix}(\breve{P} + \infty)^{m} \\ (\breve{P}r + \infty)^{m}\end{matrix}\right\}}.$$

Among the simple forms, constituting this compound one, two are immediately determined, $P + \infty$ (f) and $\breve{P}r + \infty$ (h).

If we suppose $n^{I} = 0$, $P + n^{I}$ (e) becomes $= P$, that is to say, the fundamental form. On account of the parallel edges of combination between P and $\breve{P}r + n^{III}$ (c), n^{III} is

* In the indeterminate designation both the signs (§. 192. 195.) are made use of before the forms have been determined, till it appears from the developement according to the reasons given above, which of the two is to be retained.

also $= n^{I} = 0$ (§. 152. vi.); and consequently $\breve{P}r + n^{III} = \breve{P}r$.

The faces of the horizontal prism $\bar{P}r + n^{IV}$ (*a*) appear as rhombs if combined only with P and $\breve{P}r$. This horizontal prism therefore is $= \bar{P}r - 1$ (§. 152. vii.).

The same horizontal prism produces parallel edges of combination with $P + n$ (*b*), in the place of its more obtuse terminal edges. Hence $\bar{P}r - 1$ and $P + n$ are co-ordinate forms, n is $= -1$, and $P + n = P - 1$.

The horizontal prism $\breve{P}r$ belongs to P. But at the same time it also belongs to, or produces parallel edges of combination with *d*, or that scalene four-sided pyramid of a dissimilar section with P, whose double representative sign has been expressed, either by $(\breve{P} + n^{II})^m$ or by $(\breve{P}r + n^{II})^m$.

If we suppose the corresponding finite diagonals of the horizontal prism, and the mentioned pyramid to be equal; the axes of the two forms must necessarily be also equal, and since $\breve{P}r$ belongs to P, the same applies to this fundamental pyramid; and hence we infer that the ratio of the said diagonal in the secondary pyramid to its axis is the same, which takes place in the analogous lines of the fundamental form itself.

Suppose in the pyramid, which is to be determined, the ratio of the three perpendicular lines, equal to $a' : b' : c'$. (§. 53. 6.); we have

$$a' : b' = a : b.$$

The horizontal prism $\bar{P}r - 1$ belongs to $P - 1$, but, on account of the parallel edges of combination, also to the pyramid *d*. If we proceed in comparing the axis and the diagonals as above, we find the ratio of

$$a' : c' = \tfrac{1}{2}\, a : c = a : 2.\, c;$$

and therefore the ratio of all the three lines

$$a' : b' : c' = a : b : 2.\, c.$$

If now we compare the co-efficients of this ratio with the general co-efficients for $(\breve{P} + n)^m$ (§. 94.); that is to say

$$1 : 1 : 2,$$

with

$$2^{n}.\, m : 1 : m;$$

we find $m = 2,\ n = -1$; and therefore

$$(\check{P} + n^{II})^m = (\check{P} - 1)^2.$$

By comparing them in the same way with those for $(\check{P}r + n)^m$ (§. 95.), or

$$1 \ : \ 1 \ : \ 2$$

with $$\frac{m+1}{2} \cdot 2^n : \ 1 \ : \ \frac{m+1}{m-1};$$

we have $m = 3,\ n = -1$, and

$$(\check{P}r + n^{II})^m = (\check{P}r - 1)^3.$$

In respect to the dimensions of the forms, it is quite indifferent which of the two designations we employ, for they both express exactly the same thing. Yet on account of the number of derivation 3, for the analogy with the pyramidal system, we rather prefer the latter. The vertical prisms become evident from the consideration of their belonging to the pyramids. One of them, *f*, is $P + \infty$ on account of its horizontal edges at the intersection with P; while the other, *g*, is $(\check{P}r + \infty)^3$, because the edges of combination of this prism with $(\check{P}r - 1)^3$ are horizontal.

The definite designation of this compound form will therefore be, according to the preceding developement,

$\bar{P}r - 1.$	$P - 1.$	$\bar{P}r.$	$(\check{P}r - 1)^3.$	$P.$
a	*b*	*c*	*d*	*e*

$P + \infty.$	$(\check{P}r + \infty)^3.$	$\bar{P}r + \infty.$
f	*g*	*h*

§. 156. TESSULAR COMBINATIONS.

A combination of the tessular system is more particularly said to possess a *Tessular Character*, if it contains the faces of the original forms peculiar to this system (§.121—127.), without any halves or fourths (§. 128.).

It would be superfluous to enter here into a minuter detail of the binary combinations, comprised under this head. Every thing necessary to know of them follows immediately from their derivation from the hexahedron, if we only attend to the different positions, which the faces of the forms combined assume in respect to the different axes. Hence even the angles of incidence at the edges of combination may immediately be deduced for those forms whose dimensions are invariable, their faces being perpendicular to one of the three kinds of axes (§. 40.). For these angles of intersection between the faces of such forms, and the angles at the centre produced by those axes, which are perpendicular to these faces, must be supplemental to each other. The algebraic formulæ given for the different systems of variable dimensions, may also be employed for obtaining the angles both of simple forms and of combinations of the tessular system. For this purpose, the simple forms peculiar to the tessular system, or rather parts of them contained under faces similarly situated in respect to a single axis considered as the principal one, may be considered as forms belonging to one of the preceding systems. In this case every thing applies to them, that has been above stated in respect to binary combinations. If, for instance, the hexahedron be considered as a rhombohedron $= R$; the horizontal faces of the octahedron will represent $R - \infty$, the inclined ones $R + 1$; and the combination of the hexahedron and the octahedron supposed to be a rhombohedral combination, will be expressed by $R - \infty$. R. $R + 1$. As a tessular combination, its crystallographic sign is H.O (§. 121. 124.). If we consider the octahedron as an isosceles four-sided pyramid of the pyramidal system, and designate it accordingly by P, the horizontal faces of the hexahedron will assume the situation of $P - \infty$, while the vertical ones assume that of $[P + \infty]$; thus, for the sake of applying the calculations, $P - \infty$. P. $[P + \infty]$ will express the same combination of the hexahedron and the octahedron.

This process also extends to combinations produced by more than two simple forms. Suppose, for instance, a combination of the hexahedron, the octahedron, the dodecahedron, and the first variety of the digrammic tetragonal-icositetrahedrons, to be considered as a rhombohedral combination, of which the hexahedron is the fundamental form $R = 90°$. The octahedron is consequently $= R - \infty. R + 1$; the dodecahedron $= R - 1. P + \infty$; and the digrammic tetragonal-icositetrahedron $= R - 2. (P - 1)^3. R + \infty$. The entire tessular combination expressed as a compound form of the rhombohedral system, is therefore $= R - \infty. R - 2. R - 1. R. (P - 1)^3. R + 1. R + \infty. P + \infty$. It may here be observed, that besides other forms, this combination contains four consecutive members of one series of rhombohedrons, one of which is $R = 90°$. The designation of this compound form, as belonging to the tessular system, is: H. O. D. A_1.

It is evident that this may likewise yield a method of finding the dimensions of the different varieties of such forms, as possess faces not perpendicular to any axis, of icositetrahedrons, of tetracontaoctahedrons, &c.

§. 157. SEMI-TESSULAR COMBINATIONS.

A combination of the tessular system assumes a *Semi-tessular Character*, if it contains one or more Halves. The semi-tessular combinations must farther be distinguished into semi-tessular combinations *of parallel faces*, and *of inclined faces*, according to the kind of halves which they contain (§. 128.).

Among those binary semi-tessular combinations, which contain only one Half, there are two in particular deserving of notice. The first of them is the combination of the octahedron with one of the hexahedral pentagonal-dodecahedrons. The faces of the octahedron appear as equilateral triangles in the place of the rhombohedral solid angles of the pentagonal-dodecahedron. If all the faces of the combination become triangles, the form produced is contained under eight equilateral triangles, originating from the octahedron, and twelve isosceles triangles originating from the hexahedral pentagonal-dodecahedron. This form has been called the *Icosahedron* of Mineralogy. The icosahedron, however, is not a simple form; and, on this account, it receives no particular name in the systematic nomenclature of forms (§. 49.).

The other is the combination of the hexahedron and the trigrammic tetragonal-icositetrahedron. The faces of the hexahedron appear in the figure of rhombs in the place of the prismatic solid angles of the icositetrahedron; and if they are enlarged till all the faces limiting the combination become tetragons, the result is that form which has been called the *Triacontahedron* of Mineralogy. This form is contained under six rhombs and twenty-four trapezoidal faces, the one and the other equal and similar among themselves. It is not a simple form; and therefore as little entitled to a peculiar systematic name as the icosahedron.

If two forms occur at the same time in a combination, we must attend to their position, whether they are both in the normal position, or whether one of them is in the normal while the other is in the inverse position; for the general appearance of the combination is very much influenced by this difference.

The two semi-tessular combinations of parallel faces, Figs. 75. 76., contain the same halves; with this difference only, that one of them in Fig. 76. is the inverse of the same in Fig. 75., the former being the combination $\frac{A2}{2\,ii}$ (c). $\frac{T1}{2\,ii}(f)$, and the latter the combination $-\frac{A2}{2\,ii}(c')$. $\frac{T1}{2\,ii}(f)$: and although the general appearance of the two forms is very different, yet an accurate comparison of the faces will easily shew their identity. Both of them occur in hexahedral Iron-pyrites.

The semi-tessular combinations of inclined faces, Figs. 77. 78., likewise contain similar halves, but Fig. 77. will be designated by $\frac{O}{2}(P). \frac{C_1}{2\,i}(l)$, while Fig. 78. is expressed by $\frac{O}{2}(P). - \frac{C_1}{2\,i}(r)$, since the trigonal-dodecahedron r in the latter combination is the inverse of the trigonal-dodecahedron l in the former, the tetrahedron in the normal position being common to both. These combinations occur in tetrahedral Copper-glance.

IV. OF THE IMPERFECTIONS OF CRYSTALS IN RESPECT TO THEIR FORM.

§. 158. TWO KINDS OF THIS IMPERFECTION.

The imperfections of crystals in respect to their form, originate either in the very formation of the crystals themselves, or they are the consequence of the contact of these with other minerals.

The imperfections of the crystalline forms are deviations from that regularity which has been supposed to take place in the preceding considerations of forms. This regularity requires the faces of crystals to be planes of a certain figure and extent, and the edges in which they intersect each other to be straight lines. It is very seldom met with in nature, perhaps never, if we examine the natural productions with the utmost accuracy; the deviations which it presents are founded in some cases upon the formation of the crystals themselves, if we find ourselves entitled to suppose that nothing external has had any influence upon the quality of the form; in other cases they depend upon the contact into which one crystal has come with another; for this is the means by which one of them could influence the shape of the other. The most interesting of these objects is the consideration of the first kind of these imperfections. The other will be considered more at large when treating of compound minerals; at present it is only necessary to examine in what shape an individual will appear, which is prevented from assuming its regular form by some external obstacle.

§. 159. DEVIATIONS FROM REGULARITY, DEPENDING UPON THE FORMATION OF THE INDIVIDUALS THEMSELVES.

Those deviations from the regularity of crystalline forms, which arise from the formation of the individuals themselves, refer both to simple forms and to combinations. They appear either in the size and figure, or in the physical quality of their faces.

The deviations from regularity take place in two different ways:

1, By the disproportionate and irregular enlargement or decrease of some of the faces, or

2, By their curvature, or in general by the property of not being mathematical planes.

With respect to the first, it must here be observed, that the regular enlargement of certain faces, considered above in the combinations called hemi-rhombohedral, hemi-pyramidal, &c., does not enter within the limits of our present examination; on the contrary, simple forms and combinations of that kind may, as well as any others, be subject to those deformities of which we are now treating.

We find many examples of such irregularities in simple forms. The faces of the hexahedron, for instance, very often are not squares, but oblong or rectangular figures; sometimes only two of them are squares. Of the faces of the octahedron, four become irregular tetragons, and two equiangular hexagons. The whole form of the dodecahedron is sometimes elongated or shortened in the direction of one of its axes: if this be a rhombohedral one, the form will assume the aspect of a combination of the rhombohedral system; if it be a pyramidal axis, the dodecahedron will assume the appearance of a combination of the pyramidal system; and if it be a prismatic axis, it will have the aspect of a combination of the prismatic system. The same changes are sometimes met with in the digrammic tetragonal-icositetrahedron. It is also a case not unfrequently occurring, that single faces are enlarged in the manner just described, of which the isosceles six-sided pyramids of rhombohedral Quartz may be quoted as a remarkable instance.

It will not be amiss to mention here the following precautions, in order to avoid the errors into which such irregularities might lead. First of all, those angles must be carefully examined, in which the faces of the forms intersect each other. Suppose, for instance, a form, exhibiting the aspect of a vertical oblique-angular four-sided prism, combined with a horizontal one, which belongs to the long diagonal of the former, but whose edges are all $= 109° 28' 16''$ and $70° 31' 44''$; this form will be the octahedron. If in a form representing a rhombohedral combination of $R + n$ and $P + \infty$, or in a pyramidal one of $P + n$ and $[P + \infty]$, all the edges are $= 120°$, this form will be the dodecahedron. A form, which seems to be hemi-prismatic, and composed of an oblique-angular four-sided prism, and half the number of the faces of a horizontal one, if the edges of combination prove to be equal to those of the prism, is not what it appears, but it is a rhombohedron. If, on the contrary, in a solid contained under six rhombic faces, those edges which represent the terminal edges of the rhombohedron, be not equal, the solid itself will not be a rhombohedron, but a hemi-prismatic form of the description given above.

In the second place, it is necessary to attend to those forms which enter into combinations with the one, respecting the determination of which there exists some uncertainty.

If in a right rectangular four-sided prism, instead of one or more of its solid angles, we observe equilateral triangles, the form will be the hexahedron; if these triangles be isosceles, the form will be the right rectangular four-sided prism of the pyramidal system; and should they be scalene, we have reason to suppose (§. 150.), that the form in question will be the right rectangular four-sided prism of the prismatic system.

In similar cases, the forms of cleavage (§. 167.) allow very often the same application as crystalline forms. The octahedral Fluor-haloide may serve as an example; the angles of a right rectangular four-sided prism of this species may be taken away, or broken off, by equilateral triangles, which are faces of cleavage. The figure of these triangles proves the form to be the hexahedron, although perhaps not one of its faces is a square. The following chapter will contain farther observations on cleavage.

In the third place, the quality of the faces (Chap. III.) must be considered. Nothing is more easy than to decide whether all the faces under which a form is contained, are of the same quality, or whether they differ from each other in this respect. If this quality is the same in all the faces, the form may be a simple one; if it is different, the form must be a combination of at least as many simple forms, or symmetrical halves and fourths, as there are differences existing in the qualities of the faces. A right rectangular four-sided prism, contained under faces of three different qualities, must belong to the prismatic system. If the faces present only two different qualities, the form may belong to the pyramidal system, though by this description of the faces it is not yet excluded from the prismatic system; and it may belong to the tessular system, if all its forms are exactly of the same quality. In this last case, however, the form is not excluded from any one of the other two systems.

The same is evidently applicable to combinations. It sometimes happens that some of the faces belonging to the simple forms which the combination contains, are irregularly increased, whilst other faces belonging to the same forms, diminish till even they entirely disappear. This will naturally produce differences in the figure of the faces. The method to be followed in occurrences of this kind consists in reducing the homologous faces to their regular size and figure, which is effected by giving them an equal distance from the centre of the form, and by adding, if necessary, those faces which do not appear at all in the combination; provided the observation of one or more of the same simple form entitles us to suppose this form to be one belonging to the combination. In thus completing irregularly limited compound forms, it will always be necessary to reflect on the possibility of the forms possessing a hemi-rhombohedral, a hemi-pyramidal, a semi-tessular, &c. character. If, for instance, in a hexahedron four of the solid angles are replaced by equilateral triangles corresponding to the faces of the tetrahedron, we are not entitled to suppose that the four remaining solid angles too should be truncated, because in this case it is the tetrahedron, and not the octahedron, which is contained in the combination; but if only one more of the remaining solid angles be replaced by a triangle of the same description, then we are fully entitled to add the rest of the faces required for the production of the octahedron, or perhaps of two tetrahedrons, since either the octahedron or two tetrahedrons are really contained in the combination.

Sometimes the combinations, like the simple forms, are elongated or depressed in the direction of one of their axes; and they assume the aspect of such forms as belong to the system to which the lengthened or shortened axes refer. Examples of this kind occur very frequently in the combinations of the hexahedron and the octahedron of hexahedral Lead-glance. The crystals are sometimes elongated in the direction of a prismatic axis, or depressed in the direction of a rhombohedral one. The latter assume the appearance of a rhombohedral combination, while the former possess the aspect of a combination of the prismatic system. The most common form of rhombohedral Quartz is a combination of an isosceles six-sided pyramid P with the regular six-sided prism $P + \infty$ (Vol. II. Fig. 145., abstraction being made of the faces *s* and *s'*). Very often this combination appears flattened in the direction of a prismatic axis, and then it takes the aspect of a combination of $\breve{P}r$. P. $(\breve{P}r + \infty)^3$. $\breve{P}r + \infty$ of the prismatic system, much resembling Vol. II. Fig. 16. Also in this case the means quoted above will secure us from errors which a little practice very soon teaches to avoid in imperfectly formed varieties of crystallisations.

The curvature of faces, if occurring in simple forms, in general affects all the faces at once. Thus it is in the hexahedrons of octahedral Fluor-haloide; in the dodecahedrons, icositetrahedrons, and tetracontaoctahedrons of octahedral Diamond; in the rhombohedrons of the two species of Parachrose-baryte, &c. Similar imperfections produce the lenticular forms, particularly the saddle-shaped lens of the above mentioned species; which is more correctly represented in Fig. 79. than in any of the drawings and models hitherto published, in which, for the greater part, it is given with four corners, instead of six.

In combinations, the curvature always takes place upon homologous faces, while the rest are not affected by this deformity. Examples may be found in the species of prismatoidal Gypsum-haloide, of paratomous Augite-spar, of octahedral Diamond, &c.

Curved edges are produced by the intersection of curved faces; rounded edges, too, arise evidently from the curvature of the adjoining faces.

From the preceding considerations of the irregularities of crystals, it is plain that it is necessary to observe the greatest precautions in ascertaining the measures of their angles, if we wish to obtain useful and correct results. The inaccuracies of so many of these measurements, are not always errors arising from the imperfection of the instrument, or from the operation; but very often they are the consequences of the imperfection of the crystals themselves. Small crystals are commonly less subject to these irregularities than large ones; and to this, in particular, the great advantage must be ascribed, which the Reflective Goniometer possesses over the common one, because, in applying the former, we may make use of crystals which are smaller, and therefore in general more perfectly formed, while the common goniometer requires crystals of a larger size, if it shall be applicable at all, and these are but rarely found in the necessary perfection.

The reflective goniometer must necessarily be employed in those accurate and desultory observations, upon which are founded the determinations of the angles of the fundamental form, and consequently also the dimensions of a natural-historical species; the common goniometer, however, will always be found sufficiently accurate, if our object be only to discriminate individuals, or to find out the place of a given one in the system, by the assistance of the Characteristic.

Notwithstanding all the variability in the size and in the figure of the faces depending upon it, both of which may be referred to the imperfect formation of the crystals; yet the situation of those faces towards each other will be always found constant. The faces of forms, both simple and compound, constantly intersect each other at the same angles which they would produce if the form had arrived at the highest possible degree of perfection, which depends upon the exact equality and similarity of the homologous faces. The magnitude of the angles is constant. This remarkable fact has first been ascertained and demonstrated by the celebrated Romé de l'Isle; it is the basis upon which is founded the possibility of applying crystallography to the mineral kingdom. Doubts have been raised against the correctness of that law, derived from an apparent transition of certain crystalline forms into others, by continual changes in the magnitude of the angles: these doubts, however, immediately disappear, if we consider the crystals in their greatest geometrical perfection, and not affected by any of those irregularities to which they are subject. Hence we may infer, that the crystallisation of minerals, from the simplicity and constancy of its laws, under the appearance of the greatest variability, deserves on that account to be called the most remarkable of those phenomena which inorganic nature presents to the observer.

§. 160. DEVIATIONS FROM REGULARITY, DEPENDING UPON THE CONTACT WITH OTHER INDIVIDUALS.

Crystals may either touch on all sides, those minerals to which they are adjoining, or they may adhere to them only by some of their parts.

Crystals, surrounded and inclosed by the solid mass in which they are found, or in which they have been formed, are in contact with this mass on all sides. This mass may either be homogeneous (§. 23.) to that of the crystals, or may not be homogeneous with it. In the first case, the regularity of the form is almost without any exception so much disfigured, that not even a trace of it will remain. One of the individuals prevents the other individual, by their contact, from assuming that regular form which is peculiar to it ; and, in fact, we have very often occasion to observe, that the individual really assumes this regular form, whenever a part of it emerges from the contact with other individuals.

Examples of this are frequently found in the compound varieties of rhombohedral Lime-haloide, and of other species. The individuals in these compositions are real crystals, which only have been prevented by their mutual contact, from assuming their peculiar regular forms. Very often we find cavities or empty spaces in the interior of such compound varieties ; the individuals lining these cavities present regular forms, wherever they do not touch the rest of the compound mass.

In these compositions the crystals sometimes lose only the regularity of their form, while they still continue to present its general aspect. Thus, for instance, the particles of such species whose forms belong to the tessular system, often have their three dimensions nearly equal, while many of those of other systems have two dimensions greater or less than the third. *The System of Crystallisation, however, cannot be inferred from this observation.* If one or two of the dimensions diminish much in size, the individuals become thin laminæ or fibres, of which the latter very often are much thinner than a human hair.

Nay, they may even withdraw themselves entirely from observation, if the third dimension too nearly disappears. Yet the compound mineral can never on this account be transformed into a simple one. This subject will be explained more at large in Section II.

If the mass which surrounds a crystal, and this crystal itself, are not homogeneous, the regularity of the latter is not always impaired. A crystal which, under such circumstances, has retained its regular form, is said to be formed *imbedded*, and if separated from its support, it is termed a *loose Crystal.*

Crystals of this kind may be taken out of the mass which surrounds them, and if they do not cohere with any particles of the mass, a smooth print of their form will remain. Loose crystals, if not perhaps imperfect on some other account, may be considered as the *most perfect productions* of inorganic nature. But we rarely find such crystals. Commonly they are imperfectly formed of themselves, or part of their perfection has been lost in the contact with the surrounding mass. Those individuals, whose dimensions are nearly equal, appear in this case as roundish masses, more or less spheroidal, or as angular masses, and bear the names of *Grains* or *Angular Pieces.* Both the grains and the angular pieces, therefore, are nothing else but crystals imperfectly formed.

Besides these minerals, which indeed are nothing but imperfectly formed crystals, there exist a great many others, which likewise assume more or less a spheroidal shape, or that of grains and angular pieces. These, however, must be carefully distinguished from real grains and angular pieces, because they are not simple, but compound minerals.

Crystals which are formed in an empty space, and adhere only with some of their parts to the support, which is, in most cases, different from the mass of crystals, are termed *implanted* crystals. Implanted crystals are always incomplete; because those parts are wanting in which the crystals are attached to the supporting mass. They cannot be removed from it, so as to leave behind a print of their form; they only may be broken off from the support with which they cohere more or less firmly.

Implanted crystals must be duly completed, for the purpose of a crystallographic consideration; so must also those crystals, which, by some accident, or on purpose, have been broken or rendered incomplete. The only rules we must attend to in this process are those of symmetry, by which a perfect equality and similarity is established as to the number and situation of faces between those parts of the crystal which are wanting, and those which may be observed. The most common crystallisation of rhombohedral Quartz, consists of an isosceles six-sided pyramid, which is combined in a parallel position with a regular six-sided prism. As these crystals very often occur implanted, the observation of one end of the pyramid only is possible; evidently the opposite termination of the crystal must be completed, by supposing it equal and similar to that which has been observed. Simple pyramids of rhombohedral Quartz, (and in similar cases also the forms of other minerals), if they present only one of their apices to the observer, must likewise be completed according to the rules of symmetry; and we can never be entitled to assume or consider such things as *simple* pyramids, because those do not exist among the productions of nature, nor are they obtained from the different processes of derivation (§. 80.—83.). Similar examples occur in pyramidal Garnet, in octahedral Fluor-haloide, in prismatic Hal-baryte, &c.; which must be completed according to the method explained above.

There are cases, however, in which it becomes necessary to allow of an exception of that rule. These comprehend the crystals, in which two opposite solid angles possess a different configuration (§. 147.). Evidently this difference always must remain within the range of the series of crystallisation. It has been observed, that certain crystals, part of which is differently formed from another one, which is similarly situated, likewise present differences in their electric action, on being exposed to an elevated temperature. Prismatic Zinc-baryte, prismatic Topaz, rhombohedral Tourmaline, and tetrahedral Boracite, may be quoted as examples of this peculiarity. Could this observation be established as a general law, it might prove useful in completing crystals thus imperfectly formed, though it would not indicate what faces are to be added on that termination which is opposite to the observed one.

Another case, in which the two opposite terminations of crystals are differently formed, does not refer to the present place, in as much as it is found only in compound minerals. It will be treated of more at large in §. 179.

The preceding ones are the most simple modes of the occurrences of minerals in nature.

CHAPTER II.

OF THE STRUCTURE OF MINERALS.

§. 161. EXPLANATION OF STRUCTURE.

Structure represents the mechanical connexion among the particles of a simple mineral. It may be observed, if we destroy this connexion, or separate the particles from each other.

We have to distinguish here between the *regular* and the *irregular* structure.

If we break a crystal of hexahedral Lead-glance, or of rhombohedral Lime-haloide, we observe particles detached which are contained under even, smooth, and shining faces. The property of allowing these particles to be separated, but not the particles themselves, exists in the minerals, previous to its having been rendered visible by mechanical force. The particles of a mineral therefore necessarily must stand in some regular mechanical connexion, because the faces in which they separate are parallel to faces of regular forms. It is by means of the division that we acquire a knowledge of this connexion, or of what is understood by the regular structure of individuals.

If we divide the particles of an individual in other directions than in those of regular structure, the division takes place not in even faces, but in uneven faces of different descriptions, and by this the regular structure is not rendered observable, although it does, or at least may take place in the same individual. These particles no longer possess the property of being regularly divisible parallel to faces obtained by this kind of division; and the quality of the faces therefore demonstrates, that no regular mechanical connexion of the particles can take place in these directions. The connexion between the particles in these directions, is also termed the irregular structure.

It is sometimes attended with considerable difficulty to ascertain the regular structure, and very often it is no less difficult to observe the irregular structure. The particles of certain minerals, as of hexahedral Lead-glance, of rhombohedral Lime-haloide, &c., so very readily separate in the direction of their regular structure, that it becomes almost impossible to produce any divisions in another direction; although they may allow of such a division. On the contrary, others separate with much greater facility in every direction but that of the regular structure, so that it likewise becomes difficult to observe even traces of it, though by analogy we are led to suppose the existence of the regular structure.

The regular structure of minerals is observed in their *Cleavage*, the irregular structure appears in their *Fracture*; both, fracture and cleavage are comprehended under the more general idea of *Structure*.

§. 162. CLEAVAGE.

An individual is said to be *cleavable*, or to admit of *Cleavage*, if by a mechanical separation of its particles, the *regular* structure can be rendered visible.

Certain individuals may be cleaved with great facility; and with only the blow of a hammer, they will divide into fragments contained under even faces, like those mentioned in the preceding examples. This is not the case with others, in which the mere percussion yields only irregular faces. In these, however, we are not yet forced to assume the non-existence of cleavage, but by the help of delicate chisels, or of other appropriate instruments, and by a careful examination of the resulting faces, we have to determine whether or not cleavage occurs in the individual. It is very useful to expose such faces to an intense light, the reflection of which will very soon decide which of these is the case. A certain degree of skill is required in cleaving minerals, which, however, a little practice will teach more accurately than could be done by many words; and therefore we may omit here, as superfluous, all farther particulars.

§. 163. FACES OF CLEAVAGE.

The faces obtained in cleaving a mineral, are termed its *Faces of Cleavage.*

The faces of cleavage are distinguished from each other in respect to their properties or their relative aspect.

These properties depend upon the perfection of the faces, in as far as they may be compared to mathematical planes, and upon the degree of lustre which they possess. It is very easy to tell, from the mere ocular inspection of faces, whether their quality be the same, or whether they differ more or less from each other. These differences, however, particularly in respect to lustre, will be considered more accurately on another occasion.

Faces of cleavage, which are of the same quality, and appear in one and the same individual, or in one and the same species, are said to be *homologous*; faces of cleavage of different qualities, if they appear under the same circumstances, are considered as being faces of cleavage not homologous with each other.

Sometimes faces of cleavage appear to be curved. Irregularities of this kind require to be considered, like those mentioned above in respect to the faces of crystallisation. But very often curved faces of cleavage result from the composition of several individuals in a position little different from the parallel one. Fig. 80. shews a remarkable instance in rhombohedral Lime-haloide, where the axes of the individuals diverge very little from a common centre, by which the compound product of cleavage assumes the appearance of a rhombohedron, of which three faces are convex, and the opposite ones concave.

§. 164. DIRECTION OF CLEAVAGE.

The direction in which the individuals of a species allow themselves to be cleaved, is the *Direction of Cleavage.*

The direction of the faces of cleavage is constant; its situation in respect to the fundamental form, or any other derived crystalline form of the species, is determined, and not subject to any alteration.

The directions of cleavage in every individual are not found in the same number. The individuals of most of those species which constitute the order Mica, can be cleaved only in one direction, and therefore possess only one direction of cleavage. Many species of the order Spar contain two; rhombohedral Lime-haloide and hexahedral Lead-glance contain three; octahedral Fluor-haloide contains four; dodecahedral Garnet-blende six; and in many minerals cleavage may be effected in still more directions, differing in number, and often in quality. The direction of cleavage more particularly regards such as produce the most apparent faces of cleavage. If two directions of cleavage exist at the same time in a single individual, wherever the faces corresponding to them are obtained, they always intersect each other at the same constant angles. This is a necessary consequence of the parallelism of all those faces of cleavage, which lie in one and the same direction.

§. 165. CHARACTER OF CLEAVAGE.

The *Character* of cleavage consists in the constancy of its direction (§. 164.), and in the possibility of separating the particles of individuals in this direction, as long as the acuteness of our senses, and the delicacy of our instruments will allow.

There are minerals whose particles may be separated from each other in faces which are regularly situated in respect to the crystalline forms, but which exist previous to the actual division. Those masses, however, which are contained between two such faces, allow of no farther cleavage. This property of certain minerals will be considered more particularly in §. 179. It is, however, very distinct from real cleavage, the character of which consists in the possibility of continuing it, as long as our senses may perceive it, or our instruments and contrivances may answer the purpose.

Experience shews that cleavage indeed does possess this property, since it may be effected, to whatever point of the cleavable individual the instrument is applied in the required position. If, therefore, an individual is cleavable in the direction of a certain plane, it must also be cleavable in any other plane parallel to the former, the distance of these planes being less than any given straight line.

Several minerals may be cleaved into exceedingly delicate laminæ, others do not admit of cleavage to such an extent. Among the first, several species of the genus Talc-mica are particularly remarkable. Prismatoidal Gypsum-haloide may also be cleaved into uncommonly thin laminæ; and we might succeed in attenuating them still more, if instruments could be found of sufficient delicacy. Cleavage may be continued so far in these cases, because, except that single cleavage, there are no other directions in which the minerals cleave with the same facility, or, what is the same thing, because it is very difficult to separate their particles at all in other directions. The other class of cleavable minerals comprehends the individuals of such species as present more than one direction of cleavage, or whose particles may be more easily separated in uneven irregular faces. The facility with which the particles may be separated from each other in more than one direction of cleavage, or in irregular faces, prevents the cleavage from being more apparent, and obtained with greater facility in one of the directions. Hexahedral Lead-glance, rhombohedral Lime-haloide, &c. may be quoted as examples of minerals, which cleave with equal facility in more than one direction.

§. 166. FACES OF CLEAVAGE PARALLEL TO FACES OF CRYSTALLISATION.

Every direction of cleavage (§. 164.) is parallel to the face of a form of the *Series of Crystallisation* of that species, to which the cleavable individual belongs.

In the species of octahedral Fluor-haloide, the solid angles of the hexahedron may be broken off by cleavage with the greatest facility; in the place of every one of those solid angles, there will appear an equilateral triangle, or an equiangular hexagon, which is the face of cleavage. On account of its figure, it is situated perpendicularly to a rhombohedral axis, that is to say, like a face of the octahedron, a form which in fact belongs to the series of crystallisation of octahedral Fluor-haloide. The solid angles of the octahedrons cannot in this species be broken off by faces of cleavage, but this form may be cleaved parallel to its own faces in four directions, intersecting each other at angles of 109° 28′ 16″ and 70° 31′ 44″.

The directions of the faces of cleavage in which the pyramid $(P)^3$ of rhombohedral Lime-haloide may be cleaved, are parallel to those of faces passing through every two consecutive lateral edges of the pyramid. But this is the direction peculiar to the faces of the rhombohedron R. The faces of cleavage are therefore parallel to the faces of the rhombohedron R, which is the fundamental form in the series of crystallisation of the species. In the rhombohedron, the cleavage may be continued parallel to its own faces, in three directions, intersecting each other at angles of 105° 5′ and 74° 55′.

Rhombohedral Talc-mica, pyramidal Kouphone-spar, prismatic Topaz, &c. admit of a cleavage perpendicular to their axes. The faces of cleavage will be parallel either to $R - \infty$ or to $P - \infty$, both of them being limits of their respective series of crystallisation.

It appears, from the given examples, that not every crystalline form is cleavable parallel to some one or the other of its faces. Several of them, however, shew this property. Simple forms of finite dimensions, if cleavable parallel to their own faces, have on that account by preference been chosen for fundamental forms, even though such forms should not as yet have been produced by nature among the crystalline forms of the species.

§. 167. FORM OF CLEAVAGE.

A form contained only under faces of cleavage, is termed a *Form of Cleavage*.

The forms of cleavage may be either simple forms or combinations. Both may be either perfect (finite) forms, or imperfect (infinite) forms.

If we continue cleaving an individual of octahedral Fluor-haloide, till it is comprehended on all sides within faces of cleavage, the result will be the perfect form of cleavage belonging to the species; it will be a simple form, because the faces of cleavage homologous to each other, are parallel only to the faces of the octahedron, and not to those of any other simple form. It is exactly the same with the forms of cleavage of hexahedral Lead-glance, of hexahedral Rock-salt, of octahedral Corundum, of dodecahedral Garnet-blende, and many other species.

Peritomous Ruby-blende cleaves in the direction of the regular six-sided prism $R + \infty$. The form of cleavage is therefore a simple one; but it is incomplete or infinite, and terminated in the direction of the axis, either by faces of separation, which are not faces of cleavage, or by faces of crystallisation.

Prismatic Topaz cleaves parallel to the faces of $P - \infty$. The form of cleavage is a simple one, but imperfect; and is contained in the directions parallel to the axis under faces of crystallisation, or under such faces of separation, as do not owe their existence to cleavage.

Rhombohedral Fluor-haloide cleaves parallel to the faces of $P + \infty$; but at the same time, also to those of $R - \infty$. The form of cleavage, $R - \infty. P + \infty$, is therefore a combination. But it is a perfect form, since by these faces the space is limited on all sides.

Pyramidal Feld-spar and pyramidal Garnet are cleavable in the direction of two rectangular four-sided prisms; and at the same time perpendicularly to their axis. Their form of cleavage therefore, $P - \infty. P + \infty. [P + \infty]$, is a combination and at the same time a perfect form of cleavage.

Paratomous Augite-spar, and in most cases also hemi-prismatic Augite-spar, cleave in the direction of vertical oblique-angular four-sided prisms; and at the same time in the direction of planes, passing through the axes and the diagonals of the prisms. Their forms of cleavage are expressed by the combination $(\breve{P}r + \infty)^3 . \breve{P}r + \infty . \bar{P}r + \infty$ which is imperfect, that is to say, unlimited in the direction of its axis, except by faces of crystallisation, or by faces of cleavage different from those contained in the representative sign of the compound form, or at last by faces not produced by cleavage at all.

§. 168. FORMS OF CLEAVAGE, DISTINGUISHED ACCORDING TO THE QUALITY OF THEIR FACES.

Faces of cleavage, which belong to one and the same simple form, are homologous (§. 163.). Faces of cleavage which are not homologous, belong to different simple forms of cleavage.

If we find an opportunity of comparing the form of cleavage of an individual with its form of crystallisation, it will not be difficult to decide which of the faces of cleavage belong to one, and which to different simple forms; and indeed the certainty and generality of the present proposition depends upon observations of this kind.

In many instances, however, we only can examine forms of cleavage, belonging to minerals which are not crystallised, or such individuals as present no regular external forms, and in which therefore the forms of cleavage are of still higher value for the natural-historical determination. Here the above mentioned proposition becomes of the greatest utility, since it allows the forms of cleavage to be considered in the right point of view, even though only a few of their faces should present themselves to the observer.

As to the first, we find that all the faces of cleavage in the individuals of rhombohedral Lime-haloide, of octahedral Diamond, of dodecahedral Garnet-blende, of prismatic Topaz, and of many others, are of exactly the same quality, so that they can by no means whatever be distinguished from each other; these faces being found to possess the same properties throughout the whole individual. A very remarkable difference, however, is found in several species, relative to the quality of the faces of cleavage in different varieties. One of the most striking examples of this occurs in hexahedral Iron-pyrites. Several varieties of this species cleave very readily in faces which are pretty even, shining, &c.; others only with the greatest difficulty in faces very much interrupted by asperities. In both cases, however, the resultant form of cleavage is the hexahedron; and the faces under which it is contained are of the same perfection and quality, at least in each individual taken separately, as it must be on account of their belonging to one and the same simple form.

The circumstance that two or more faces of cleavage possess exactly the same properties, cannot be considered as a proof of their belonging to one and the same simple form. An example of this may be taken from prismatic Gypsum-haloide. The form of cleavage of this species is represented by the combination $P - \infty . \breve{P}r + \infty . \bar{P}r + \infty$: the two latter, although faces of two different forms, shew almost exactly the same quality; and yet the same two forms in prismatoidal Gypsum-haloide, where they likewise appear as forms of cleavage, differ very much in their aspect.

On the contrary, in respect to the second part of the proposition, the difference of the simple forms to which they belong, follows in every instance from the different quality of the faces of cleavage, or from their not being homologous. In the above mentioned compound form of cleavage of pyramidal Feld-spar, the face perpendicular to the axis appears very different from those parallel to it, being less even, &c.: in a similar way in rhombohedral Emerald, the face perpendicular to the axis is more perfect and even than those parallel to it. As a very remarkable example, the form of cleavage of prismatoidal Gypsum-haloide deserves our particular attention. The form considered by itself appears as a right oblique-angular four-sided prism. It is more natural, however, to consider it in reference to its crystalline forms, as the hemi-prismatic oblique-rectangular four-sided prism (§. 153.), similar to Vol. II.

Fig. 46., of the dimensions given in the Characteristic. From this point of view it becomes at once evident, that only those pairs of parallel faces which are of the same quality, can belong to the same simple forms, which could not agree with the hypothesis of a right oblique-angular four-sided prism. The crystalline forms being always supposed to have been previously brought into an upright position, the most even, smooth, and shining face of cleavage will correspond to the form $\breve{P}r + \infty$. The other two forms, likewise very different from each other, though less so than from $\breve{P}r + \infty$, correspond to $\bar{P}r + \infty$ and to $-\frac{\bar{P}r.}{2}$ Many individuals, containing only one very apparent face of cleavage, as, for instance, the one perpendicular to the axis in pyramidal Euchlore-mica, in prismatic Topaz, &c. besides shew traces of other forms of cleavage, which here likewise may serve as examples.

But the most interesting of all the consequences to be drawn from these and similar examples, is the full confirmation of the theory of crystalline forms, as it has been given in the preceding chapter. According to that theory, there exist forms which, although they appear in a single face, or in two or more, in such directions that they cannot include the space from all sides, yet must be considered as peculiar simple forms. The physical quality of the faces of cleavage in the individuals incontestibly proves the correctness of that method of considering forms, which nevertheless has been merely the consequence of geometrical inquiries.

§. 169. FORMS OF CLEAVAGE MEMBERS OF THE SERIES OF CRYSTALLISATION.

The forms of cleavage represent members of the series of crystallisation of those species, from the individuals of which they have been extracted.

The demonstration of this proposition follows immediately from §. 166. For, if the faces containing a form of cleavage are parallel to the faces of a crystalline form in the same species, the form obtained by cleavage must itself necessarily be similar to the form of crystallisation ; it must possess the same dimensions and relations, and therefore be capable of being substituted for the member in the series. This is applicable to both simple and compound forms of cleavage ; because nature produces combinations of such simple forms only as are members of the same series (§. 139.). Cleavage therefore extends the application of crystallography to the productions of the mineral kingdom, and enables us not only to determine the system, but also very often even the series of crystallisation of such individuals or species, in which crystals are either not known at all, or at least are not the immediate object of our observation. Hence the study of cleavage is particularly recommended to those who intend to apply the Characteristic to nature, and to acquire that degree of skill which is required for determining with facility and certainty the productions of the mineral kingdom by the assistance of the Characteristic.

An accurate knowledge of the peculiarities of cleavage is moreover very useful for recognizing and completing such crystalline forms as occur indistinct, imperfect, cohering with others, &c. Some single faces, fissures in the interior, striæ arising from the superposition of laminæ and other observations of that kind, are very often sufficient for assigning the true position to a crystalline form, and by this means to acquire a correct knowledge of its nature.

The preceding observations indisputably shew, that cleavage in itself is a highly remarkable phenomenon of inorganic nature. Yet this will appear still more strikingly, if we consider it in connexion with the forms of crystallisation, in as far as they both refer to the natural-historical species. If we attend also to the less apparent directions of cleavage, or those faces which are less distinct, though subject to the same laws, we are led to the conclusion that cleavage represents the phenomenon of crystallisation to its full extent. The latter is therefore not a mere accident of the exterior form, but it is a property intimately related to the existence of the species itself; and every one of the forms which it is capable of assuming, is deeply founded in its interior, or in the regular structure of the mineral; so that a face of crystallisation is parallel to every direction of cleavage, and a direction of cleavage, more or less distinct, parallel to every face of crystallisation of the species. The first of these results has already been perfectly confirmed by observation. More accurate information with respect to the second will be obtained by future investigations of this interesting subject.

§. 170. DESIGNATION AND NOMENCLATURE OF THE FORMS OF CLEAVAGE.

The forms of cleavage are designated like those of crystallisation; several particular cases of its occurrence have been provided with appropriate verbal expressions, for the purpose of the Systematical Nomenclature and of the Charăcteristic.

Since the crystallographic designation of forms of crystallisation and of forms of cleavage is exactly identical; it becomes necessary to indicate whether the sign refers to cleavage or to crystallisation.

In the systems of variable dimension, the cleavage is said to be *Axotomous**, if it consists of a single face perpendicular to the axis, or parallel to the base of the fundamental form. The same expression may be employed, although, beside this single cleavage, others should appear parallel to the axis, or including an angle with it, yet it is always required, that such faces should be less distinct, and thus form a contrast with the single one. This observation extends also to the following expressions relative to some other peculiarities of cleavage.

* From, ἄξων, the axis, and τέμνω, I cut or cleave.

Cleavage is termed *Prismatoïdal*, if it takes place in a single direction parallel to the axis, whatever may be the simple form corresponding to this direction.

It is said in general to be *Monotomous**, if it consists of a single face, which may be perpendicular, or inclined, or parallel to the axis.

The term *Paratomous*† refers to faces of cleavage of an indeterminate number, parallel to the faces of a finite form, and which therefore are not vertical, nor perpendicular to the axis of the fundamental form. This expression may also be employed if the directions of cleavage correspond only to half the number of faces of a simple form.

Cleavage is termed *Peritomous* ‡, if it takes place in more than one direction parallel to the axis, and if the faces are all of the same quality and perfection. The result of this cleavage is a vertical prism.

If the directions of cleavage are parallel to the faces of a vertical oblique-angular four-sided prism, and at the same time to those of a horizontal one, the cleavage is said to be *Di-prismatic.* This di-prismatic cleavage and the prismatoidal cleavage is confined to prismatic, hemi-prismatic, and tetarto-prismatic forms. The rest may occur in any of the systems of variable dimensions, which, if necessary, is expressed by *Cleavage, rhombohedral and paratomous*, &c. In the peritomous cleavage, the form obtained *may* be a combination; it *is* a combination in the di-prismatic cleavage.

The expressions employed for the designation of the particulars of cleavage in the tessular system, are evident from themselves, and therefore need no farther explanation.

§. 171. FRACTURE.

To *break* a mineral, in order to obtain its *Frac-*

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* From μόνος, single, and τέμνω, I cleave.
† From παρὰ, about, and τέμνω, I cleave.
‡ From περὶ, round, and τέμνω, I cleave.

ture, is to make its *irregular* structure appear, by a mechanical separation of its particles.

Every individual, cleavable or not, may be broken. Irregular structure, that is to say, fracture, may be much more generally observed than cleavage. Fracture, however, on account of the want of regularity essential to it, can only be of very limited use in the Natural History of the Mineral Kingdom. It may here be useful, from this observation, to derive the important consequence, *that it is not a generality or a variety, but only a regularity and constancy in the differences occurring in a natural-historical property, which renders it applicable, and determines its value as a distinctive character in Natural History.*

Fracture is considered here as a property of individuals, or of simple minerals in general, agreeably to the principles of Natural History. The greater part of the varieties of fracture, quoted in books on Mineralogy, on that account must here be excluded, because they refer to compound minerals, which in fact also may be broken in pieces. These, however, will be considered in another more convenient place.

§. 172. FACES OF FRACTURE.

The faces in which the particles of the individuals separate when broken, are termed *Faces of Fracture.*

The kinds of fracture are determined according to the quality of its faces. The irregularities of these faces are either round or angular. The first sometimes represent the aspect of the inside of a shell. That kind of fracture which is formed by such faces, is termed the *Conchoidal* fracture, and provided with peculiar adjectives referring to the size, concavity, lustre, &c.; all this, however, without any useful consequences. The angular irregularities cannot be compared to any thing at all. The kind of fracture formed by these asperities, has received the denomination of the *Uneven* fracture, and has been divided again according to differences in the size of its grain, but this too is of very little value. There exists an immediate transition between the two kinds of fracture, in which only arbitrary limits can be fixed. The other kinds of fracture, the *even* fracture, the *fibrous* fracture, the *splintery* fracture, either do not refer to simple minerals, or they do not belong to structure at all. The *hackly* fracture is not produced by breaking, but by tearing a mineral in pieces. *Foliated* fracture is cleavage; the *passage of the folia* means the same as the directions of cleavage; *regular fragments* are the forms of cleavage; and the varieties of *radiated* fracture are the same, only referring to compound minerals.

§. 173. CHARACTER OF FRACTURE.

The faces of fracture preserve *no* constant direction. In this particular, fracture is essentially different from cleavage.

Individuals without cleavage, or with a very indistinct cleavage, or such individuals whose particles possess only a small degree of coherence, may be broken in any direction, that is to say, no direction could be indicated beforehand, and determined in reference to the situation of any line or plane, in or parallel to which the mineral would break. Fracture, therefore, has no constant or determined direction; neither can it be continued according to parallel planes, although, by a continual diminution in size, we may carry on the separation of the particles to an indefinite extent.

The same applies also to those individuals which allow of cleavage, if they be broken, that is to say, if their particles be separated in any other directions than those of cleavage.

The irregular structure is very often observable, along with the regular structure in the same individuals.

CHAPTER III.

OF SURFACE.

§. 174. SURFACE IN GENERAL.

The *Faces of Crystallisation* (§. 28.) are the most interesting and useful among all those faces which limit the forms occurring in the productions of the mineral kingdom.

Beside the faces of crystallisation, Terminology considers the *Faces of Cleavage*, *of Fracture*, and *of Composition.* An explanation of the three first has been given in §. 28. 163. 172. *Faces of Composition* are those in which several individuals touch one another; they belong to the individuals (§. 158.), and therefore require to be considered in this part of the work. Besides, it is necessary to attend to the difference existing between these faces of composition, and those of crystallisation, of cleavage, and of fracture, in order to distinguish forms contained only under faces of this kind, from such as are contained under faces of crystallisation, of cleavage, or of fracture. This will in particular be necessary if we have to consider individuals, limited merely by faces of composition, by themselves or singly, and in their original connexion with others.

Of all sorts of faces, the most interesting ones are those which are even, since, in the mineral kingdom, the uneven faces are not subject to any constant law, not being curved like the surface of the geometrical solids, the sphere, the cone, or the cylinder. The only even faces are the faces of crystallisation and the faces of cleavage. The latter exhibit very little remarkable differences in their quality; while the former shew a peculiarity, the more deserving of consideration, because it is closely connected with the phenomenon of crystallisation itself. The faces of crystallisation, therefore, will form the most interesting subject of our present inquiries.

The different qualities of even faces consist in their being either *smooth*, without elevations or depressions; or in their being provided with certain elevations and depressions, which, however, are so faint, that the general appearance of evenness and continuity of the faces is not affected by their occurrence. Smooth faces also are called perfect, particularly if they refer to cleavage; and cleavage is said to be the more perfect, the more its faces possess this property. This is at the same time the reason why smooth faces of cleavage are commonly much more easily obtained, than such as shew opposite properties.

The quality of irregular faces likewise depends upon the greater or less degree of smoothness of its inequalities; and according to this measure in particular, the conchoidal fracture is said to be *more* or *less perfect*. The insensible diminution of this perfect smoothness produces the passage from the conchoidal fracture into the uneven fracture (§. 172.).

The intensity or the degree of lustre of the faces, is proportional to the degree of their perfection.

Those faces which are not smooth, may be *striated*, or *rough*, or *drusy*. The most remarkable of these are the striated faces.

§. 175. STRIATED FACES OF CRYSTALLISATION.

The *Striæ* upon the faces of crystallisation are produced by the alternating re-appearance of the faces of those simple forms, which are contained in a compound one, and they are always parallel to edges of combination.

One of the forms most commonly occurring in rhombohedral Quartz, is the combination of the regular six-sided prism, with an isosceles six-sided pyramid, P. $P + \infty$. The faces of the prism are streaked horizontally, if the forms be brought into the upright position.

These striæ are produced in the following way. Instead of the faces of the pyramid, *P*, *z*, Fig. 73., which in the perfect combination would continue without being interrupted from the edges of combination to the apices, the faces of the prism *r*, *r'* re-appear. These faces, however, do not reach very far, but are again exchanged for the faces of the pyramid, which in their turn must yield to the faces of the prism, and this alternately, till at last the faces of the pyramid meet in the apices, as it is represented in Fig. 73. If now we suppose the faces altogether, and more particularly those of the pyramid, to become very narrow, we may form an idea of those delicate striæ, which are so often met with in nature. It is, however, not very rare, to be able to observe immediately the formation of the striæ, on the large scale now described, which perfectly confirms the above explanation.

It is evident, that the striæ thus produced must be parallel to the edges of combination between the faces of the pyramid and those of the prism; because the faces of those forms alternate in their edges of combination.

The situation of the faces (viz. those of the prism), is indeed altered by these striæ; yet this is inconsiderable if the striæ become very delicate, and the more so, if those faces which belong to the opposite apex of the pyramid, likewise alternate with the former, and exercise their influence upon the streaking, as may be seen in Fig. 74. It will require some attention if we have to apply measuring instruments to crystalline faces disfigured by these striæ.

Another very remarkable example of striated faces occurs in hexahedral Iron-pyrites, in the combination of the hexahedron, and of the hexahedral pentagonal-dodecahedron. Vol. II. Fig. 165. In this example, the striæ are parallel to the edges of combination; hence they are parallel to each other upon parallel faces, and perpendicular to each other upon such faces as are not parallel. If, instead of the hexahedron, the form combined with the dodecahedron is a trigrammic tetragonal-icositetrahedron, whose characteristic angle is equal to the characteristic edge of the dodecahedron, the striæ produced upon the faces of the latter, are parallel to perpendicular lines, which may be drawn from the single plane angle of the faces towards the opposite edge; upon the faces of the icositetrahedron, the striæ are parallel to the longer one of those edges, which join two different solid angles formed by four faces. It would be superfluous to mention here any more examples; for every case referring to this subject may be explained with as great facility as the preceding combinations of rhombohedral Quartz, and of hexahedral Iron-pyrites.

The curvature of the faces (§. 159.) sometimes is produced by streaking. Thus, in rhombohedral Tourmaline, the three-sided prisms, with convex faces, are produced by numerous striæ between the faces of $\frac{P+\infty}{2}(l)$ and $P+\infty\,(s)$. Vol. II. Fig. 138.

The streaking of the faces may be very useful in finding out those which are homologous, since homologous faces always shew similar occurrences of this phenomenon; a fact proved by numerous examples in rhombohedral Lime-haloide, in dodecahedral Garnet, &c.

§. 176. DIVERSE QUALITIES OF THE FACES OF CRYSTALLISATION.

The property of the surface of crystalline forms, designated by the terms *rough* and *drusy*, arises from elevations projecting from the faces of crystals. They differ from each other only in the size of the elevated particles.

In the species of octahedral Fluor-haloide, octahedral crystals, sometimes of considerable size, seem entirely to consist of minute hexahedrons. The faces of such octahedrons cannot be planes; but they consist of the faces of the hexahedrons, which are perpendicular upon each other; in such situations, that a plane passing through their solid angles is parallel to the face of the octahedron. The more the size of these hexahedrons diminishes, and consequently the more their number increases, the more the faces themselves will assume the appearance of exact planes. They are said to be *drusy*, if the asperities upon the faces are still easily distinguishable; they are termed *rough*, if they may only be perceived with difficulty, or if the existence of such asperities merely can be inferred from the want of lustre of the resultant faces.

The faces of the hexahedron exhibit in the same species a phenomenon connected with the former, which in many respects is very remarkable. They are sometimes covered as it were with small very flat four-sided pyramids, whose lateral edges are parallel to the edges of the hexahedron; and of which only the upper part is visible. The faces of these pyramids are the faces of the hexahedral trigonal-icositetrahedron, a form not uncommon in this species. If they become very minute, they may render the faces drusy or rough, or they may produce striæ parallel to the edges of the hexahedron.

Another very remarkable fact, which in particular is very often met with in rhombohedral Quartz, must be classed along with the preceding peculiarities. Many crystals of the same form, P. P + ∞, appear as if grouped parallel to each other, or round a larger crystal of the same form. Exact parallelism is here understood, as it is in every occurrence of this kind. Sometimes, however, the aggregation of crystals of rhombohedral Quartz depends upon regular composition (§. 179.). The preceding observations, in respect to striated surfaces, as to the quality of homologous faces, applies likewise to such as are rough or drusy.

Those particles which project from the faces of the crystals, must not be considered as single individuals; and crystals with drusy faces therefore are not compound minerals. They indicate rather the gradual progress of the formation of crystals, from the interruption in which they arise. If we suppose in the octahedrons of octahedral Fluor-haloide, homogeneous matter of the crystal to fill up the interstices between the faces of those minute hexahedrons, so that the formation is terminated; no character remains by which a crystal thus formed could be distinguished from a simple mineral. We may imagine that, in the progress of the formation, a simple mineral may give rise to a compound one; but it is utterly impossible that a simple one could be formed out of a compound mineral. Crystals with drusy faces may consequently be simple minerals.

§. 177. FACES OF COMPOSITION.

The quality of the faces of composition is accidental.

The faces of composition sometimes are even, yet this is very rare. Even faces of composition may easily be distinguished from faces of cleavage, because those particles which are contained between two faces of composition, can no more be cleaved in the same direction; provided they do not possess besides a cleavage of that kind; in this case, however, the quality of the two kinds of faces would suffice for their distinction.

They are rarely smooth; and if this happens, we find it only in single, not continuous parts of the faces. More commonly they are streaked; but the striæ are irregular, without any determined or constant direction. Very often we meet with rough faces of composition, their lustre being of a very low degree, or even sometimes entirely wanting; this may be used as a distinctive character between the faces of cleavage and those of composition, if, in a mineral, these two kinds of faces should happen to be parallel. Lastly, they often are uneven, or contain more or less considerable elevations and depressions. Faces of this kind must not be confounded with uneven faces of fracture; this, however, may be very easily avoided by comparing them with real faces of fracture in the same individual.

The character by which the faces of composition essentially differ from those of crystallisation and of cleavage, consists in the circumstance, that generally they preserve no determined direction, and do not produce any regular forms. Here we must except those faces, in which parallel individuals touch one another, or those which depend upon regular composition, and which will afterwards be considered more at large (§. 179.). These faces, indeed, preserve a constant and determined direction, and are occasionally of a peculiar degree of evenness, although, in other respects, they possess all the properties peculiar to faces of composition.

Very often the individuals cohere so very strongly in their faces of composition, that they will rather separate in faces of cleavage or of fracture, than in those of composition. If the individuals, on account of their minuteness, withdraw themselves from observation, or become impalpable, the faces of composition likewise disappear. It is evident, from the preceding observations, that by this process a compound mineral can never be transformed into a simple one. At all events, we must carefully distinguish between the faces of composition and those of crystallisation and cleavage, the first of which are present in every compound mineral.

SECTION II.

THE NATURAL-HISTORICAL PROPERTIES OF COMPOUND MINERALS.

§. 178. REGULAR AND IRREGULAR COMPOSITION.

The mode of composition in which the individuals of the mineral kingdom appear, is said to be *regular*, if the form produced by their connexion is a regular one, and if this regularity is a necessary consequence of the composition; if the contrary takes place, the composition is said to be *irregular*.

If two or more homogeneous individuals join in a compound form, regularly and symmetrically, at least if duly completed; the composition is in every respect perfectly determined. For we may indicate, with the greatest accuracy, in which faces of the simple forms, or in which plane the individuals cohere, even though this plane should not be parallel to a face of any simple form of that species to which the individuals belong. In general we may obtain the situation of the individuals required or necessary in order to produce the compound form. A composition of this kind is said to be regular.

The composition is irregular, if the forms are not connected in the manner now described, and if, therefore, they do not produce any regular or symmetrical forms. Two or more individuals joined in this way, are said to be merely *aggregated*, an expression which intimates, that there is no regularity in their composition.

There are compound minerals, which affect regular external forms, although their composition is in fact irregular. The regularity of the form in such cases evidently does not follow from the composition, but it must originate from something which is foreign to the mineral. Compositions of this kind cannot be called regular in the sense of the word now explained.

§. 179. REGULAR COMPOSITION. TWIN-CRYSTALS.

The regular composition of two homogeneous individuals, joined in one crystalline form, has been designated by the name of a *Twin-Crystal.*

It is unnecessary to consider by themselves regular compositions of three, four, five, or more individuals, because, although compositions of that kind, according to several laws, should take place in nature, yet they always can be reduced to, and explained by the regular composition of two individuals.

The property peculiar to the twin-crystals consists in the close and exact connexion of the *Face of Composition* (§. 177.), or that in which the individuals join, with the series of crystallisation of the species. The face of composition is either parallel to the face of a form belonging to this series, or it is perpendicular to a certain edge. The situation of the two individuals themselves is obtained, if we first suppose both to be in the parallel position, and then turn one of them round a certain line, likewise of a determined direction, under an angle of 180°, while the other remains unmoved. This line is termed the *Axis of Revolution.* It is either perpendicular to the face of composition, or it coincides with this face, while it is parallel to a crystallographical axis of the individual. The angle of 180° is the *Angle of Revolution.*

These properties are necessary to, and characteristic of, the twin-crystals, which are very easily distinguished from any other compositions occurring in nature, in which the junction of two or more individuals takes place in other faces and in other directions, the aggregation of the individuals being accidental.

It is moreover necessary that the individuals be homogeneous. Two individuals not homogeneous, even though they were supposed to possess the same form, and to join according to some rule, can never produce a twin-crystal; they cannot be considered as compound, but must be classed among the mixed (§. 14.) minerals.

The forms of the single individuals of a twin-crystal must therefore contain members of *one and the same* series of crystallisation: They may be simple forms or combinations. Commonly they either are or both contain *the same* members, so that in most cases the forms of the two individuals may be considered merely as parts of *one and the same* crystalline form, part of which only has assumed an extraordinary yet determined situation.

This has been the foundation of another method employed for explaining the form of regularly composed individuals. A plane is imagined to bisect in a determined situation the form of the simple mineral, and one of the halves to be turned in this plane through a certain number of degrees, while the other remains in its former situation. The number of degrees is equal to half the circumference, or = 180°, hence twin-crystals considered in this point of view, have been called *Hemitrope Crystals.*

The change in the situation, which is produced in the parts of the combined individuals, occasions, under certain circumstances, the production of angles greater than 180°, which are not to be met with in the forms of simple individuals as considered above. These angles are said to be *re-entering* or *re-entrant;* and they are commonly taken for a character of a twin-crystal, or of a hemitrope one.

Angles of this kind, however, greater than 180°, may arise if two individuals of a species of the same or of different forms, are joined in a parallel position; as, for instance, in rhombohedral Quartz, where it sometimes happens that two crystals of the common form have all their faces parallel, and the axis of the one in the prolongation of the axis of the other. The product, nevertheless, must not be considered as a twin-crystal, or as a hemitrope one. The two apparent individuals in a composition of this kind, form parts of a single one; and the whole is nothing else but a particular case of those mentioned in the explanation of the striæ (§. 175.), that is to say, it is an alternating repetition of the faces of the simple forms contained in the combination.

On the other hand, we find real hemitrope or twin-crystals, which yet do not contain any re-entering angles, because the existence of these depends upon the situation of the faces of the simple forms. It is not the re-entering angles, therefore, that form the essential character of the twin-crystals, but the situation of their different parts, which cannot be explained without assuming that they are formed from the composition of two individuals.

The law according to which twin-crystals are formed, may also be expressed by crystallographic signs; since it is required only to mention the situation of the axis of revolution and the plane of composition in reference to the crystalline forms occurring in a species. For this purpose the crystallographic sign of the face, parallel to which the regular composition takes place, is included in braces; the direction of the axis of revolution is added to it, and separated by the sign :, if it should not be perpendicular to the face of composition.

After this general consideration of twin-crystals, a few examples will be sufficient for illustration, in so far as is required for our purpose, and for exemplifying the employment of the crystallographic signs.

Suppose *the face of composition to be parallel to a face of crystallisation, and the axis of revolution, perpendicular to it, to be at the same time an axis of crystallisation.*

If we join two octahedrons in a parallel position in such situations, that they come into contact with their own faces, the face of composition will be parallel to one of the faces of the octahedrons, and one of the rhombohedral axes will be common to both these forms. Although the faces of one of them produce re-entering angles with some of the faces of the other, yet the assemblage of the two octahedrons cannot be considered as a twin-crystal, because the faces of the two forms are exactly parallel to each other, This must be referred to the case explained above in the examples of octahedral Fluor-haloide, and rhombohedral Quartz.

Turn now one of these octahedrons round the common rhombohedral axis through an angle of 180°, this axis being considered as the axis of revolution, while the other remains unmoved; the face of composition being perpendicular to the axis of revolution. A twin-crystal will now be formed, because the individuals can no more be considered the one as the continuation of the other, since their respective homologous parts have assumed a different, yet determined situation towards each other, which is the peculiar character of a twin-crystal. The faces of the one produce re-entering angles with those of the other, equal to double the edge of the octahedron, or $= 218° 56' 32'' = 360° - 141° 3' 28''$.

The same result is obtained, if we bisect an octahedron by a plane through its centre, parallel to two of its faces, or perpendicular to one of its rhombohedral axes, and allow one of the halves to make a revolution of 180° round that rhombohedral axis, upon which the section is perpendicular, while the other half remains unmoved. The plane of the section is the face of composition itself.

In the preceding case, the term twin-crystal is more appropriate to the first, that of a hemitrope crystal, more to the last mode of explanation. The first supposes that every twin-crystal consists of two different individuals, which require to be joined in a certain determined situation, in order to produce the compound crystal; the other supposes only one individual, in which the situation of some of its parts has undergone a regular change by an operation which can never have been employed by nature. This is the reason why the first, and the name twin-crystal referring to it, has been preferred in the present work, to that of a hemitrope crystal, particularly since it is applicable to many cases, where the latter term would produce an erroneous idea. The expression regular composition being more general than either of them, is very often useful in its application.

The above mentioned twin-crystals occur in octahedral Iron-ore, in octahedral and dodecahedral Corundum, &c. Vol. II. Fig. 156. Its crystallographic sign is $O, \left\{\frac{0}{4}\right\}$.

Instead of the octahedrons, we may substitute two hexahedrons, without changing any thing in the law of composition; the resulting twin-crystal will have its face of composition parallel to a face of the octahedron, and that rhombohedral axis, which is common to the individuals, for its axis of revolution. The re-entering angles are $= 250° 31' 44'' = 360° - 109° 28' 16''$.

Regular compositions of this kind are found in hexahedral Lead-glance, also such twin-crystals as result from combinations of the hexahedron and the octahedron, or other forms of the species. The crystallographic signs of the first are $H, \left\{\frac{0}{4}\right\}$; of the second $H.O, \left\{\frac{0}{4}\right\}$.

The dodecahedron, if substituted instead of the octahedron, produces similar twin-crystals, which are found in octahedral Diamond, dodecahedral Garnet-blende, &c. Vol. II. Fig. 163. Their designation is $D, \left\{\frac{0}{4}\right\}$.

If we join two rhombohedrons in a transverse position, the principal axis being the axis of revolution, the twin-crystal produced will be of the same kind as the preceding ones, which becomes evident, if we imagine a rhombohedron to take the place of the above mentioned hexahedron. The face of composition, which is perpendicular to the rhombohedral axis, and, as such, analogous to that of the octahedron, is parallel to $R - \infty$. We may now substitute any finite rhombohedral form, for instance $(P)^3$, or any combination of rhombohedrons, pyramids, and prisms, instead of the rhombohedron; and the result will yield forms of the same kind, of which many examples are found in rhombohedral Lime-haloide. Vol. II. Fig. 129. The crystallographic sign of this regular composition is $(P)^3, \left\{R - \infty\right\}$.

Let every thing be as before, only *the axis of revolution not parallel to an axis of crystallisation.*

Twin-crystals of this kind are produced, if two rhombohedrons of the form R + n join in their own faces, so that their axes include an angle. They are brought into this situation by supposing the axis of revolution to be perpendicular, the face of composition parallel, to one of the faces of R + n. Of this composition, the sign is $R, \left\{\frac{R}{3}\right\}$. The law of composition remaining the same, we may substitute in the place of R + n, any one of the forms R + n + 1, R + n — 1, R + ∞, &c.; in short, every member of the series of crystallisation derived from R + n, and every combination which they possibly may produce. On the other hand, we may likewise alter the situation of the face in which the individuals join; and, provided it does not coincide with R — ∞, the resultant twin-crystals still belong to the present section. Nature produces many examples of this law; for instance, in rhombohedral Lime-haloide, R — 1 composed in the face of R, of which the crystallographic sign is $R-1, \left\{\frac{R}{3}\right\}$; the combination R — ∞. R + ∞, composed in the face of R, Vol. II. Fig. 132., or in the face of R — 1, Vol. II. Fig. 133., the crystallographic sign of the former being $R-\infty.\ R+\infty, \left\{\frac{R}{3}\right\}$; that of the latter, $R-\infty.\ R+\infty, \left\{\frac{R-1}{3}\right\}$; R, in the face of R — 1, designated by $R \left\{\frac{R-1}{3}\right\}$. Vol. II. Fig. 130.; $(P)^3$, in the face of R + 1, which is designated by $(P)^3, \left\{\frac{R+1}{3}\right\}$; &c.

The present law is not confined to the rhombohedral system. The pyramid P in pyramidal Copper-pyrites, if composed in one of the faces of P, produces a twin-crystal, very much resembling that of the octahedron in octahedral Iron-ore, but differing from it, in as much as the axis of revolution in pyramidal Copper-pyrites, is not at the same time an axis of crystallisation. Its sign is $P, \left\{\frac{P}{4}\right\}$. In the common twin-crystals of pyramidal Tin-ore two individuals of the crystalline form P + 1, or P + 1. [P + ∞], are joined in a face of P; the axis of revolution is perpendicular to this face, and is not an axis of crystallisation. The crystallographic sign becomes $P + 1 \left\{\frac{P}{4}\right\}$, or $P + 1. [P + \infty], \left\{\frac{P}{4}\right\}$.

Sometimes the faces of infinite forms are faces of composition. Thus, in paratomous Augite-spar the crystalline form $\frac{P}{2}. (\breve{P}r + \infty)^3. \bar{P}r + \infty$ is composed in the face of $\breve{P}r + \infty$, the axis of revolution being perpendicular to this face. The re-entering angles are produced by the hemi-prismatic form $\frac{P}{2}$. The crystallographic sign of this regular composition is $\frac{P}{2}. (\breve{P}r + \infty)^3. \bar{P}r + \infty, \left\{\breve{P}r + \infty\right\}$.

Hemi-prismatic Augite-spar gives a similar example. The form is $\frac{P}{2}. - \frac{\breve{P}r}{2}. (\breve{P}r + \infty)^3. \bar{P}r + \infty$, the face of composition is parallel to $\bar{P}r + \infty$, and the axis of revolution perpendicular to it. This composition, however, by a curious anomaly, commonly does not present any re-entering angles, although they should occur, both on account of $\frac{P}{2}$ and of $- \frac{\breve{P}r}{2}$.

To this class likewise belong the well known cruciform twin-crystals of paratomous Kouphone-spar and prismatoidal Garnet. The crystalline form of the first is $P. \breve{P}r + \infty. \bar{P}r + \infty$; the face of composition is $P + \infty$, and the axis of revolution is perpendicular to it, Vol. II. Fig. 40., without the faces *t*. The form of the other is $P - \infty. P + \infty. \breve{P}r + \infty$; the face of composition is one of the faces of $\frac{3}{4}\breve{P}r$, and the axis of revolution is perpendicular to it.

We may observe, in respect to the latter two twin-crystallisations, that they represent in some measure the double of what we have seen in the preceding cases, so that a more symmetrical assemblage of the two individuals is produced, which is not the case with the greater number of the other twin-crystals. This peculiarity is owing to the circumstance that the individuals do not terminate at the face of composition, but that they are continued beyond it, which causes their cruciform aspect. In the crystallographic sign of similar compositions, the number 2 is prefixed to the sign of the face of composition. Thus the twin-crystal of paratomous Kouphone-spar is expressed by $P.\ \breve{P}r + \infty.\ \bar{P}r + \infty,\ 2\left\{\frac{P + \infty}{2}\right\}$; that of prismatoidal Garnet by $P - \infty.\ P + \infty.\ \breve{P}r + \infty,\ 2\left\{\frac{\frac{3}{4}\breve{P}r}{2}\right\}$. Other systems present similar occurrences, as, for instance, one of the most remarkable in the hexahedral pentagonal-dodecahedrons of hexahedral Iron-pyrites.

The crystallographic sign of the cruciform pentagonal-dodecahedron is $\frac{A_2}{2},\ 2\left\{\frac{D}{6}\right\}$; the face of composition being one of the faces of the dodecahedron (§. 63.).

Another kind of twin-crystals results, *if the face of composition is perpendicular to an edge of the crystalline form, and the axis of revolution parallel to this edge*, or, which is the same thing, perpendicular to the face of composition. Rhombohedral Ruby-blende affords examples of this law; the crystalline form $R - 1.\ P + \infty$ is composed in a face perpendicular to one of the terminal edges of $R - 1$, or to the inclined diagonal of $R - 2$, the axis of revolution being parallel to this line. Vol. II. Fig. 139. The crystallographic sign is $R - 1.\ P + \infty, \left\{\frac{R - 2.\ R - 1}{3}\right\}$; where $R - 2.\ R - 1$ indicates those edges, which at the same time lie in the planes of $R - 1$, and of $R - 2$.

According to this law may be explained the twin-crystals of di-prismatic Lead-baryte, prismatic Lime-haloide, prismatic Melane-glance, &c., whenever they assume a cruciform aspect. Under these circumstances, the present law is as it were complementary to the preceding one, and either of them may be applied, although in many cases the preceding one will be found more simple. The law of the composition of di-prismatic Lead-baryte, Vol. II. Fig. 38., may therefore be expressed either by $2\{\breve{P}r\}$, or by

$2\{\text{P}.\breve{\text{P}}\text{r}\}$; of which the former indicates the face of composition to be parallel to both the faces of the horizontal prism $\breve{\text{P}}\text{r}$, while the latter refer to faces of composition perpendicular to those edges of P, in which the pyramid is touched by the faces of $\breve{\text{P}}\text{r}$. The continuation of the individuals beyond the faces of composition produces the identity of the results of the two laws.

Prismatic Feld-spar gives an interesting example of another law of regular composition, in which *the face of composition is parallel to a face of crystallisation, but the axis of revolution lies in the same face, and coincides with an axis of the crystalline form.* Its crystalline form is the combination $\frac{\frac{3}{4}\breve{\text{P}}\text{r}+2}{2}(y).\ -\frac{\breve{\text{P}}\text{r}}{2}(P).\ (\breve{\text{P}}\text{r}+\infty)^3\,(T,l).\ \bar{\text{P}}\text{r}+\infty\,(M).$ Vol. II. Fig. 61. The face of composition is $\bar{\text{P}}\text{r}+\infty$; the axis of revolution is situated in it, and parallel to the principal axis of P. After having placed two crystals of this form in an upright and parallel position, we must turn the one round its vertical axis, while the other remains unmoved. In the position thus produced, the junction of the two individuals will produce a twin-crystal. But we may join them either with those faces of $\bar{\text{P}}\text{r}+\infty\,(M)$, which are situated to the right, or with those which are situated to the left of the face y, which is conceived to be turned towards the observer, and contiguous to the upper apex of the form. The product in the first case will be the twin-crystal represented, Vol. II. Fig. 80., and its crystallographic sign $\frac{\frac{3}{4}\breve{\text{P}}\text{r}+2}{2}.\ -\frac{\breve{\text{P}}\text{r}}{2}.\ (\breve{\text{P}}\text{r}+\infty)^3,\ \bar{\text{P}}\text{r}+\infty,$ $\{\text{r}, \bar{\text{P}}\text{r}+\infty : \breve{\text{P}}\text{r}+\infty.\ \bar{\text{P}}\text{r}+\infty\}$; the product of the second is the twin-crystal, Vol. II. Fig. 81., and its sign $\frac{\frac{3}{4}\breve{\text{P}}\text{r}+2}{2}.$ $-\frac{\breve{\text{P}}\text{r}}{2}(\breve{\text{P}}\text{r}+\infty)^3.\ \bar{\text{P}}\text{r}+\infty, \{\text{l}, \bar{\text{P}}\text{r}+\infty : \breve{\text{P}}\text{r}+\infty.\ \bar{\text{P}}\text{r}+\infty\}.$ The first part of the expression within the braces, indicates the right or the left face of $\bar{\text{P}}\text{r}+\infty$ to be the face of composition, while the second expresses the direction of the axis of revolution by the edge produced between $\breve{P}r + \infty$ and $\bar{P}r + \infty$, which is parallel to the axis of P.

Cleavage confirms to the full extent of its application, every thing that has been said here on the subject of twin-crystals. For it is possible to extract from the twin-crystals compound forms of cleavage, in which those parts which belong to one individual represent the real forms of cleavage of the species.

§. 180. IRREGULAR COMPOSITION. GROUPE AND GEODE OF CRYSTALS.

If several loose or imbedded crystals are merely aggregated (§. 178.), so that the one becomes the support of the other, while there exists no general support; the assemblage is termed a *Groupe of Crystals*; if, however, several crystals of that kind are fixed to a common basis, so as to produce a general support for them all, the assemblage is said to be a *Geode of Crystals.*

The difference between these two sorts of assemblages is the same as that existing between an imbedded and an implanted crystal.

There is sometimes a certain order observable in these groupes of crystals, although this is never geometrical regularity (§. 178.); and no regular form is produced in the assemblage. Both the groupes and the geodes refer only to compound minerals, never to such as are mixed.

These compositions consist of individuals of considerable size, which, therefore may be very easily recognized; they assume always their regular form, as soon as they are disengaged, or cease to touch other individuals. Upon these assemblages is founded the explanation of several forms of compound minerals found in nature.

§. 181. IMITATIVE SHAPES.

The shape of a compound mineral is called an *imitative* or *particular external Shape*, if it bears some resemblance to the shape of another natural or artificial body. Some of these forms are produced in a space not incumbered with matter, and depend upon the properties peculiar to the minerals themselves, without being influenced by any contiguous matter; others owe their shape to that extraneous or foreign matter, with which they are surrounded. The latter of these have been called *extraneous imitative Shapes.*

The groupes and geodes are the simplest modes in which the irregularly compound minerals appear in nature. If the individuals thus connected are diminished in size, and if their number at the same time increases, imitative forms are produced from the groupes of crystals; which, although they are founded in the nature of the individuals themselves, yet cannot be employed to any useful purpose in Natural History. The extraneous imitative forms cannot be brought into any connection with these groupes at all; they do not depend upon the natural forms of the individuals; on the contrary, in most cases where they are observable, we find them quite contradictory to the nature of the individuals which they contain. For in these the form depends entirely upon the shape of the space previously existing, and is accordingly entirely accidental.

§. 182. IMITATIVE SHAPES ORIGINATING IN THE GROUPES OF CRYSTALS.

The imitative shapes which originate from the groupes of crystals, are loose or imbedded, and more or less regular, globular or spheroidal masses.

If the individuals connected with each other become very small, but join in a great number into a groupe of crystals, *globular* forms result, which are sometimes perfect, sometimes very imperfect. Their surface is drusy, or covered with asperities, where it has not been disfigured in its formation, or by subsequent accidental circumstances. In their interior we may still discover the direction of the constituent individuals, which, in most cases, corresponds to the direction of the radii of a sphere; they begin in the centre, and terminate at the surface. Imbedded globular shapes, like imbedded crystals, are complete on all sides, and leave an impression of their form in the mass from which they have been detached.

Several globular masses of this kind, if attached to one another, may produce reniform and botryoidal shapes, which, however, require to be distinguished from those described in §. 183.

The loose or imbedded globular shapes differ from grains and angular masses (§. 160.), in as much as they are not simple minerals. Examples of imbedded globular shapes occur in prismatic Iron-pyrites, in prismatic Azure-malachite, and other species; the same Malachite presents also reniform and botryoidal shapes formed from imbedded crystals.

§. 183. IMITATIVE SHAPES ARISING OUT OF THE GEODES OF CRYSTALS.

There are three different kinds of imitative shapes resulting from geodes of crystals: 1. Those in which the individuals spring from, or are attached to a common point of support; 2. Those in which the individuals form one the support of the other; and 3. Those in which the support is cylindrical, sometimes a simple line, sometimes a tube.

Among those of the first division we find the implanted *globular shapes*. They arise, if very thin, capillary crystals, or in general, such as have one of their dimensions considerably surpassing the others, are fixed with one of their ends to a common point of support, from which they diverge in every direction. The mode of the formation of such globular shapes is more apparent, if the number of the individuals is not so great that they touch each other on all sides. The implanted globules must necessarily be incomplete, because the implanted crystals of which they consist, are themselves incomplete, and therefore they leave no impression when detached from their support. Globular shapes of this kind occur very frequently in prismatic Kouphone-spar, in macrotypous Lime-haloide, in prismatic Hal-baryte, &c.

If, during the formation of several globules, they come into contact with each other, there will arise *reniform* and *botryoidal* shapes, which therefore are nothing else than several implanted globules joined together. The single globules are separated from each other by faces of composition. Rhombohedral Iron-ore very often affects shapes of this description, in which species they are known under the name of *Hæmatites.* They occur also in the varieties of rhombohedral Quartz, called Calcedony. In these very often the individuals are so delicate, that they withdraw themselves from observation.

Into the present class belong also the *fruticose* shapes, which possess some resemblance with parts of certain plants, and most of those commonly called *dendritic*, the latter of which may penetrate throughout the whole mass, or only be superficial.

The second division contains, among others, the *dentiform*, the *filiform*, and the *capillary* shapes. These arise, if one implanted crystal is the support of another, this of a third, and so on; so that rows of such crystals are produced, as we may observe them very often differently bent in hexahedral Silver, in octahedral Copper, and also in octahedral Iron; in the last of these, however, they have not yet been found disengaged. If the crystals join so very intimately, that it is no longer possible to distinguish them from each other, those imitative shapes result, which are not unfrequently met with in the above mentioned species.

Sometimes several rows of individuals thus composed join within one and the same plane in certain constant directions, so that the individuals of the one of these series do not join with those of the other, but remain separate. Thus the *dendritic* shapes are produced, of which most distinct varieties occur in hexahedral Silver and hexahedral Gold. The same minerals evidently shew the formation of the dendritic, as also that of the dentiform, filiform, and capillary shapes, in those varieties where the individuals constituting them still may be distinguished in the compositions. To this division belong also some of the superficial dendritic shapes formed in fissures.

If the rows of individuals, thus arranged, approach so near each other that they at last meet, so as to form a continuous mass, they are said to occur in the external shape of *leaves* or *membranes*, which are among the most common shapes found in hexahedral Gold, where they exhibit various modifications. In some of them we may still discern the individuals ; in others there are striæ in certain directions, indicating their composition, so that their mode of formation does not remain doubtful. From external shapes of this description, we may infer that those likewise whose smooth surfaces no longer present any traces of composition, are yet owing to the aggregation of several individuals.

Compound minerals, like those now described, may again join in a new composition, in which consequently the individuals are arranged in the direction of different planes ; in most cases at right angles to each other. Thus the *reticulated* shapes arise, of which the most distinct specimens occur in octahedral Cobalt-pyrites. In these very often the composition itself is still observable. Some of the reticulated forms, however, allow of a different explanation, if, instead of rows of individuals, they consist of capillary crystals, like those of peritomous Titanium-ore.

The third division comprehends the *stalactitic* and *coralloidal* shapes. The first of these consist of individuals which are *perpendicular* to every point of a straight cylindrical or linear support in its whole circumference, as appears from many examples in prismatic Iron-ore, in prismatic Iron-pyrites, where the composition commonly is still observable, and in the varieties of rhombohedral Quartz, called Calcedony, where the individuals no longer can be distinguished. On a very large scale they are not uncommon in limestone caves, and consist of varieties of rhombohedral Lime-haloide. The coralloidal shapes consist of individuals *inclined* at an angle to their support, which, although linear, is not straight; they are fixed upon this support in every part of the circumference, exactly as is the case in the stalactitic shapes. This sort of imitative shapes occurs not unfrequently in prismatic Lime-haloide, particularly in those varieties which have been called *Flos-ferri.*

There occur many more imitative shapes in nature than those contained in the preceding examples, and many more have been distinguished and described by mineralogists. But these few examples will be sufficient to shew the method of explaining all forms similarly composed.

§. 184. AMORPHOUS COMPOSITIONS.

If the mass, formed by the junction of several individuals, is not only of an irregular shape, but if even in this we cannot trace any resemblance with the shape of another body, the mineral is said to be *massive.*

Massive minerals are amorphous irregular compositions of individuals of the same species, which are in contact with each other on all sides. The difference between massive minerals, and those forms resulting from the groupes of crystals, which deviate more or less from the spheroidal shape, consists merely in the strong adhesion of the former to the surrounding masses of other species. It is formed, however, and assumes a shape corresponding to its own inherent powers, and does not depend upon its support, in as much as we are led to suppose both of them to be of contemporaneous origin.

Massive minerals of a smaller size are also called *disseminated* minerals, which have again been subdivided according to the size of the particles. Very large masses of amorphous minerals sometimes enter into the composition of rocks, as rhombohedral Lime-haloide and prismatoidal Gypsum-haloide, several varieties of Iron-ores, &c. Under these circumstances they assume the shape of beds, &c., the consideration of which is no longer an object of Natural History.

§. 185. ACCIDENTAL IMITATIVE SHAPES.

The accidental imitative shapes presuppose an empty space, which has been filled up by the individuals of compound minerals, to which is transferred the form of the pre-existing space.

In this case, the shape which the mineral assumes is not a consequence of the properties inherent in the mineral, or peculiar to its nature, but it merely belongs to that space, in which the formation takes place. The sides of this space serve as support for the individuals. Thus, at first a coating is formed, which consists of small, but in many cases very perceptible crystals, whose apices are turned towards the inside of the empty space. This accounts for the hollowness of many imitative forms of this kind, of which the cavities are still lined with crystals. Sometimes also we find in the interior of such specimens, implanted globular, reniform or botryoidal shapes, &c., in short, imitative forms depending upon the crystallisation of the mineral itself. Yet the external shape of the whole, or of the compound mineral, must always be considered as an accidental imitative shape.

If the whole of the space is entirely filled up, there remains nothing else but the mode of composition of the individuals, from which we may judge, without giving any attention to the surrounding mass, whether the imitative shape be the result of crystallisation, or whether it has been influenced by other circumstances.

The space in which the accidental imitative shapes are formed, may be either regular or irregular. A regular space cannot be produced except by crystallisation; and this may be either in the interior of a real crystal, or it is the cast of a crystal in the surrounding mass (§. 186.). The first is not uncommon, particularly in large crystals of rhombohedral Quartz, where part of the space of the crystals has remained empty, and is regularly limited by the surrounding crystalline mass.

The irregular spaces sometimes consist of accidental fissures, cracks, and other similar openings: sometimes they depend upon the structure of the surrounding mass, which in many instances belongs to the class of rocks; others at last arise from moulds of various minerals, and also of organic bodies.

The different description of the space in which compound minerals are formed, produces a distinction of their forms into *regular* and *irregular* accidental imitative forms.

§. 186. REGULAR ACCIDENTAL IMITATIVE SHAPES. PSEUDOMORPHOSES.

The regular imitative shapes of the preceding paragraph have been called *Pseudomorphoses*, or *Supposititious Crystals*. The latter denomination seems to be rather improper, since they share so very little in the properties of real crystals.

No pseudomorphoses are formed in such impressions as originate from imbedded crystals, which can be separated from their surrounding mass (§. 160.); at least experience has not as yet furnished us with any well authenticated instances which could not be explained, but on that supposition.

But if an implanted crystal (§. 160.) is covered over by the mass of another mineral, which has been formed after the production of the first, the deposite of new individuals will at first constitute a *Coating*, which consists of minute crystals, and through which the form of the implanted crystal still continues to be perceptible; the mineral may yet proceed in its formation, and become massive, or it may assume any other imitative shape, in which the form of the original implanted crystal entirely disappears. The crystal is moulded in this mass; and, if it be taken away, or decomposed, it will leave an *Impression* of its form. Rhombohedral Quartz, and many other minerals, present instances of similar impressions. From the form of the impression, we may very often infer by what mineral they have been occasioned. What has been called the *ramose* shape of the octahedral Iron from Siberia, is nothing else but the result of impressions produced by crystals and grains of prismatic Chrysolite.

The crystals sometimes are again decomposed in the place of their formation, and thus leave the impressions of their form on other minerals, which have escaped this destruction. If in these empty spaces a new compound mineral is formed, it must necessarily assume the shape of the space already existing, since the sides of this become the support of the individuals newly formed. Thus pseudomorphoses are formed, which appear in the shape of implanted crystals, if the mass containing the impressions, by whatever circumstances, happens to disappear.

All the peculiarities of the pseudomorphoses can easily be explained from the mode of their formation now described.

The form of the pseudomorphoses has no relation at all to the nature of the mineral in which it occurs. For it is entirely accidental, from what mineral the impression is derived in which the new individuals have been deposited. Hence it is not derived from the composition of the individuals contained in the pseudomorphosis (§. 178.). Thus in rhombohedral Quartz we meet with forms originating from rhombohedral Lime-haloide, from octahedral Fluor-haloide, from prismatoidal Gypsum-haloide, &c. ; which is sufficient to prove, that the forms of the pseudomorphoses cannot by any means be members in the series of crystallisation of those species (§. 136.), to which they belong.

The quality of the surface of the pseudomorphosis depends only upon its form, and not upon its substance or its mode of composition. For the elevations and depressions of the mould are likewise expressed in the cast, which in this case is the pseudomorphosis. The quality of this surface tends very often to indicate the mineral, from which the form is derived, particularly if these forms belong to the tessular system, which occurs in several species. Thus rhombohedral Quartz presents not unfrequently pseudomorphic hexahedrons, which as such may originate from various minerals. But on a closer inspection, we observe upon the faces of some of them, the obtuse apices of isosceles tetragonal pyramids, which, as it is mentioned in §. 176., belong to the hexahedral tetragonal-icositetrahedron, a form not uncommon in octahedral Fluor-haloide. We may hence infer that the form of the pseudomorphosis of rhombohedral Quartz now under consideration, is owing to the species of octahedral Fluor-haloide.

The surface of the pseudomorphosis is never drusy in the sense of §. 176., but only in the way described above. But sometimes it bears a new coating of very minute crystals, of the species of which the pseudomorphosis consists. This is not at all uncommon in many of those pseudomorphoses of rhombohedral Quartz, which affect the form of rhombohedral Lime-haloide. In general the surface of the pseudomorphoses is less smooth and shining than that of real crystals of the species. This, however, is merely accidental, and does not deserve to be classed among the peculiar and constant characters of pseudomorphoses.

The pseudomorphoses are very often hollow in their interior; the cavities are lined with crystals, or with reniform and other imitative shapes of that species, which constitutes the pseudomorphoses. There are crystals which contain cavities, either empty or filled with water and other fluids. These are always in close relation to the external form of the crystals themselves, which is not the case in the pseudomorphoses. Another class of openings in the interior, occasioned by other minerals being included, must be referred to the impressions.

The pseudomorphoses are compound minerals, even though, on account of the minuteness of the individuals, the composition should no longer be perceptible. But they are also very often mixed, since several species may be deposited in an impression at the same time, in the same way in which several species may enter into the composition of a geode.

The pseudomorphoses cohere immediately with the adjacent mass, and therefore seem only to be implanted.

This also is the case in certain real crystals; but here the crystals form only those parts of the individuals constituting the support, which have reached the free space, and which for that reason have assumed a regular form.

Mere coatings of crystals must not be enumerated along with the pseudomorphoses, since the latter are produced by the process of subsequent formation in a mould, as it has been explained before. Nor can it be allowed to consider decomposed or otherwise destroyed varieties of one species, as pseudomorphoses of another (§. 21.). Thus the decomposed varieties of hexahedral Iron-pyrites can never become pseudomorphoses of prismatic Iron-ore, nor those of paratomous Augite-spar pseudomorphoses of Green-earth, the latter being a variety of prismatic Talc-mica.

The origin of another remarkable appearance, is so very nearly related to that of the pseudomorphoses, that there is no place more adapted than this for its explanation.

Sometimes it happens that also the regular structure of a simple mineral is impressed into the mass of another, which enters into fissures parallel or dependent upon this structure. If now the simple mineral, by whatever accident, is decomposed, the remaining compound one will represent a shape which entirely depends on the structure of the decomposed individuals. The same takes place if the individuals of compound minerals do not cohere from all sides, so that they allow of the interposition of foreign matter. Thus the *cellular* shapes arise, of which again the former have been called *regular*, and the latter *irregular* cellular shapes. The sides of the alveolæ again are sometimes lined with minute crystals of a third mineral, and this among others is the case in what has been called the cellular Pyrites. In that mineral the sides of the alveolæ are perpendicular to each other, because they express the structure of hexahedral Lead-glance; they consist of rhombohedral Quartz, and are lined with crystals sometimes of hexahedral, sometimes of prismatic Iron-pyrites. On this account it is necessary to refer some varieties of cellular pyrites to the one, some to the other species of the genus Iron-pyrites.

The crystals of Steatite are considered as real crystals by some mineralogists, by others as pseudomorphoses: nothing decisive, however, has been brought forward in respect to this point; and they require therefore a very accurate examination, to prevent us from forming an erroneous opinion of their nature.

§. 187. IRREGULAR ACCIDENTAL IMITATIVE SHAPES.

According to the quality of the space, in which these imitative shapes have been formed, they may be distinguished into: 1, such whose form is entirely accidental; 2, such whose form depends upon particular openings in other minerals, which are not simple ones; and 3, such whose form depends upon bodies, not belonging to the mineral kingdom.

In the mass of rocks, and in that of beds and veins, we very often meet with cracks or fissures, which seem to have once been open, or which still continue so. Commonly this appearance is explained by supposing them to be real fissures, or that the coherence of the particles in the rocky mass has, in their place, by whatever means, been resolved. If a mineral is formed in a fissure of that kind, it must necessarily assume its form; and the mineral appearing in this shape, is said to occur *in Plates.* Mixed minerals likewise may affect this shape; and the veins themselves might be quoted as examples, if their consideration did not belong to another science. These fissures sometimes are so very narrow, that a fluid can scarcely enter between their sides; a mineral formed in such a space is said to occur *superficial*, which in fact is nothing else, but a very thin plate. Examples occur in hexahedral Silver, octahedral Copper, &c. both of plates, and of superficial varieties.

There are instances where the sides of these fissures are nearly even, and possess a certain degree of polish. Fissures of that description very seldom seem to have been filled up with other minerals: on the contrary, the sides are commonly in immediate contact with each other. Minerals are said to occur *specular*, if specimens of them shew part of such polished sides of fissures. The specular faces sometimes shew a particular sort of striæ, which would deserve very well to be noticed by geologists. As examples, we may quote specular hexahedral Iron-pyrites, hexahedral Lead-glance, brachytypous Parachrose-baryte, &c.

Several rocks contain vesicular cavities. In these cavities minerals are formed, which consequently must assume their shape, and appear as more or less spheroidal masses. Such globules very often consist of the varieties of more than one species, and are sometimes hollow inside. They must be accurately distinguished from the grains (§. 160.), and from the globules described above (§. 183.). Among the present we must class also the Agate-balls, and the balls of other varieties of rhombohedral Quartz, as of flint, of Egyptian jasper, &c.

If this kind of globular concretions is not hollow inside, and at the same time very irregular, so as to exhibit some resemblance with the roots of certain plants, the forms arising are called *tuberose;* of which flint is one of the most common examples.

To this class also we must refer the irregular cellular shapes (§. 186.). The present subject is of little value in scientific Mineralogy, and does not require, therefore, very nice divisions.

Those shapes which depend upon forms foreign to the mineral kingdom, are the *petrifactions.* There is no difference between the formation of the greater part of petrifactions, and of the pseudomorphoses or the accidental imitative forms, and it does not require any particular explanation. *Mineralized* organic substances cannot be classed among real petrifactions. These are not formed like pseudomorphoses, in which the space left empty by the decomposition of one body is filled up by another, but the organic mass is metamorphosed or changed into that of the mineral. Mineralized organic bodies, besides their original shape, also may retain their original structure, as numerous varieties, particularly of bituminous Mineral-coal.

Several minerals, even after their formation, assume other forms, which, however, are quite accidental. Thus *Pebbles* are formed when fragments of minerals are carried along by water, till, by attrition, they acquire a more or less roundish or globular shape. Simple, compound, and mixed minerals, are found in the shape of pebbles.

§. 188. PARTICLES OF COMPOSITION.

The individuals of which a compound mineral consists are called its *Particles of Composition.*

The particles of composition are true crystals, which, by their contact, have prevented each other from assuming their regular form (§. 160.).

The particles of composition have also been called *Distinct Concretions.* The other expression, however, is by far preferable, in as much as by itself it intimates, that they can only appear in compound minerals. It conveys likewise more readily the idea of individuals, since distinct concretions might also refer to simple minerals. The idea of the individuality of minerals is one of those which have for a long time remained unsettled, and yet the possibility of a scientific mineralogical method greatly depends upon this idea.

The particles of composition are distinguished, according to their length, breadth, and thickness, into *granular*, *columnar*, and *lamellar* particles of composition. The granular particles have all their dimensions nearly equal, or at least not very different. We may omit here all those distinctions which mineralogists have introduced, in respect to the particular shape of these granular particles, because this being not a regular one, it signifies but little in the mineral kingdom. Granular limestone (rhombohedral Lime-haloide), Coccolite (paratomous Augite-spar), dodecahedral Garnet, &c. contain many examples of granular particles of composition.

In the columnar particles the length is greater than both breadth and thickness. Sometimes they are rather thicker on one end; sometimes also they are broad. This, however, for the above mentioned reasons, does not occasion any farther difference. As to their direction, they are either parallel or diverging. Examples of columnar particles of composition we find in rhombohedral Lime-haloide, in a variety of prismatic Topaz called Picnite, in rhombohedral and prismatic Iron-ore, &c.

In the lamellar particles, the length and breadth surpass the thickness. These likewise are sometimes thicker on one end, and thus approach to the columnar particles of composition; in general, these three kinds of particles of composition are not contained within precise limits, but they pass insensibly into each other. There are straight and curved lamellar particles of composition. The latter are not individuals, but of themselves they are already composed, which distinguishes them from the former, even in case they should themselves be straight, while the others are curved or bent. Straight lamellar particles, or such as in themselves are simple, are found in prismatic Hal-baryte, axotomous Kouphone-spar, rhombohedral Lime-haloide (the varieties called Slate-spar), &c.

The *size* of the particles of composition varies considerably. Sometimes they are so minute that they entirely withdraw themselves from observation. Yet a compound mineral, consisting of such impalpable individuals, does not for that reason cease to be a compound one. This subject requires our particular attention. A specimen of hexahedral Lead-glance being given, which consists of considerably large particles of composition, we may very easily find another, in which these particles are smaller, and a third, in which the size of the particles is still more diminished. These specimens differ only in the size of their constituent individuals. We may continue, and discover a fourth variety, a fifth, &c., every one of them being in a similar relation to that which immediately precedes it; very soon we arrive at such varieties, whose individuals are scarcely perceptible to the naked eye. Yet the immediate connexion with the other varieties, and a magnifying glass, demonstrate that they are all the same mineral, viz. varieties of hexahedral Lead-glance. No reason can be assigned why these varieties should be the limit of the series; the members immediately following will be the varieties of compact Lead-glance; and in these very often the individuals are so minute, that they withdraw themselves from observation even through a magnifying glass. Compact Lead-glance, therefore, is not a simple mineral, but a compound one. From the same point of view, we must consider the compact varieties of rhombohedral Lime-haloide (compact Limestone), of octahedral Fluor-haloide (compact Fluor), of prismatic Hal-baryte (compact Heavy-spar), of rhombohedral Quartz (Flint, Hornstone, Chrysoprase, &c.), and of other species. The composition which really takes place, cannot be observed only on account of the minuteness of the particles of composition.

The columnar and lamellar particles are exactly in the same case. The columnar particles may still very easily be traced in the stalactitic and reniform shapes, called brown Hematite. But in compact brown Iron-stone, they have entirely disappeared. Of this vanishing and impalpable composition, we have a very remarkable example in those varieties of rhombohedral Quartz, which have been called Calcedony, and occur in reniform and stalactitic shapes. Commonly there is not a trace of their composition left in the interior; but in some of its varieties this composition is still observable. Among these, the fibrous Carnelian is one of the most well known instances.

It has been mentioned above (§§. 171. 172.), that the columnar composition has sometimes been confounded with certain relations of structure. *Fibrous* fracture is always columnar composition; and the difference between what has been called the *foliated* and the *radiated* fracture, consists in nothing else but that the first refers to simple minerals or granular compositions, while the second is confined to columnar compositions.

§. 189. SINGLE AND MULTIPLE COMPOSITION.

The single composition takes place, if a compound mineral consists of individuals; but if the particles of composition are again composed, then the composition is multiple.

The compositions treated of in the preceding paragraphs, are single compositions.

But there exist granular particles of composition, which are again composed of granular particles; and these only are real individuals. They join into those masses, which again on a larger scale produce a granular composition. Macrotypous Lime-haloide exhibits examples of this composition. Sometimes the granular particles consist of columnar particles, diverging from the centre, or from one of the corners of the former. Among the first are several varieties of rhombohedral Lime-haloide, called Oolite, of hemi-prismatic Augite-spar, called Actinolite, &c.; among the second the varieties of several species of Kouphone-spar, of Wood-tin, a variety of pyramidal Tin-ore, &c. In other cases, the granular particles of composition again consist of lamellar particles, as in prismatic Hal-baryte and axotomous Kouphone-spar.

Columnar particles of composition sometimes consist again of columnar ones, as in several varieties of prismatic Iron-ore. The preceding observations will suffice for explaining many occurrences of this kind, among which the curved lamellar particles of composition are the most remarkable. The curvature of their surface corresponds to that of reniform or globular shapes, so that the quantity of its deviation from a mathematical plane depends upon the radius of curvature. Commonly they consist of columnar, sometimes of lamellar and granular particles. They occur in octahedral Fluor-haloide; in prismatic Iron-ore; in prismatic Hal-baryte, called curved lamellar Heavy-spar; in rhombohedral Quartz, called Calcedony; in rhombohedral Lime-haloide, in rhombohedral Antimony, &c.

The composition is very often still more complicated; we may dispense, however, with entering into a greater detail, since the given examples are perfectly sufficient for explaining every other case.

§. 190. CHARACTERISTIC MARKS OF COMPOSITION.

Imitative shapes, and the want of cleavage, are the chief characters, from the presence of which composition may be inferred, if this should not be observable at first sight.

An individual formed under such circumstances as to be beyond the reach of any foreign influence, will always assume a regular form.

If, therefore, we meet with minerals which evidently have not been acted upon by any such circumstance, and which nevertheless do not present any regular form, we may infer with perfect security that the mineral is not a simple one, but that it is a compound of several individuals. This proposition is demonstrated on one side by all crystals which are simple, on the other by all the imitative shapes dependent upon the nature of the mineral itself (§. 182. 183.), which are compound.

With regard to the accidental imitative shapes, it is evident, that not even those which are regular, can be the forms of simple minerals, because they are altogether accidental, whereas the forms of simple minerals are founded in the nature of the individuals themselves. Hence, the imitative shapes, of whatever kind they may be, are, in every instance, infallible characters from which the composition of the minerals may be inferred. But we could suppose that a compound mineral might consist of particles in a perfectly parallel position, but so small, that on account of their minuteness, the composition can no longer be observed, so that the directions of cleavage of the single particles or supposed individuals in one of them are the continuation of those in the other. In this case, the whole mass will be cleavable, and the whole will therefore be a single individual, and not a composition, agreeably to the definitions in §. 176. Hence cleavable minerals are simple; and the want of cleavage in varieties of such species as commonly allow of cleavage, is a mark of their composition; because here one individual assumes a situation different from that of another, so that their respective faces of cleavage can have no continuity among one another. From the same reason, compact Limestone, compact Fluor, compact Heavy-spar, compact Lead-glance, are not cleavable, although the simple varieties of the same species may be cleaved with the greatest facility.

The same applies to the pseudomorphoses.

Among the other characters of composition, we may mention, that compound minerals, in which the composition can no longer be observed, are most intimately connected in all their properties with those in which it is still visible, and that commonly they possess lower degrees of transparency and lustre, than simple varieties of the same species. Examples occur in hexahedral Lead-glance, rhombohedral Lime-haloide, rhombohedral Quartz, &c.

The following observations will furnish characters in most cases perfectly sufficient for distinguishing mixed minerals and compound minerals, in both of which the particles disappear on account of their minuteness.

The different ingredients of the mixture are sometimes found separated from the rest in more or less pure masses, by which the mixture ceases to be uniform. If we find an opportunity for observing mixed masses of this kind on a larger scale, we may very often find those particles entirely disengaged, or separated from each other, as is the case with rhombohedral Iron-ore, and rhombohedral Quartz, in the original repositories of Iron-flint, which is an intimate mixture of these two species. Thus we infer Basalt to consist of several species of the genus Feld-spar, and hemi-prismatic or paratomous Augite-spar, because Greenstone and the Syenitic rocks in which the particles of mixture have only more extension, really do consist of individuals of the above mentioned species, and differ from Basalt merely by their coarser grain.

Moreover the mixed minerals partly possess the properties of the one, partly also those of the other of the simple minerals of which they consist, without entirely agreeing with any of them, as, for instance, Iron-flint, which possesses some of the properties of rhombohedral Quartz, &c. or they assume such properties as never occur in simple minerals, as, for instance, the columnar shapes of Basalt, of Porphyry, the globular concretions of Green-stone, of Syenite, &c., which by themselves prove those minerals to be compound, even though the component individuals should no longer be perceptible.

§. 191. STRUCTURE OF COMPOUND MINERALS.

That kind of structure which has been considered above in the simple minerals, does not occur in the compound ones. If we break them, however, we produce what has been called their *Fracture;* and the particles of the mineral separate in the *Faces of Fracture.*

If the particles are still distinguishable as individuals, they must be considered according to their respective regular or irregular structure, to their faces of composition, and to every other character which they present to the observer; in short, they must be considered as simple minerals. In the present place, therefore, only those compound minerals shall be treated of, in which, on account of their minuteness, the individuals are no longer distinguishable. In these the following kinds of fracture have been distinguished.

1, The *Conchoidal Fracture*, together with its various modifications, which depend upon size, perfection, relative depression (§. 172.).

2, The *Uneven Fracture*, which has been subdivided according to the size of the asperities, into coarse-grained, small-grained, and fine-grained uneven fracture.

3, The *Even Fracture*, which arises if the elevations and depressions upon the face of separation nearly approach to evenness. These even parts of the fracture must not be confounded with faces of cleavage, because they do not keep a constant direction, and are only observable in compound minerals. This variety of fracture is so very rare, that it is difficult to quote good examples of it. Sometimes it occurs in compact Lead-glance.

4, The *Splintery Fracture*, which is produced if upon the face of separation, detached scaly particles remain, joined to the mass by their thicker end. These particles are rendered visible by that portion of light which passes through them, and the splintery fracture therefore does not occur in perfectly opaque minerals. It may occur at the same time with the conchoidal, or another kind of fracture. This has been expressed by the phrases: *conchoidal in the great, splintery in the small*, &c.

5. The *Hackly Fracture*, which has been sufficiently explained above (§. 172.).

6. The *Slaty Fracture*, which resembles imperfect faces of cleavage (§. 163.), and partly arises from it. It is met with in the different kinds of *Slate*, which, for the greater part, are compound minerals, or even mixed, although they appear to be simple. The slaty fracture keeps a constant direction, and is in this respect analogous to cleavage.

7. The *Earthy Fracture*, which is the same as the uneven fracture, except that it occurs in decomposed minerals.

The particles into which compound minerals may be broken, are termed *Fragments*, and their shape is irregular. According to the quality of their edges, they have been divided into *sharp-edged* and *blunt-edged* fragments. Slaty fracture produces *tabular* fragments; thin columnar composition produces *splintery* fragments.

§. 192. COMPOSITION IS OF LITTLE VALUE IN NATURAL HISTORY.

It is impossible to derive characteristic terms for the determination of the natural-historical species, from the occurrences of composition. Hence they are of use in the Natural History of the Mineral Kingdom, only in so far as their knowledge is necessary for ascertaining the existence of the simple mineral in the compound.

Among the various impediments that have retarded the progress of the science, and, in particular, the correct determination of the natural-historical species, one of the most conspicuous was the occurrence of individuals in compound varieties, as long as this composition had not been explained, nor the individuals themselves traced in it, whose diminutive size very often considerably adds to the difficulty. If we consider the varieties of rhombohedral Iron-ore, or of many other species, and if we do not possess an exact knowledge of the individual in the mineral kingdom, that is to say, if we do not sufficiently distinguish between simple and compound minerals; we find some of them so very different, that it would seem to be in opposition to all the principles of Natural History, if nevertheless we would unite them into one species, since they differ in almost every one of their properties. This has indeed been the reason why Specular Iron-ore, Red Iron-ore, and many others, have really been considered and distinguished as particular species. At that time the transitions (§. 221.), which might have led to the knowledge of errors committed in these determinations, had also not yet been distinctly developed; and it must be avowed that those mineralogists who have escaped similar errors, owe this to inquiries and considerations of minerals, very different from such as are carried on according to the principles of Natural History.

A few examples, once given of the dismemberment of a natural-historical species, could not but produce many similar errors in subsequent instances. The consequences of this mode of proceeding, have been numberless species, or aggregates erroneously so called, being connected with each other by transitions. Incapable of being characterised, or distinguished from one another, they have only served to incumber the nomenclature, and to degrade the systems into mere registers of words. If one single erroneous idea is capable of producing such confusion, we must bestow all possible attention on the establishment of correct ideas, in a science in which they are or might be so pure and simple, as in the Natural History of the Mineral Kingdom.

Zoology and Botany have not been subject to similar errors. They are almost impossible in the first, and in the second they would be obvious at first sight. It would be an error of this kind, if a person should consider a corn-field or a bank of trees for an individual, and establish accordingly these corn-fields and banks of trees into particular species. A corn-field or a bank of trees is exactly the same as Red Hematite (the fibrous Red Iron-ore, a compound variety of rhombohedral Iron-ore), if compared with real individuals of their respective species.

From the preceding observations we may infer, that from the composition of minerals, the Natural History of the Mineral Kingdom cannot derive any characteristic properties for the determination of the species. This would be an error sufficiently powerful to shake it to the very foundations, and to degrade it from the rank it assumes in the sciences as a part of Natural History.

SECTION III.

THE NATURAL-HISTORICAL PROPERTIES, COMMON TO BOTH, SIMPLE AND COMPOUND MINERALS.

§. 193. DIVISION.

Those natural-historical properties, which are common to both, the simple and the compound minerals, may be divided into the *Optical Properties*, and into the *Physical Properties* of minerals, or such as refer more particularly to their *Mass* or *Substance*.

CHAPTER I.

OF THE OPTICAL PROPERTIES OF MINERALS.

§. 194. EXPLANATION.

Optical properties are such as *depend* upon light, and are not observable except in its presence.

The consideration of the natural-historical properties in general, presupposes the presence of light. Yet all of them do not *depend* upon its presence. This is the case, however, with the properties to be considered in this chapter. We cannot maintain that a mineral possesses in the dark that same colour, lustre, or transparency, which it exhibits when observed under the influence of light. This character, therefore, is sufficient for distinguishing the optical properties from all other properties of the minerals.

§. 195. LUSTRE, COLOUR, TRANSPARENCY.

The phenomena observable in minerals, with respect to reflected and transmitted light, are comprehended under the heads of *Lustre*, *Colour*, and *Transparency*.

These subjects are treated of in Natural History, only in so far as they allow some application to its own peculiar purposes, of discriminating and describing minerals. Their explanation belongs to Natural Philosophy.

In order to employ lustre, colour, and transparency, agreeably to the purposes of the Natural History of the Mineral Kingdom, it is necessary to determine and to provide with certain denominations, those differences which may be distinguished in these properties, in respect to both, their kind and their degrees of intensity. This will require us to fix a certain impression upon our mind, and always to designate this impression with the same name, or to recal it to our memory, whenever we read this name, or hear it uttered. It is necessary therefore to have experienced these impressions upon our own mind, and explanations or descriptions cannot be successfully substituted in their place.

An acquaintance with the colours occurring in the Mineral Kingdom, with the different kinds of lustre, &c., may be acquired from the consideration of bodies, which are not minerals. It may indeed be effected in more than one way; but it seems to answer the purpose best, if we employ these bodies themselves, this being not only a more sure, but also an easier way of fulfilling our intention. To those who do not intend to go any farther than to become capable of making use of the Characteristic for discriminating varieties occurring in nature, a very small number of specimens will suffice, the choice of which is not attended with any difficulties.

The relative value we have to attach to the employment of these properties, depends upon their generality and constancy, and consequently upon the possibility of their application. The properties most important, in Determinative Mineralogy (§. 11.) are the different kinds of lustre, the metallic colours, and particularly the phenomena of simple and double refraction, as observed in common and polarised light. As yet we cannot dispense in the characters with the first and the second, although they are of comparatively less value. The third are of great consequence: in many cases they are sufficient for determining the system of crystallisation, in such varieties as do not present any regular form, or any traces of cleavage; they very often yield neat and decisive characters, where the minerals are much resembling each other in the rest of their properties, and they are most useful in ascertaining whether a mineral is simple or compound. For the Descriptive Mineralogy (§. 11.) all the optical properties of the minerals are of equal consequence, and at the same time not inferior to any other properties. These descriptions have not the purpose of distinguishing objects, but that of producing an image of them; and to this the colours confer no less than the forms, the knowledge of lustre and transparency no less than the degrees of hardness or of specific gravity.

The optical properties must therefore by no means be neglected, although many of them are of less influence than those deriving from the forms to the scientific progress of Mineralogy. Very often by their assistance we may dispense with the use of the Characteristic, because they are very well calculated for recalling to our mind such varieties of the same species as we have already determined. They are obvious at first sight, and therefore easily observed and determined under whatever circumstances they may be found, they are in particular recommended to those who wish to acquire an extensive *general* knowledge of the productions of the mineral kingdom.

§. 196. KIND AND INTENSITY OF LUSTRE.

The lustre of minerals is considered in respect to its *Kind*, and in respect to its *Intensity*.

The kinds of lustre are:

1, *Metallic lustre*,
2, *Adamantine lustre*,
3, *Resinous lustre*,
4, *Vitreous lustre*,
5, *Pearly lustre*.

Metallic lustre is subdivided into *perfect*, and *imperfect* metallic lustre. The first occurs in all the species of the orders Metal, Pyrites, and Glance, and in some of those of the order Ore. The same lustre occurs in wrought metals and metallic alloys, as silver, brass, copper, &c. The second is found in several ores, as Tantalum-ore, Uranium-ore, &c.

Adamantine lustre is subdivided into *metallic* adamantine lustre, and *common* adamantine lustre. Examples of the first may be seen in the dark-coloured varieties of several species of the order Blende, and in some, particularly the grey varieties of di-prismatic Lead-baryte; the common adamantine lustre is peculiar to octahedral Diamond, to the pale-coloured varieties of Ruby-blende and Garnet-blende, and to some varieties of di-prismatic Lead-baryte.

Resinous lustre is that which a body presents, if besmeared with oil or fat. It occurs in dodecahedral Garnet, still more distinctly in pyramidal Garnet; also in the varieties of empyrodox Quartz, called Pitchstone.

Vitreous lustre is that of common glass, and may be observed in rhombohedral Quartz, rhombohedral Emerald, prismatic Chrysolite, and in several other species of the order Gem.

Pearly lustre is divided into *common*, and *metallic* pearly lustre. The first occurs in prismatoidal and hemi-prismatic Kouphone-spar, in prismatic Disthene-spar, in rhombohedral Talc-mica, and in other species of the order Mica; the second is found in several species of Schiller-spar, and in several varieties of rhombohedral Talc-mica.

As to the intensity of lustre, we have to distinguish the following degrees:

1, *Splendent*,
2, *Shining*,
3, *Glistening*,
4, *Glimmering*,
5, *Dull*.

Splendent faces, or those which possess the highest degree of lustre in the whole mineral kingdom, produce distinct and well defined images of the objects, provided they possess the required extension and evenness. Such faces are contained in many varieties of dodecahedral Garnet-blende, of rhombohedral Iron-ore, of rhombohedral Quartz, and other species.

Shining is a less degree of lustre; it is still lively, but does not produce a distinct image. This degree is very common in several species of the orders Spar, Haloide, Baryte, &c.

Glistening reflects light still more disorderly; but although it does not any longer produce an image, yet it reflects it in pretty well defined patches. This degree of lustre is found in most of those compound minerals, in which the particles of composition are still observable, or at least in which they have not yet entirely disappeared. Examples are pyramidal Copper-pyrites, tetrahedral Copper-glance, &c.

Glimmering does not reflect defined patches of light, but a mass of undefined light seems spread over the glimmering surface. This degree of lustre is peculiar to the very thin columnar composition (commonly called fibrous fracture), and to several other compound minerals, in which the composition disappears, as in the varieties of rhombohedral Quartz, called flint, calcedony, hornstone, in compact hexahedral Lead-glance, and other minerals. Commonly this degree is a sign of a compound mineral, the individuals of which are so very small, as nearly to disappear. It is produced by the reflection of light from every one of the impalpable component particles.

Dull possesses no lustre at all. This perfect absence of lustre is almost entirely confined to decomposed minerals, as Porcelain-earth, which is a decomposed Feld-spar, and to some compound minerals, as Chalk, where it depends upon some particular circumstances in the formation.

§. 197. SERIES IN THE DIFFERENCES OF LUSTRE.

The gradations in the kinds and in the degrees of lustre in the varieties of *one and the same* natural-historical species, produce *continuous series.*

In general, neither the kinds nor the degrees of lustre admit of rigorous limits. It is necessary to determine them in some particularly distinct examples, and to compare with them such as are less distinct.

If there occur several kinds or degrees of lustre in the varieties of a species, these will be in an uninterrupted connexion, and they will pass insensibly into one another, so that in no place we are capable of observing any interruption or want of continuity.

From the succession in these gradations, the above mentioned series arise. Rhombohedral Ruby-blende presents a striking example of a similar series. In some of its varieties the lustre is nearly metallic, in others decidedly adamantine. Between these there are a great number of gradations of metallic adamantine lustre, by which the two kinds of lustre are so closely connected, that it is impossible to say where the one begins, and where the other terminates.

The series in the gradations of lustre allow of the same application as the series in the varieties of colour mentioned in §. 202. The results of crystallography in respect to simple forms, are exactly confirmed by the occurrences of lustre in single individuals, as to the existence of forms, which, although they appear with a number of faces, not sufficient to limit the space from all sides, yet must be considered as simple forms (§. 168.). For if we abstract from what is merely accidental, homologous faces entirely agree as to the kind and to the degrees of intensity of their lustre; and, *vice versa*, such faces which do not agree with each other in the same respects, are not homologous, or they do not belong to one and the same simple form. This is equally applicable to faces of crystallisation, and to faces of cleavage, as is shewn by numerous examples in prismatoidal Gypsum-haloide, in several species of the order Mica, in several species of the genus Kouphone-spar, &c. Pearly lustre is the most remarkable among the different kinds, since, in a high state of perfection, it appears in simple minerals only upon *single* faces of crystallisation, as well as of cleavage. Such faces, therefore, are parallel either to the axis, or to the base of the fundamental form of the species. A single face which possesses a distinct pearly lustre, if it is a face of cleavage, is also termed *Eminent*. The pearly lustre of compound minerals very often is a consequence of this composition.

§. 198. COLOUR, PROPERLY SO CALLED, AND STREAK.

We have to distinguish between the colour of the entire mineral and that of its powder. The first is the *Colour* of the mineral, properly so called, while the second has been designated by the name of the *Streak*.

§. 199. DIVISION OF COLOURS.

The colours have been divided into *metallic* and into *non-metallic* colours.

This division is not rigorously correct, because the difference does not so much lie in the colours themselves, as in the kinds of lustre joined to them. It is, however, very useful, since it separates what is as yet indispensable, from what is merely useful in the process of discriminating minerals.

For the sake of a better distinction of the colours, eight principal colours have been assumed by the celebrated

WERNER, to whose labours this part of Terminology is particularly indebted. These colours are, *White*, *Grey*, *Black*, *Blue*, *Green*, *Yellow*, *Red*, and *Brown*. Each of them comprehends several varieties, of which again the one considered the purest, is called the *characteristic* colour. The names or denomination of the varieties are either derived from such bodies in which they are found often or by preference, or they are formed by composition. Examples of the first are, *rose-red*, *apple-green*, *gold-yellow* ; of the latter, *reddish-brown*, *yellowish-brown*, *greyish-white*, &c.

The Wernerian method in the determination of colours is as generally introduced and received as it deserves. It is not advisable to change or alter any thing without the most urgent necessity, even though, from other reasons, these alterations should be improvements ; since there is nothing required, but to recal a certain impression upon our mind : and the best plan therefore will be to keep to such expressions to which we have been accustomed.

§. 200. METALLIC COLOURS.

The metallic colours are: 1. *Copper-red* ; 2. *Bronze-yellow* ; 3. *Brass-yellow*, and 4. *Gold-yellow* ; 5. *Silver-white*, and 6. *Tin-white* ; 7. *Lead-grey*, and 8. *Steel-grey*, and 9. *Iron-black*.

1. *Copper-red*, the colour of metallic copper. *Examples*, Octahedral Copper ; less distinct prismatic Nickel-pyrites.

2. *Bronze-yellow*, the colour of several metallic alloys called Bronze and *Speise*, in particular the alloy of copper and tin. Very distinct in hexahedral and prismatic Iron-pyrites.

3. *Brass-yellow*, the colour of brass. *Ex.* Pyramidal Copper-pyrites. This colour is never found in hexahedral Gold.

4. *Gold-yellow*, the colour of pure gold. Distinct, but exclusively in hexahedral Gold. The gold-yellow colour sometimes becomes pale, and then approaches to silver-white.

5. *Silver-white*, the colour of pure silver. Distinct in hexahedral Silver; less distinct in prismatic Arsenical-pyrites; inclining to red in hexahedral Cobalt-pyrites.

6. *Tin-white*, the colour of pure tin, particularly not mixed with lead. *Ex.* Fluid Mercury; rhombohedral Antimony, and also native Arsenic, in this, however, inclining a little to lead-grey.

7. *Lead-grey*, the colour of metallic lead. Of this colour three different shades have been distinguished.

a, *Whitish* lead-grey; b, *pure* lead-grey; and c, *blackish* lead-grey. Whitish lead-grey is found in the compact varieties of hexahedral Lead-glance; pure lead-grey in the common varieties of the same species, which consist of larger individuals than the former, also in rhombohedral Molybdena-glance, &c.; blackish lead-grey in hexahedral Silver-glance, in prismatic Copper-glance, &c.

8. *Steel-grey*, nearly the colour of fine grained steel upon a recent fracture. *Ex.* Native Platina and prismatic Antimony-glance.

9. *Iron black*, nearly the colour of highly carboniferous cast iron. *Ex.* Octahedral Iron-ore; less distinct rhombohedral Iron-ore, and tetrahedral Copper-glance.

§. 201. NON-METALLIC COLOURS.

The non-metallic colours are considered in the consecutive order of the principal kinds (§. 199.), which represent the *general* series of colours.

The following are the non-metallic colours:

A. *White*.

1. *Snow-white.* The purest white colour. Nearly the colour of newly fallen snow. *Ex.* Rhombohedral Lime-haloide (Carrara marble), prismatic Lime-haloide (Flosferri).

2. *Reddish-white.* White (though not always of the purest tint) inclining somewhat to red. *Ex.* Several varieties of rhombohedral and macrotypous Lime-haloide, of rhombohedral Quartz, &c.

3. *Yellowish-white.* White (though not always of the purest) inclining to yellow. *Ex.* Several varieties of rhombohedral Lime-haloide and of uncleavable Quartz.

4. *Greyish-white.* White, inclining to grey. Common in rhombohedral Lime-haloide, particularly in the compound varieties called granular Limestone, and in rhombohedral Quartz, particularly in Common Quartz.

5. *Greenish-white.* White, somewhat inclining to green. Very distinct in several varieties of hemi-prismatic Augite-spar, particularly in Amiantus, and in the varieties of prismatic Talc-mica, called Common Talc.

6. *Milk-white.* White somewhat inclining to blue, the colour of skimmed milk. *Ex.* Several varieties of uncleavable Quartz, called Common Opal.

B. *Grey.*

1. *Blueish-grey.* Grey inclining to a dirty blue colour. Seldom distinct. Sometimes in the varieties of rhombohedral Quartz, called splintery Hornstone, and in several compound varieties of rhombohedral Lime-haloide.

2. *Pearl-grey.* Grey, mixed with red and blue. In the pearls this colour is very pale. Sometimes it is very distinct in hexahedral Pearl-kerate; less distinct in several varieties of rhombohedral Quartz, and of prismatic Hal-baryte.

3. *Smoke-grey.* Grey mixed with brown; the colour of thick smoke. This colour occurs particularly in the dark varieties of Flint, which belong to the species of rhombohedral Quartz.

4. *Greenish-grey.* Grey mixed with green. *Ex.* Several varieties of rhombohedral Quartz, particularly Cats-eye; several varieties of rhombohedral Talc-mica, &c.

5. *Yellowish-grey.* Grey mixed with yellow. This colour is not uncommon in several compound varieties of rhombohedral Lime-haloide (compact Limestone) and of rhombohedral Quartz (Flint).

6. *Ash-grey.* The purest grey colour, a mixture of white and black. It is seldom distinct. *Ex.* Prismatoidal

Augite-spar, called *Zoisite*, and trapezoidal Kouphone-spar.

C. *Black.*

1. *Greyish-black.* Black mixed with grey (without any green, brown, or blue tints). *Ex.* Basalt; Lydian-stone, which is an impure variety of rhombohedral Quartz; Anthrakolite, an impure variety of rhombohedral Lime-haloide.

2. *Velvet-black.* The purest black colour. It is the colour of black velvet. *Ex.* Empyrodox Quartz, called Obsidian; rhombohedral Tourmaline, called Schörl.

3. *Greenish-black.* Black mixed with green. A very common colour in several species of the genus Augite-spar.

4. *Brownish-black.* Black mixed with brown. *Ex.* Several varieties of rhombohedral Talc-mica; bituminous Mineral-coal.

5. *Blueish-black.* Black mixed with blue. It is a rare colour, and scarce ever found except in the botryoidal and reniform varieties of Black Cobalt from Saalfeld in Thuringen.

D. *Blue.*

1. *Blackish-blue.* Blue mixed with black. *Ex.* The dark coloured varieties of prismatic Azure-malachite.

2. *Azure-blue.* A very bright blue colour, mixed with a little red. *Ex.* The pale varieties of prismatic Azure-malachite, and the bright varieties of Lapis lazuli.

3. *Violet-blue.* Blue mixed with red. *Ex.* Rhombohedral Quartz (Amethyste), and octahedral Fluor-haloide.

4. *Lavender-blue.* Blue with a little red, and a great deal of grey. *Ex.* Lithomarge, and some varieties of Porcelain Jasper.

5. *Plum-blue.* A colour inclining somewhat to brown, and very difficult to describe. It is something like the colour of certain varieties of plums. It is very rare, and occurs only in a few varieties of dodecahedral Corundum, and of octahedral Fluor-haloide.

6. *Prussian-blue*, or Berlin-blue. The purest blue colour.

Ex. Rhombohedral Corundum (the bright coloured varieties of Sapphire); prismatic Disthene-spar; and hexahedral Rock-salt.

7. *Smalt-blue.* The colour of a pale coloured sort of smalt, called *Eschel.* *Ex.* Several varieties of prismatoidal Gypsum-haloide.

8. *Indigo-blue.* Blue mixed with black and green. The colour of several coarser sorts of indigo. *Ex.* Prismatic Iron-mica, particularly the decomposed or imperfectly formed varieties called Blue Iron-earth.

9. *Duck-blue.* Blue, with a great deal of green, and a little black. *Ex.* Several varieties of dodecahedral Corundum, called Ceylanite; also several varieties of prismatic Talc-mica, under the denomination of common Talc.

10. *Sky-blue.* A pale blue colour, with a little green. It is called Mountain blue by painters; it is the colour of the clear sky. *Ex.* Prismatic Lirocone-malachite, sometimes also octahedral Fluor-haloide.

E. *Green.*

1. *Verdigris-green.* A green colour, very much inclining to blue. It is the colour of verdigris (Acetite of Copper). *Ex.* Amazon-stone, which is a variety of prismatic Feld-spar, and prismatic Lirocone-malachite.

2. *Celandine-green.* A green colour, mixed with blue and grey. *Ex.* Prismatic Talc-mica, called Green-earth; several varieties of rhombohedral Emerald.

3. *Mountain-green.* Green, with a great proportion of blue. *Ex.* Rhombohedral Emerald; prismatic Topaz, the oriental *Aqua-marine.*

4. *Leek-green.* Green, with a little brown; the colour of the leaves of garlick. Very distinct in rhombohedral Quartz, called Prasem.

5. *Emerald-green.* The purest green colour. Very distinct in rhombohedral Emerald; less characteristic in some varieties of hemi-prismatic Habroneme-malachite.

6. *Apple-green.* A light green colour, with a trace of yellow. Very distinct in rhombohedral Quartz, called Chrysoprase.

7. *Grass-green.* Green, mixed with a little more of yellow; the fresh colour of grass. Very distinct in Green Diallage, and other varieties of paratomous and hemi-prismatic Augite-spar; sometimes in pyramidal Euchlore-mica and in hemi-prismatic Habroneme-malachite.

8. *Pistachio-green.* Green, with yellow and a little brown. *Ex.* Prismatic Chrysolite; sometimes prismatoidal Augite-spar.

9. *Asparagus-green.* Pale green, with a great proportion of yellow. *Ex.* Prismatic Corundum and the varieties of rhombohedral Fluor-haloide from Spain and Salzburg, called Asparagus-stone.

10. *Blackish-green.* Green, with black. *Ex.* Paratomous Augite-spar; sometimes also Serpentine.

11. *Olive-green.* Pale green, with a great deal of brown and yellow. *Ex.* Prismatic Chrysolite, the varieties called Olivine; several varieties of dodecahedral Garnet; hexahedral Lirocone-malachite, and Pitchstone, a variety of empyrodox Quartz.

12. *Oil-green.* A green colour, still lighter, with more of yellow, and less of brown. The colour of olive oil. *Ex.* Dodecahedral Garnet-blende; rhombohedral Emerald; empyrodox Quartz, called Pitchstone.

13. *Siskin-green.* A light green colour, very much inclining to yellow. Very distinct in pyramidal Euchlore-mica; also in some varieties of rhombohedral Lead-baryte.

F. *Yellow.*

1. *Sulphur-yellow.* The colour of pure sulphur. *Ex.* Prismatic Sulphur.

2. *Straw-yellow.* A rare colour; light yellow, with a little grey. Nearly the colour of straw. *Ex.* Some varieties of prismatic Topaz, called Pycnite.

3. *Wax-yellow.* Yellow, with grey and a little brown. The colour of pure yellow wax. *Ex.* Pyramidal Lead-baryte; several varieties of uncleavable Quartz, called common Opal.

4. *Honey-yellow.* Yellow, with a little red and brown; the dark colour of honey. *Ex.* Rhombohedral Lime-haloide; octahedral Fluor-haloide; pyramidal Melichrone-resin.

5. *Lemon-yellow.* The purest yellow colour. *Ex.* Prismatoidal Sulphur, and the decomposed varieties of uncleavable Uranium-ore, called Urane-ochre.

6. *Ochre-yellow.* Yellow, with brown. *Ex.* The varieties of rhombohedral Quartz and of uncleavable Quartz, if mixed with oxide of iron, from which this colour is derived.

7. *Wine-yellow.* A pale yellow colour, with a little red and grey. The colour of several sorts of white wine. *Ex.* Prismatic Topaz from Saxony and from Asia Minor; octahedral Fluor-haloide.

8. *Cream-yellow.* A pale yellow colour, with a little red and very little brown. Rare. Sometimes in Lithomarge, and Bolus from Strigau in Silesia.

9. *Orange-yellow.* Yellow, very much inclining to red. The colour of ripe oranges. *Ex.* Several varieties of pyramidal Lead-baryte from Hungary and Carinthia.

G. *Red.*

1. *Aurora-red.* Red with a great deal of yellow. Very distinct in several varieties of hemi-prismatic Sulphur.

2. *Hyacinth-red.* Red with yellow and a little brown. *Ex.* Pyramidal Zircon, called Hyacinth; dodecahedral Garnet.

3. *Brick-red.* Red with yellow, brown, and grey. The colour of newly baked bricks. *Ex.* Hemi-prismatic and prismatoidal Kouphone-spar; also Porcelain-jasper, and other varieties of burnt clay.

4. *Scarlet-red.* The brightest red colour, but not without a tint of yellow. It is the colour of Cinnabar, or of the streak of peritomous Ruby-blende.

5. *Blood-red.* Red with a little of yellow and black. The colour of blood. *Ex.* Dodecahedral Garnet, called Pyrope.

6. *Flesh-red.* A pale red colour, with grey and a little yellow. *Ex.* Prismatic Hal-baryte.

7. *Carmine-red.* The purest red colour, that of carmine. Rare. *Ex.* Dodecahedral Corundum. This colour occurs less distinct in the capillary varieties of octahedral Copper-ore.

8. *Cochineal-red.* Red with a little blue and grey. *Ex.* Rhombohedral Ruby-blende; dodecahedral Garnet.

9. *Rose-red.* A pale red colour, mixed with white and a little grey, the colour of the flowers of *rosa centifolia.* *Ex.* Rhombohedral Quartz, called Rose-quartz, macrotypous Parachrose-baryte.

10. *Crimson-red.* Red with a little blue, a particularly fine colour. *Ex.* Rhombohedral Corundum (the bright coloured varieties of Ruby); prismatic Cobalt-mica.

11. *Peachblossom-red.* Red with white and more of grey than rose-red. The colour of peachblossom. *Ex.* Prismatic Cobalt-mica; also Lepidolite.

12. *Columbine-red.* Red with a little blue, and a great deal of black. Distinct in dodecahedral Garnet.

13. *Cherry-red.* A dark red colour, mixed with a great deal of blue, and a little brown and black. *Ex.* Prismatic Purple-blende.

14. *Brownish-red.* Red with a great deal of brown. The colour of reddle, a well known substance for drawing. *Ex.* Iron-flint, a mixture of rhombohedral Quartz and oxide of iron. This colour occurs besides almost exclusively in undeterminable varieties of rhombohedral Iron-ore.

H. *Brown.*

1. *Reddish-brown.* Brown mixed with a great deal of red. *Ex.* Dodecahedral Garnet-blende; pyramidal Zircon.

2. *Clove-brown.* Brown with red and a little blue. Very distinct in prismatic Axinite; also in several varieties of rhombohedral Quartz.

3. *Hair-brown.* Brown with a little yellow and grey. *Ex.* Prismatic Iron-ore; several varieties of uncleavable Quartz, called Wood-opal.

4. *Broccoli-brown.* A brown colour mixed with blue, red, and grey, hardly to be defined, and scarcely ever to be met with, except in pyramidal Zircon.

5. *Chesnut-brown.* The purest brown colour. *Ex.* Egyptian Jasper, a variety of rhombohedral Quartz, mixed with oxide of iron.

6. *Yellowish-brown.* Brown with a great deal of yellow. *Ex.* Iron-flint and common Jasper, both varieties of rhombohedral Quartz, mixed with oxide of iron.

7. *Pinchbeck-brown.* Yellowish-brown, with a metallic lustre, or with a metallic-pearly lustre. *Ex.* Several varieties of rhombohedral Talc-mica. In this mineral, at least, pinchbeck-brown does not deserve the name of a metallic colour, since it is only superficial, and is changed in the streak into white or grey.

8. *Wood-brown.* Brown with yellow and grey. The colour of old, nearly rotten wood. Very distinct in several varieties of hemi-prismatic Augite-spar, called Mountain Wood; sometimes also in Bituminous Wood.

9. *Liver-brown.* Brown with grey and a little green. *Ex.* Common Jasper, a variety of rhombohedral Quartz, mixed with oxide of iron and clay; it occurs also in brown Earthy Cobalt, which is oxide of cobalt mixed with clay.

10. *Blackish-brown.* Brown with a great deal of black. *Ex.* Several varieties of black Mineral-resin, and of bituminous Mineral-coal, called Brown-coal.

The mentioned varieties of colours represent as many fixed points, between which there exist in nature a great number of shades or varieties. Such colours are expressed by the indications of those two, with which they agree nearest. It is not necessary, however, that these two colours should be consecutive ones in the above series; they may even be varieties of different principal colours. If the occurring colour differs but little from one of these fixed points, it is said to represent, or to be that colour, only *inclining*, or *passing* into another.

Colours may be different in their intensity, though belonging to one and the same variety. Differences of this kind are indicated by the expressions, *pale*, *light*, *deep*, *dark*, which expressions do not require any further explanation.

§. 202. SERIES OF COLOURS.

The varieties of colours occurring in the individuals of one and the same species, form an uninterrupted series, which is called the *Series of Colours of that Species.*

If we consider the colours occurring in a species, which is pretty complete in this respect, we find that they insensibly pass into each other, or that every one of them is intermediate between two others. Thus they represent an uninterrupted succession of the shades of colours, and this it is what is meant by the series of colours.

The occurrence of the series of colours is the most important fact in respect to the present subject for the use of Natural History. In order to obtain these series, by abstracting from the rest of the natural-historical properties of the individuals in any complete species, it is necessary to exclude all those colours, which are derived from a mixture with heterogeneous minerals; as, for instance, the lemon-yellow, and the blood-red colours of prismatic Hal-baryte, the existence of which is owing to an admixture of certain species of the order Sulphur; or the yellowish-brown and reddish-brown colours of rhombohedral and uncleavable Quartz, arising from oxides of iron, &c.

The series of colours cannot be described; they must necessarily be studied from nature; but the little trouble which this requires will be amply rewarded. In the species of the order Gem, the most striking examples occur. Octahedral Diamond, rhombohedral Corundum, prismatic Topaz, rhombohedral Emerald, dodecahedral Garnet, and rhombohedral Tourmaline, may serve as examples for the illustration of these series. The series of colours of octahedral Fluor-haloide is one of the most common, and very easily completed, at least to a certain extent. This series comprehends a great variety of colours, and resembles, in some respect, the series of colours of octahedral Diamond and of rhombohedral Fluor-haloide.

It is deserving of notice, that very often all the species of one genus possess nearly the same series of colours. Thus the several species of the genus Garnet entirely agree with each other; the three most common species of the genus Augite-spar, and more particularly the hemi-prismatic and the paratomous one, almost entirely coincide in their series of colours.

The metallic colours do not form any series at all, or at least, the series which they form are very limited. This is the reason why they are more applicable in the Characteristic than the non-metallic colour, the employment of which is almost entirely confined to the Descriptive part of Mineralogy. There are series containing both metallic and non-metallic colours, as, for instance, those of rhombohedral Ruby-blende, of rhombohedral Iron-ore, and others.

The series of colours differ very essentially from the series of homogeneous forms (§. 85.) or from the series of crystallisation themselves (§. 136.). The latter are derivable from a single form, which is given or has been observed, and can be obtained in their whole extent between their limits; whereas the former arise by the interpolation of new members between known ones; and these consequently cannot be produced with security beyond that extent, which is given by immediate observation.

§. 203. SEVERAL OTHER PECULIARITIES IN THE OCCURRENCE OF COLOURS.

The *Play of Colours*, the *Change of Colours*, the *Opalescence*, the *Iridescence*, the *Tarnish*, and the *Delineations of Colours*, must be considered as properties very remarkable in themselves, though of comparatively little use in Natural History.

The only use that can be made of these properties, is in the Descriptive part of Natural History, and even here it is very limited.

The *Play of Colours* is produced, if the mineral in certain directions reflects as it were coloured points of great intensity, which change with the position of the mineral, or with the direction of the rays of light. Of this property, octahedral Diamond, if cut, and precious Opal, a variety of uncleavable Quartz, both cut and in its natural state, are quoted as examples. The play of colours in octahedral Diamond depends upon the reflexion of refracted light, occasioned by the artificial facets; in precious Opal it is more analogous to the change of colours and the opalescence.

The *Change of Colours* consists in the reflection of bright hues of colour, in certain directions depending upon the structure of the mineral. The mineral which presents the change of colour in the most remarkable degree, is Labradore felspar, a species of the genus Feld-spar.

The *Opalescence* consists in a kind of milky light, which certain minerals reflect, either if cut *en cabochon*, or upon plane faces both natural and artificial. It is, like the preceding property, analogous to the play of colours in uncleavable Quartz. In the varieties of rhombohedral Quartz, called Cats eye, it depends upon composition; in prismatic Corundum, and in the transparent varieties of prismatic Feldspar, called Moonstone, it depends upon the crystalline structure. Upon this structure it likewise depends in rhombohedral Corundum, and in dodecahedral Garnet: this appears in particular in the six-sided and four-sided stars of light, from which the varieties of the former have received the name of Asteria.

The *Iridescence* shews the colours of the rainbow, similar to those produced by the refraction of light, through a prism of glass. It presupposes fissures or separations in the interior of the minerals, which may depend on structure or on composition, or which may even be entirely accidental. The included space, not filled up by the mineral, shews the phenomenon of the coloured rings, sometimes very bright as in rock-crystal, a variety of rhombohedral Quartz, where it is occasioned by accidental fissures in the interior. Another variety of the same species, called the

Rainbow Calcedony, shews similar colours but more faint, and here they depend upon composition.

Another remarkable property of certain minerals is, that they shew *different colours*, if examined by transmitted light in different *determined* directions, which demonstrates that it is intimately connected with their forms and structure. This property of minerals has been called their *Dichroism*. Rhombohedral Tourmaline, prismatic Quartz, rhombohedral and prismatic Talc-mica, are among the most distinct examples. Several varieties of the first are nearly opaque in the direction of the axis, while they shew different degrees of transparency, and different green, brown, and blue colours, in the directions perpendicular to it. Prismatic Quartz is blue in the direction of the axis, yellowish-grey perpendicular to it. Prismatic Talc-mica is sometimes green in the direction of the axis, and brown perpendicular to this line, &c. The application of this property is greatly extended by examining minerals in polarised light, where many minerals shew dichroism, which exhibit in common light the same colour in every direction.

The *Tarnish* consists in the alteration of the colour of a mineral upon its surface. It is necessary to be acquainted with this peculiarity of certain minerals, in order to avoid confounding it with their real colours. Minerals with a perfect metallic lustre are almost the only ones subject to become tarnished; in these it produces many shades of bright colours, the further distinction of which, however, is of very little use in Natural History. Several minerals become tarnished in a very short time, if a new fracture has been effected. Among these we observe native Arsenic.

Simple minerals very seldom present more than one colour at a time. There are, however, examples of the occurrence of two colours, as in rhombohedral Corundum, prismatic Topaz, rhombohedral Tourmaline, prismatic Disthene-spar, and a few others. Compound minerals, on the contrary, are very often variegated; and the *Delineation of Colours* comprises the figures which the different colours produce. It is superfluous to enter, in this respect, into a minute detail. With regard to the *dendritic* delineations, however, it is worth noticing that they are real imitative forms (§. 183.), and that therefore they do not refer to the mineral upon which they are found: they may be only superficial, or be distributed throughout the whole mass of the specimen.

The delineations of the Florentine ruin marble, a compound variety of rhombohedral Lime-haloide, represent on a small scale a very interesting phenomenon, which occurs very often in nature on a larger scale; this, however, is a subject foreign to the Natural History of the Mineral Kingdom.

§. 204. THE STREAK.

If we scratch a mineral with a sharp instrument, either a powder will be produced, or the scratched place assumes a higher degree of lustre. Both these phenomena are comprehended under the general expression of the *Streak.*

The lustre is increased by the streak in malleable metals, in several species of the order Glance, and in several varieties of black Mineral-resin. This is likewise the case with clay, and with several other decomposed minerals.

The best method for observing the colour of the powder, is to rub the mineral upon a plate of porcelain biscuit, or upon a file, till the powder appears. In those minerals which are too hard for a process of this kind, the streak itself is of no great consequence.

Some minerals retain their colour in the streak; others change it. Among the former are most of those belonging to the orders Glance, Haloide, Spar, and all those of a white colour; among the latter, several of the orders Ore, Pyrites, Blende, &c. The former are said to be unchanged in the streak; of the latter, the alteration of the colour in the streak is indicated. A white or grey streak of minerals is said to be *uncoloured.*

§. 205. DEGREES OF TRANSPARENCY.

With regard to the transparency of minerals, we have to observe the relative quantity of light which is transmitted through their substance. The use of the degrees of transparency is confined to the Descriptive part of Natural History.

These degrees are,

1, *Transparent*, if the light is transmitted in a sufficient quantity to enable us to distinguish small objects placed behind the mineral.

2, *Semi-transparent*, if it is possible to see an object behind the mineral, without, however, being able to distinguish more of it, than its general outline.

3, *Translucent*, if a small quantity of light only falls into the mineral, but without allowing an object behind it to be seen, except in so far as it in general may prevent the light from falling upon the mineral.

4, *Translucent on the edges*, if only the most acute edges of a mineral receive some light, while the interior remains perfectly dark. This degree of transparency has moreover been distinguished into strongly and feebly translucent on the edges; and it is upon distinctions of this kind that are founded the differences between the varieties of rhombohedral Quartz, called flint, hornstone, jasper, &c.

5, *Opaque*, if a mineral transmits no light at all.

The species of the orders Metal, Glance, and Pyrites, consequently most of those which possess a perfect metallic lustre, are entirely opaque. This is, however, not quite general for all the minerals of a metallic appearance, as for instance the lamellar varieties of rhombohedral Iron-ore, which transmit sometimes in the sun a very bright red colour.

Minerals of a non-metallic appearance, are not entirely opaque, a few species of the order Ore, perhaps excepted. Yet accidental impurities influence so much their transparency, that this property becomes almost entirely useless in the Determinative part of Natural History. The best employment to be made of it seems to be, in the distinction of compound varieties from simple ones, if the minuteness of the particles of composition prevents them from being observed immediately. Commonly in the same species (§. 190.) the compound varieties possess a less degree of transparency than the simple ones. A very distinct example of this we have in the varieties of rhombohedral Quartz. Almost all its single individuals, provided they are not impure, shew higher degrees of transparency than flint, hornstone, calcedony, and other compound varieties.

CHAPTER II.

OF THE PHYSICAL PROPERTIES OF MINERALS.

§. 206. EXPLANATION.

The properties of the substance of minerals, or those which have by preference been termed their physical properties, comprehend all those which neither depend upon their form and the space which they fill up, nor upon the presence or absence of light.

Among these are the *State of Aggregation*, *Hardness*, *Specific Gravity*, *Magnetism*, *Electricity*, *Taste*, and *Odour*.

It is almost unnecessary to observe, that the expressions mass, or substance, must not be conceived in the chemical sense of the word, and that these properties are not meant to be more essential to the minerals than any of those which have been considered above, which perhaps might be inferred from the consideration, that these properties refer to the minerals themselves, even though we should abstract from their geometrical and optical properties.

§. 207. STATE OF AGGREGATION.

In respect to the state of aggregation, we distinguish *solid* and *fluid* minerals. The former are either *brittle*, or *sectile*, or *malleable*, or *flexible*, or *elastic*; the latter are either *liquid* or *expansible*.

A solid mineral is said to be—

1, *Brittle*, if in the experiment of detaching small particles of it with a knife or a file, these particles lose their coherence, and separate with a grating noise, while they fly about in the state of powder. The particles therefore cannot alter their respective situations without separating entirely. *Ex.* All the species of the orders Gem, Spar, Pyrites, several of those of Ore, Haloide, &c.

2, *Malleable*, if the particles detached by the knife, do not lose their connexion. From a malleable mineral, we may detach slices as we do from metallic lead. *Ex.* Many metals, hexahedral Pearl-kerate, hexahedral Silver-glance, and several varieties of black Mineral-resin.

3, *Sectile*, if in the above experiment the particles lose their connexion, and do not allow the separation of any slices; but if at the same time they do not fly about with a noise, but quietly remain upon the instrument we have applied. The sectile minerals form an intermediate stage between the malleable and the brittle ones. This state of aggregation is commonly not distinguished by natural philosophers, though it is very useful in the characters of several species. Examples of sectile minerals we have in most of the species of the orders Mica and Glance, in some of the orders Haloide, Baryte, &c.

4, *Ductile*, if it can be wrought into sheets or wire; so that by the application of a greater or lesser force, the particles of the mineral may change their relative situation, without absolutely losing their connexion. *Ex.* Several metals, as hexahedral Gold and hexahedral Silver.

5, *Flexible*, if the particles, whose relative situation has been changed, do not resume their former situation. There are flexible minerals, which are neither ductile nor malleable. *Ex.* Several metals, hexahedral Silver-glance, and several varieties of prismatic Talc-mica.

6, *Elastic*, if the particles, whose relative situation has been changed, resume their former situation. *Ex.* Several varieties of rhombohedral Talc-mica, and of black Mineral-resin.

A fluid mineral is more particularly said to be—

1, *Liquid*, if in pouring it out from a vessel, perfect round drops are formed. *Ex.* Water, several Acids, fluid Mercury, and several varieties of black Mineral-resin.

2, *Viscid*, if the drops are not round, but ropy. *Ex.* Several varieties of black Mineral-resin.

Expansible minerals do not shew any further differences in this respect. They comprehend the Gases and some Acids.

It is evident that all these properties are subject to small variations, and that they pass into each other by insensible gradations.

§. 208. HARDNESS.

Hardness in general may be defined to be the resistance of solid minerals to the displacement of their particles. The magnitude of this resistance is their *Degree of Hardness*.

Hardness is a very useful property in the Natural History of the Mineral Kingdom, particularly so in its determinative part.

Nothing is attended with greater difficulties, than the establishment of an accurate scale for the degrees of hardness. It is necessary therefore to endeavour, even without an accurate scale of that kind, to become capable of ascertaining and indicating these differences, at least with a degree of accuracy and certainty sufficient for the wants of the Natural History of the Mineral Kingdom.

The existence of differences in the degrees of hardness among the minerals, is very easily discovered, by the simple experiment of scratching one of them by the other. A sharp corner of rhombohedral Quartz will produce a deep cut in the mass of rhombohedral Lime-haloide; whilst a sharp corner of the latter species does not injure the surface of the former. Hence we infer, that rhombohedral Quartz possesses a higher degree of hardness than rhombohedral Lime-haloide; and in general, that of two minerals, the harder one scratches the other, but cannot inversely be scratched by it. Some precautions, however, are necessary in drawing general inferences from these observations.

If we proceed upon this principle, we may obtain a *Scale* for the degrees of hardness, answering in every respect the purposes of Mineralogy. This is effected by choosing a certain number of suitable minerals, of which every preceding one is scratched by that which follows it, while the latter does not scratch the former; taking care always that the intervals between every two members of the scale be not so disproportionate, as either to render its employment more difficult, or to hinder it altogether.

The following scale possesses these properties:

1, *Prismatic Talc-mica*, the common Talc of mineralogists, of a whitish or greenish colour.

2, *Prismatoidal Gypsum-haloide*, a variety imperfectly cleavable, of an inferior degree of transparency, and not crystallised; crystals and perfectly transparent varieties being rather too soft. This degree of hardness is exactly that of *hexahedral Rock-salt*, which mineral therefore may be very useful, either in being immediately employed in the determination of hardness, or at least in finding out such varieties of the above-mentioned species, as exactly possess the required degree of hardness.

3, *Rhombohedral Lime-haloide.* Any cleavable variety. The minerals called Brown-spar (macrotypous Lime-haloide), or Rhomb-spar (macrotypous and brachytypous Lime-haloide), cannot be employed in its place, the hardness of these being considerably higher.

4, *Octahedral Fluor-haloide.* Any cleavable variety.

5, *Rhombohedral Fluor-haloide.* The variety from Salzburg called Asparagus-stone, possessing a conchoidal fracture. The Apatite from Saxony or Bohemia, will seldom be found to answer the purpose, though of exactly the same degree of hardness.

6, *Prismatic Feld-spar.* A perfectly cleavable variety of Adularia.

7, *Rhombohedral Quartz.* Limpid and transparent.

8, *Prismatic Topaz.* Any simple variety.

9, *Rhombohedral Corundum.* The easily cleavable variety from Bengal, called Corundum-stone.

10, *Octahedral Diamond.*

The minerals, which represent the units of this scale, have been chosen among those species, which may be most easily acquired with the necessary qualifications, excepting perhaps only rhombohedral Fluor-haloide. Yet it has been impossible to find out another which might be as useful in its place.

The intervals between the members of the scale are not everywhere of the same magnitude. Octahedral Diamond is evidently much harder, if compared with rhombohedral Corundum, than octahedral Fluor-haloide, if compared with rhombohedral Lime-haloide. This, however, is of no consequence in the case above mentioned; for there exists no mineral of a hardness intermediate between the degrees represented by the two first of these species. But the interval between rhombohedral Fluor-haloide and prismatic Feld-spar is likewise greater than it should be. In this case it would be very desirable to have another mineral which might allow of being employed instead of rhombohedral Fluor-haloide. But in general it is very difficult to ascertain the perfect equality of the intervals between the different degrees of hardness, and on that account also it is very difficult to be obtained. Yet all these imperfections are by no means prejudicial to the useful employment of the scale.

The degrees of hardness are expressed by means of those numbers which in the above enumeration are prefixed to them. Thus the hardness of rhombohedral Lime-haloide is = 3, that of rhombohedral Corundum = 9.

The intervals between each two subsequent members may be divided into *ten* equal parts; and these tenths determined by estimate. It will very seldom be required to value the hardness to more or less than 0. 5; but it will always be possible to proceed so far as we find it necessary to answer our purpose.

The state of liquidity may be considered as the zero of the scale.

If, in employing the scale, we endeavour to find the degree of hardness of a given mineral, by trying which member of the series is scratched by it, and which of them injures the surface of the given one, it will appear that the specimens employed should possess certain properties, in many cases difficult to be found. They should all have faces perfectly smooth and even, and solid angles or corners of the same form, and be equally durable.

As to the faces, those produced by cleavage seem the most eligible, if they possess a pretty high degree of perfection. Faces of crystallisation are commonly uneven or streaked; cut and polished faces, however, in many instances shew a less degree of hardness than the mineral really possesses.

It is still more difficult to obtain the corners with the constant quality which is required. Even in a determined form these are sometimes liable to be so much influenced by structure, that they give very uncertain results. In this respect, the solid angles of the tetrahedron, and those of the octahedron of octahedral Fluor-haloide, shew quite different phenomena. The corners of compound varieties, in which the individuals become impalpable or disappear, such as Calcedony, Flint, and others, are commonly found very powerful, much more so than similarly formed corners of simple varieties. But if the composition is still observable, the particles very often separate in the experiment of scratching another mineral, and the corner of a compound mineral cannot produce the effect of that of the simple mineral. The application of the edges is subject to similar difficulties.

Numerous experiments of determining the degree of hardness, by the mere scratching of one substance with the other, have completely established, that this process alone is not sufficient, if we intend to make a more sure and extensive application of the characters that may be taken from hardness, than that which has hitherto been common in Mineralogy.

But if we take several specimens of one and the same mineral, and pass them over a fine file, we shall find that an equal force will everywhere produce an equal effect, provided that the parts of the mineral in contact with the file be of a similar size, so that the one does not present to the file a very sharp corner, while the other is applied to it by a broad face. It is necessary also that *the force applied in this experiment, be always the least possible.*

Every person, however little accustomed, will experience a very marked difference, if comparatively trying in this way any two subsequent members of the above scale, and thus the difference in their hardness will be easily perceived. A short practice is sufficient for rendering these perceptions more delicate and perfect, so that in a short time it is possible to determine differences in the hardness very much less than those between two subsequent members of the scale.

Upon these observations is founded the application of the scale, the general principle of which consists in this, that the degree of hardness of the given mineral is compared with the degrees of hardness of the members of the scale, not immediately, by their mutual scratching, but mediately, *through the File*, and determined accordingly.

The process of this determination is as follows :

First we try, with a corner of the given mineral, to scratch the members of the scale, beginning from above, in order that we may not waste unnecessarily the specimens representing lower members. After having thus arrived at the first, which is distinctly scratched by the given mineral, we have recourse to the file, and compare upon it the hardness of this degree, that of the next higher degree, and of the given mineral. Care must be taken to employ specimens of each of them nearly agreeing in form and size, and also as much as possible in the quality of their angles. From the resistance these bodies oppose to the file, and from the noise occasioned by their passing over it, we argue with perfect security upon their mutual relations in respect to hardness. The experiment is repeated with all the alterations thought necessary, till we may consider ourselves arrived at a fair estimate, which is at last expressed by the number of that degree with which it has been found to agree nearest, the decimals being likewise added, if required.

The files answering best for the purpose are fine and very hard ones. Their absolute hardness is of no consequence; hence every file will be applicable, whose hardness is in the necessary relation with that of the mineral. For it is not the hardness of the file with which we have to compare that of the minerals, but the hardness of another mineral, by the medium of the file. From this observation it appears, that the application of the file widely differs from the methods of determining the hardness of minerals which have hitherto been in use; as scratching glass, striking fire with steel, cutting with a knife, scratching with the nail, &c.

Besides an appropriate form, there is another necessary property of the minerals to be determined, consisting in their state of purity. Neither the degree of hardness, nor that of specific gravity, can be correctly ascertained, if we employ impure substances. For the same reason it would be wrong to make use of minerals which have undergone a total or even partial decomposition; and in general every circumstance which might influence the hardness, must be duly attended to, if we intend to arrive at a useful and correct result.

Minerals that cleave with particular facility in only one direction, very often shew a less degree of hardness upon the perfect face of cleavage, than in other directions. Prismatic Disthene-spar sometimes is scratched by octahedral Fluor-haloide upon the eminent face of cleavage, whilst an angle of the very same individual scratches not only rhombohedral Fluor-haloide, but even sometimes prismatic Feld-spar. If we intend to determine a mineral of this description by the help of the Characteristic, it will be the best plan to take a mean term between the two degrees measured, or rather to keep nearer to the higher one. It would be wrong to receive them into a scale of hardness, like the preceding one, since this would betray a want of acquaintance with the scale itself, and with its employment.

Supposing all the precautions necessary in determining the degrees of hardness to have been taken, and the circumstances well attended to, which might have exercised some influence; we find that those individuals which belong to one and the same species, admirably agree with each other in respect to this property; and that deviations from an exact coincidence, if they happen to occur, do not take place *per saltum*, but that they are joined with each other by intermediate members. These members produce a series, in most cases between very narrow limits. This observation seems to be contradicted by the authority of several mineralogical works. But there are indeed few subjects with regard to the properties of minerals, which have been treated with more indifference or even carelessness than their hardness, and on this account little or no credit is due to what most of the mineralogical works contain of its indications.

Kirwan, De la Metherie, and Romé de L'Isle, have each endeavoured to construct scales of hardness. A comparative table of the hardness of different substances is contained in the works of HAÜY. A glance at them will suffice for enabling us to form an idea of their applicability.

§. 209. SPECIFIC GRAVITY.

If we suppose the absolute weight of one of two bodies, possessing the same volume, to be = 1; the ratio of the absolute weight of the other to this unit, is termed its *Specific Gravity.*

The determination of the specific gravity depends upon the comparison between absolute weights and volumes. They cannot be instituted at all, or at least not to a sufficient degree of accuracy, merely by sight or estimate. We must avail ourselves of the assistance of appropriate instruments, if we wish the determination to be of use.

The instruments intended for ascertaining the specific gravity of solid bodies, are the Hydrostatic Balance and Nicholson's Aræometer. That of a liquid is determined by weighing in it a solid body, whose specific gravity we know, and which is not soluble in the liquid. Instruments have likewise been constructed for this purpose. The determination of the specific gravity of expansible fluids requires very delicate operations, and instruments that are not within the reach of every body.

The arrangement of the two above mentioned instruments, their use, and the whole process of taking the specific gravity of bodies, are very generally known, or at least they may be found described at large in every treatise on Natural Philosophy. Each of them possesses particular advantages.

The hydrostatic balance allows of a high degree of accuracy, and is very convenient in its use. The instrument being correct in itself, and delicate as a common balance, its delicacy as a hydrostatic one will depend upon the thinness of the thread by which the vessel is suspended, which bears the body immersed in the water. A human hair is sufficiently strong for supporting a weight of three hundred grains, and therefore very useful in taking the specific gravity of such bodies as do not possess any great absolute weight.

The pin, which supports the uppermost cup of the aræometer, destined for the reception of the weights and of the body to be weighed, must possess a certain diameter, since it acts not only as a supporter, but also in the capacity of a real weight, according to the depth to which it is immersed in the water. This diameter, however, must remain within certain limits, if the instrument shall not lose its niceness.

The hydrostatic balance will therefore be more eligible for more accurate inquiries, either for obtaining a greater number of decimal figures, or for determining the gravity of a very small specimen: hence it must always be employed, if our object is to fix the limits of the range in the specific gravities of a natural-historical species, for the sake of the determination of its varieties; and this has been done in the species contained in the subsequent Characteristic. For the common use of determining the specific gravity of minerals, in order to find out their denominations by the assistance of this Characteristic, the aræometer will be found both sufficient and preferable, because in this case we may acquiesce in most cases in the first decimal figure of the specific gravity. The instrument is besides recommendable, on account of its being cheap and portable. The size of the specimens, the specific gravity of which may be taken by help of the aræometer, cannot exceed certain limits, determined on one side by the absolute weight it will bear, till it be immersed to the sign marked upon the pin, on the other by the niceness of the instrument itself.

In taking the specific gravity, we must likewise observe the degree of temperature. The changes of temperature render it necessary to determine the normal weight, or that which is required for depressing the aræometer to a certain point, at every experiment, in the same way as it is necessary in the hydrostatic balance, which likewise, previous to every experiment, must be brought into equilibrium.

The minerals, of which we intend to take the specific gravity, must be perfectly pure. The greatest care therefore must be taken in removing as much as possible whatever heterogeneous substances may adhere to them, or at least, if this should be impossible, not to neglect considering the influence of such an admixture upon the correctness of the results. Moreover, all the vacuities or empty spaces within the specimens, must carefully be opened. In order to get rid of these, the minerals ought to be broken down, till, even by the assistance of a microscope, we can no longer detect a want of continuity in the fragments. Compound varieties are more subject to contain similar vacuities than simple minerals; for this reason the composition must be removed, at least in so far that it cannot have any more influence upon the accuracy of our results. Yet the minerals must not be too much reduced in size, since this might lead into an opposite error, in supposing those minerals lighter than water, which swim upon it, when reduced to an impalpable powder.

These precautions have been very often neglected in taking many of those specific gravities quoted in mineralogical works, and thus numberless erroneous and inaccurate statements have been introduced, which render their employment at least uncertain, and on that account useless for Mineralogy. Another source of error, for which many examples might be quoted, consists in the incorrect determination of the natural-historical species to which the specific gravities have been referred, and which have passed from one work into another.

A certain degree of attention is required, both in selecting the specimens and in the operation of ascertaining the specific gravity. But from this it will appear that the results of the single experiments made upon specimens of homogeneous minerals, coincide in a remarkable degree; and thus we may argue upon the great importance of the application which this property will allow in the Natural History of the Mineral Kingdom.

§. 210. MAGNETISM.

Some minerals act upon the magnetic needle, if they are brought within the sphere of its attraction. Others become magnets themselves. These phenomena are made use of as characters, under the name of *Magnetism.*

The only minerals hitherto known, which exercise a considerable action upon the magnetic needle, are the octahedral Iron, and the octahedral Iron-ore. Rhombohedral Iron-ore, rhombohedral Iron-pyrites, and several others, likewise act upon it, but with less energy.

Instead of a needle, the magnetic bars may be applied in examining minerals, which in this case must be converted into a fine powder, in order to extract from them such particles as possess magnetic properties.

§. 211. ELECTRICITY.

Several minerals produce electric phenomena; some of them by friction, others by pressure, others by communication, and others by heat. Some are idio-electric; others are conductors of electricity. These phenomena may be usefully applied as characters of minerals.

Vitreous electricity is produced by friction in most minerals of the orders Gem, Spar, Mica, Baryte, &c. in several Haloides, and even in Salts. In the same way those of the orders Sulphur, Resin, and Coal, shew the phenomena of resinous electricity. As conductors of electricity, we may notice the minerals of the orders Metal, Pyrites, and Glance. Those of the orders Blende, Ore, and several others, do not appear quite uniform in this respect.

Heat produces electric phenomena in prismatic Topaz, in rhombohedral Tourmaline, in prismatic Kouphone-spar, in axotomous Triphane-spar, in prismatic Zinc-baryte, &c. The opposite extremities of the crystals assume in these species opposite kinds of electricity, and they possess therefore electric axes. Tetrahedral Boracite shews four electric axes, coinciding with the rhombohedral axes of the hexahedron. This difference in the electric phenomena is very often accompanied by a different configuration of the opposite terminations of crystals (§. 160.).

The processes employed in producing and observing the electric phenomena, and the small apparatus required, may be found in many works, both described and illustrated by figures. Perhaps these phenomena will prove in future more useful for the purposes of Natural History, than has hitherto been the case, since, in respect to minerals, they have been too generally considered as mere physical curiosities.

§. 212. TASTE.

Several minerals, solid as well as fluid, produce a sensible taste. Most of the solid ones are tasteless. This difference yields very useful general characters.

All the Acids and Salts produce some taste. The salts found in nature, commonly not shewing any of the characters required for their exact determination, their taste is almost the only one left to which we possibly may recur; and for this reason the differences in the kinds of taste have been provided with particular denominations. The following expressions have been employed:

1, *Astringent* for the taste of vitriol;
2, *Sweetish* for the taste of alum;
3, *Saline* for the taste of common salt;
4, *Alkaline* for the taste of soda;
5, *Cooling* for the taste of saltpetre;
6, *Bitter* for the taste of epsom salt;

7, *Urinous* for the taste of sal ammoniac;

8, *Sour* for the taste of sulphuric acid, or of carbonic acid.

Besides, the intensity or other peculiarities of several kinds of taste may be indicated, which is sufficiently plain by the manner in which it is effected in this work.

Pure artificial salts are most eligible as examples for the different kinds of taste. Some caution is required in ascertaining this character in unknown minerals. Since in most cases it is quite sufficient to know, whether or not a mineral excites some taste, we may also dissolve them in water, because all sapid minerals are soluble in a small quantity of this fluid.

§. 213. ODOUR.

There are minerals which, either spontaneously or when rubbed, emit some odour, which likewise in particular cases may afford useful characters.

Several varieties of black Mineral-resin possess a bituminous odour. The species of the genus Iron-pyrites emit a sulphureous odour, when strongly rubbed, as takes place in striking fire. The Arsenical-pyrites under the same circumstances yield an arsenical or garlick smell. Several varieties of rhombohedral Lime-haloide, of prismatic Hal-baryte, of prismatoidal Gypsum-haloide, &c., if rubbed with hard substances, emit an empyreumatic odour; pebbles of rhombohedral Quartz, and other hard bodies, if rubbed against each other. Several Resins produce a peculiar odour, if rubbed against soft substances.

Certain species of Gas, of expansible Acids, possess a peculiar kind of odour; that of rotten eggs, of rotten fish, of burning sulphur, &c.

Besides the characters treated of till now, there are still some more phenomena which have been employed as such, by mineralogists. Among these, the *Adhesion to the tongue* is almost exclusively met with in decomposed minerals;

the *Unctuous and Meagre touch* are used for distinguishing certain friable minerals; and the *Phosphorescence* produced by heat, is also employed in those minerals in which the natural-historical properties properly so called are not observable. It would be superfluous to dwell any longer upon these subjects; the more so since every Treatise on Mineralogy may be consulted for all their particulars.

PART II.

THEORY OF THE SYSTEM.

§. 214. IDENTITY.

Natural productions, which do not differ from each other in any of their natural-historical properties, are *identical* (§. 14.).

This proposition is self-evident; and it is the foundation of the whole Theory of the System in Natural History.

By considering in this science two bodies as *identical*, it is meant that every one of them may be substituted in the place of the other in every natural-historical respect; so that if the one belongs to a certain class, to a certain order, genus, or species, the other likewise must necessarily belong to the same class, to the same order, genus, and species.

In considering the identity of two bodies, we must abstract all accidental differences (§. 25.). Such are, besides the size of crystals, also the disproportionate enlargement of some of their faces (§. 159.), their junction with other individuals, their being implanted or imbedded, &c. Individuals, which differ only in properties of this kind, must be taken for identical ones, as well as those which agree also in respect to these accidental circumstances.

§. 215. DIFFERENCE.

Individuals, which do not agree in all their natural-historical properties, are not identical.

This proposition is an immediate consequence of the preceding one.

If two individuals agree in every one of their properties, except in their crystalline form, or in colour, or in hardness, or in specific gravity, &c., so as to differ only in one of these properties, nevertheless they will not be identical. For the above mentioned properties are natural-historical ones, and upon these depends their identity or their difference (§. 214.). Hence the difference among individuals may be produced by a difference, however small, in any one of their natural-historical properties; and in this case, from the one nothing can be argued in respect to the other. It is almost superfluous to observe, that accidental differences also in the present place cannot have any influence upon the difference or identity of bodies.

The Natural History of the Mineral Kingdom does not require any foreign assistance for determining what is accidental or not, and agrees in this respect with the Natural History of the Vegetable and of the Animal Kingdoms.

§. 216. DEGREES OF DIFFERENCE.

The difference among those individuals which are not identical (§. 215.), does not everywhere take place in the same degree.

Suppose two crystals of dodecahedral Garnet, to agree in all their natural-historical properties except in their crystalline forms; the form of the one being the dodecahedron, while that of the other is a digrammic tetragonal-icositetrahedron. These individuals are evidently different. Now, suppose one of those crystals again, to be compared with a crystal of hexahedral Gold. There is also a difference between these two individuals; and nobody will hesitate in pronouncing the degree of difference in the latter case to be higher than that in the preceding one; even though the form of the crystal of hexahedral Gold should be exactly the same as that of the crystal of dodecahedral Garnet. Many examples of this kind might be quoted, which indubitably demonstrate the degrees of difference not to be the same in every two different individuals. We may very easily perceive what influence this must have upon the further consideration of mineral productions, and what would be the consequence if, on the contrary, we should meet everywhere with exactly the same degrees of difference.

These degrees of difference must not be valued according to the number or the kind of properties in which those individuals differ which are not identical. They depend rather upon certain relations of these properties with each other, which will be explained afterwards. A crystal of hexahedral Iron-pyrites is much more different from a crystal of prismatic Iron-pyrites, though they should exactly agree in every property except in the form, and what depends upon it, than one crystal of rhombohedral Corundum, of the variety called Sapphire, from another of the same species called Adamantine spar. And yet the form of the first is an isosceles six-sided pyramid, that of the other a regular six-sided prism. The first presents almost no trace of cleavage, while the other cleaves very easily parallel to the faces of a rhombohedron: they differ moreover in colour, in transparency, and in many other characters. Such examples are common; and whoever therefore would determine the degrees of the natural-historical affinity according to the number, or even to the kind of properties not agreeing, considering the one as more essential than the other, would act contrary to the principles of Natural History.

If it were possible to invent a scale for measuring with accuracy the degrees of difference among the non-identical individuals, this would afford most useful assistance in classifying the productions of inorganic nature. But there exists no such scale. We must contrive, therefore, to collect several of the non-identical individuals, adapted to this purpose in respect to their properties, and *to bring them under the idea of identity* (§. 214.). This will enable us to extend the inferences which may be drawn from identical individuals, to such as by themselves do not exactly agree in all their properties. Without this process of extending the idea of identity, it would be impossible to derive from it a sufficiently useful employment in Natural

History. In order to effect this, however, some preparations will be necessary.

§. 217. MUTUAL RELATIONS OF THE NATURAL-HISTORICAL PROPERTIES IN CERTAIN INDIVIDUALS.

Individuals, which are different from each other in their natural-historical properties, so that their differences constitute members of *one and the same* series, may thoroughly agree with each other in the rest of their properties.

Among the different cases comprised under this head, there is one more remarkable than the rest, if the differences of the individuals consist in the forms, and if these forms are members of a series. It deserves to be considered more in particular, since all the others may very easily be explained upon a similar principle.

Experience confirms, in numerous examples, that forms, which are different members of the same series, may, in other respects, possess properties entirely equal and similar. A demonstration, however, may be given of this proposition with more generality, and which therefore will receive a greater degree of evidence than that which it could acquire by any number or description of examples quoted.

It has been observed above (§. 139.), that such forms, as are members of one series, may enter into combination with each other, and inversely, that all the combinations produced by nature contain only such simple forms as belong to, or represent members of, one and the same series.

An individual appearing in a compound form, appears at the same time in as many simple forms as the combination contains; and, in respect to these, it may be considered as representing as many individuals at a time (§. 138.). But with every one of these simple forms we find connected the rest of the natural-historical properties of the individual under consideration. These assemblages of properties represent individuals, which, in so far as their forms are members of the same series, differ only in these, and in none of their other properties. Every combination occurring in nature confirms, therefore, the above proposition; and if we are capable of deriving useful arguments from it, these will be perfectly general, on account of the perfect generality of the laws according to which combinations are formed (§. 139. 140.).

The preceding observations are not limited to the series of crystallisation; but they extend to every natural-historical property, by the differences or gradations of which series are produced. It applies, however, equally to any natural-historical property whatever: their gradations may produce series or not; because those which give no series at all, or at least not in every instance, may yet be considered as series of equal members. In the present inquiry the series of forms have been chosen by preference, because they allow of a mathematical mode of treatment, and therefore impart a full evidence to the arguments derived from them. Together with the arguments, also, this evidence is transferred to other series of properties treated in the same manner.

§. 218. INDIVIDUALS BROUGHT UNDER THE IDEA OF IDENTITY.

Individuals, whose forms are members of a series, their remaining natural-historical properties being entirely coincident, may be brought under the idea of identity.

It cannot be liable to any objection, that every individual, not excepting those which appear in compound forms, is identical with itself. But if in this combination we allow all the simple forms to disappear, except one, and continue this process with every simple form contained in the combination; we develope a *series of individuals*, each of which is exactly in the same relation to the idea of identity as the fundamental individual, and which therefore exactly agree in respect to this idea, although they are not absolutely identical. It is indifferent which of the simple forms we may ascribe to the one or to the other; and thus *we may arbitrarily exchange these forms with each other, without in the least altering any thing in respect to that relation.* Two or more of these individuals become absolutely identical (§. 214.), if we suppose them to possess one and the same form. Herein consists the process by which, under the supposed circumstances, individuals, though not absolutely identical, may yet be brought under the idea of identity.

If, on the contrary, a number of different individuals is given, agreeing in every natural-historical property except the forms of crystallisation, and if these forms are members of the same series, we are entitled to consider all those individuals as a *single* one, whose form is a combination of the different simple forms of the single individuals, with which the rest of the properties exactly agree. The individual in the compound form is identical with itself, and the single individuals contained in it will consequently be in an exactly similar relation to the idea of identity. This proves that by the above mentioned process, that is to say, by substituting one form instead of the other, they may be collected under the idea of identity.

In order to explain this by an example, let us suppose a crystal of octahedral Fluor-haloide to possess the form of a hexahedron. If we substitute the octahedron in its stead, the relation of the individual to the idea of identity is not altered, because the two forms, the hexahedron and the octahedron, are members of the same series of crystallisation. Notwithstanding this and other similar substitutions, the individual does not cease to be octahedral Fluor-haloide. But if, instead of the hexahedron, we suppose a rhombohedron, or any other form not belonging to the tessular system, to take the place of the hexahedron, the relation of the individual to the idea of identity is indeed altered, and it can no longer be maintained that the individual still remains octahedral Fluor-haloide. For if we suppose a number of individuals, to agree with each other in all their properties, excepting their forms, these however not being members of the same series; we are not entitled to apply to them the above process; because the differences existing among them cannot be removed or made to disappear by the idea of a series, and accordingly the individuals themselves cannot be brought under the idea of identity. The degree of difference (§. 216.) between such individuals is therefore much higher than it would be, if under the same circumstances the forms were members of the same series.

As an example of the latter case, we may quote the hexahedral and the prismatic Iron-pyrites. There exist individuals in these two species exactly agreeing with each other in every one of their natural-historical properties, except their crystalline forms. But as these forms belong to different systems, and are therefore incompatible with each other, the difference between the individuals appears greater than it would be, if the forms should belong to the same series.

The process explained in the members of the series of crystalline forms, naturally applies likewise to every property, from the gradual differences of which series arise. Thus we are provided with the means of discovering such individuals, as, though not absolutely identical, may yet be brought under the idea of identity, and of separating them from all the rest. Such individuals might be collected under particular ideas; but such ideas would be of little use, on account of their very limited application to experience. Nevertheless they lead the way to that idea, which it is the particular object of the present inquiry, to develope according to the principles of Natural History.

§. 219. CONNEXION OF SEVERAL SERIES OF INDIVIDUALS.

An individual which, on account of its form and the rest of its natural-historical properties, is a member of a series of individuals (§. 218.), differing in nothing but their forms, may *at the same time* be a member of another series of individuals, differing only in the gradations of their colours, &c., the rest of the natural-historical properties being supposed exactly to agree.

Let A, B, C, D, . . .

represent a *Series of Individuals*, in which the forms and the colours are not yet determined, so that every one of those letters signifies the aggregate of the remaining properties, which are exactly the same in all of them. Hence in this respect they differ from each other only by their succession, that is to say, by their not being one and the same thing. Suppose, now, every individual to have the same colour a, but different forms, without the latter of which they would not be different individuals. According to our supposition, these forms must be members of the same series, and may therefore be expressed by

$$X,\ X+1,\ X+2,\ X+3,$$

where X may denote any fundamental form whatever. The designation of the series of individuals, as above, only including their forms and colours, will therefore be

I. A. a. X; B. a. (X + 1); C. a. (X + 2); D. a. (X + 3); ...

A fragment of another series of individuals may, under the same restrictions as those mentioned above, be designated by

. . . P, Q, R, S, . . .

P, Q, R, &c. being similar aggregates of properties, as A, B, C, &c. in the preceding series. Suppose the difference among the individuals to consist only in their colours, which, according to the supposition, are members of one series of colours. The colours of the individuals

. . . b, c, d, e, . . .

which represent members of a series of colours, may be joined to a form X + n, common to them all, and to the above mentioned aggregates, so that the entire designation of the series of individuals, every property being taken into consideration, becomes

II. . . . P. b. (X + n); Q. c. (X + n); R. d. (X + n); S. e. (X + n); . . .

The individual connecting the two series, or that which at the same time is a member of the series I. and a member of the series II.; must necessarily possess the form (X + n) and the colour a, the rest of its properties coinciding exactly with those of the two series. If, for the individual above mentioned, we designate that aggregate by N, the individual itself will be:

N. a. (X + n).

For under these circumstances its properties, excepting the form, agree entirely with those of series I.; and this form is a member of the series X, X + 1, &c.; whilst in the same manner, excepting its colour, it agrees exactly with series II.; its colour being a member of the series b, c, . . .

We are led by experience to assume such relations as those mentioned above. Suppose, for instance, A, B, C, ... P, Q, R, ... in the above signification of the letters, to be varieties of octahedral Fluor-haloide. Let the members of the series of forms in I. be * the dodecahedron (D), the octahedron (O), a digrammic tetragonal-icositetrahedron (I), a tetraconta-octahedron (T) ... and the colour grass-green (gg); the series of individuals will be

A. gg. O; B. gg. D; C. gg. I; D. gg. T . . .

In the series II. the series of colours may be apple-green (ag), mountain-green (mg), verdigris-green (vg), sky-blue (sb)... and their form the hexahedron (H); the series of individuals therefore

. . . P. ag. H; Q. mg. H; R. vg. H; S. sb. H . . .

The above mentioned forms and colours have not only

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* We may choose whatever forms of the series of crystallisation, and whatever varieties of the series of colours of octahedral Fluor-haloide; we shall always derive the same results.

been really observed in octahedral Fluor-haloide, and acknowledged as members of their respective series; but we know also, from §. 218., that without in the least affecting the relation to the idea of identity, they may arbitrarily be exchanged with each other, and that we were even entitled to *produce* or *suppose* the members of the two series I. and II. if we had not had any occasion of observing them in nature. The individual, whose remaining quality is expressed by N, becomes thus = N. gg. H; and this likewise is either an object of our immediate observation, or it may be produced by connecting single observations.* Thus experience confirms to its full extent, that several series of individuals may be connected with each other in the manner described.

An individual N. y. (X + n), can therefore be at the same time a member of two different series, only under the following conditions. Those of its properties which have not been mentioned by name, and which are here expressed by N, must agree with the properties analogous to them in the two series; and those which have been named (in the preceding case, forms, and colours), must be members of the respective series, produced by the properties in the two series of individuals. Under these circumstances, N. y. (X + n) may be brought under the notion of identity (§. 218.) with the members of the first, but at the same time also with the members of the second series. From this we draw the inference, that *all the members of one of these series may be brought under the idea of identity, with all the members of the other.*

If we continue this process, and extend it upon all those properties which form series by their gradations, we may include the assemblage of all those individuals, which, notwithstanding their differences, may yet be brought under the idea of identity. At the same time those individuals which do not allow the process to be applied to them, are excluded

* It is evident, that if it be found necessary thus to obtain a determined object, the individuals of every well determined natural-historical species may serve as examples.

with perfect distinctness and accuracy. An assemblage of individuals formed in this way does not contain any thing foreign, nor does it want any thing capable of being united with it on account of its natural-historical properties.

§. 220. SPECIES.

An assemblage of individuals, brought under the idea of identity by the process of §. 219., is termed a *Species;* and the individuals belonging to it are *homogeneous* individuals.

This is the pure natural-historical and invariable idea of the species in the Mineral Kingdom. The series of characters do not every one of them allow of a mathematical treatment. This, however, has no influence upon their application for producing the idea of the natural-historical species; and nothing is lost of the peculiar evidence of this idea, which immediately flows from the mode of its formation (§. 218. 219.). Under these circumstances, the idea of the species is capable of becoming a certain foundation to the whole scientific Mineralogy; it must likewise be the fixed point, from which every inquiry has to start, whose object it is to procure some knowledge of the productions of the Mineral Kingdom, of whatever kind this knowledge may be, if we wish to preserve a certain unity in the acquirement of our information.

We must not pass over unnoticed any of the series, in short none of the natural-historical properties, in producing the idea of the natural-historical species; because this would render the idea itself incomplete; the variety of nature could not be explained sufficiently and to its full extent, nor could it be demonstrated, by a general developement, that *we are really entitled to consider certain bodies under the required circumstances, as belonging to one and the same species, although they differ in their natural-historical properties;* and this nevertheless is the very proposition which was to be proved by the preceding considerations. It would be contrary to the principles of Natural History, to determine the idea of the species according to *single* properties, of whatever kind these may be. An idea thus determined is not scientific, and cannot be but incomplete. It will never be found sufficient in its application, and thus open the way to the introduction of other considerations, foreign to Natural History, in producing the idea of the species. This has been the source of the contamination which the science has suffered through the introduction of heterogeneous principles, the disagreeable consequences of which have long ago been sufficiently conspicuous. Moreover, the determination, according to single characters, will unavoidably introduce a distinction among *essential* and *accidental* properties, which cannot be allowed to take place, either in developing the idea of the species, or in considering the identity of individuals (§. 215.).

The species itself is the proper object of classification, or the thing which is to be classified. The idea of the species, therefore, cannot be produced by the classification, as some naturalists seem to believe, who begin and terminate their classification, without previously having produced the idea of the species. This idea is constant everywhere, in all sciences, concerning the productions of the Mineral Kingdom ; and it must be the foundation of every system, whatever may be the principles followed in its construction. The correct determination of the natural-historical species thus appears to be of the greatest moment in the Mineral Kingdom.

§. 221. TRANSITIONS.

The progress of the gradations in the properties of homogeneous individuals is termed a *Transition* or *Passage ;* and we say of individuals, in which such a progress may be demonstrated, that *they pass into each other.*

The transitions arise from the series of characters. *Simple* transitions take place only in one character, *compound* transitions in more than one character at the same time. The simple transitions are very evident, but comparatively rare. The compound ones are more common, but they must be followed up in every simple transition of which they consist, if we wish to draw consequences upon which we may rely. This is effected by supposing the differences in all the properties to disappear, except in the single one, in which the transition is to be considered. If these differences constitute members of *one and the same* series, there exists a transition in this property; if they cannot be joined in *one and the same* series, we are not entitled to assume a transition. After having thus followed up and demonstrated the simple transitions in every one of those properties which present differences in a number of individuals, we may consider the compound transition, with the greatest security, as really existing, and the individuals themselves as passing into each other.

§. 222. HOMOGENEITY FROM THE TRANSITIONS.

Individuals, connected by transitions, are homogeneous, or belong to one and the same species.

A transition in a single character, for instance in the forms of crystallisation, arises, if these forms of the individuals are members of the same series of crystallisation, all the remaining properties being equal. Under these circumstances the individuals are homogeneous.

The colours form a transition, if, in several individuals, exactly agreeing in the rest of their characters, they represent members of the same series of colours. But in this case again the individuals are homogeneous.

Hence the individuals also are homogeneous, if they are joined by compound transitions.

It is not necessary that the members of the series of crystallisation representing the transition, be such as immediately follow each other, or produce among themselves a coherent fragment of the series. Thus, not only R, $R + 1$, $R + 2$ ... but also R ... $R + n$ ... $(P + n)^m$ ... $R + \infty$ will answer the idea of what is meant by a transition. It is the same with the transitions in any other series, for instance, in the shades of colour, &c.; though in these it must be applied with the necessary degree of circumspection. In the forms of crystallisation, this likewise becomes necessary in respect to limiting forms, since these are common to some of the series, in some cases even to all the series of the same system of crystallisation.

Transitions exist only within the species, as it evidently follows from the preceding considerations; hence *there can be no transition* from one species into another.

Many examples of this kind of transitions may be found, not in nature, but in several mineralogical books. Of these it may be maintained, that wherever the transition is correct, the determination of the species is erroneous, and *vice versa*, that the transition is falsely stated, if the determination of the species be correct.

From the continuity of the transitions, or of the series of characters from which they depend, we may infer, that there is a remarkable connexion within the natural-historical species, by which all the differences occurring in its individuals may be joined into a whole. Thus we become capable of comprehending the variety of inorganic nature. For the very same reason also, it is contrary to the real interest of Mineralogy to divide or subdivide the species, and to distinguish sub-species and kinds. The purpose of such divisions is to facilitate the general survey of the species; but this indeed would rather be assisted by establishing the connexion between its individuals, if this should happen to be still wanting, than by such divisions, which render it less evident.

With those divisions into sub-species and kinds, which have hitherto been in use, it has very often been the case, that an individual was really found to belong to the species, and yet to none of its sub-species. This has been the consequence of the divisions not having been effected in a single series of characters, but in several series at once.

Let the forms of several individuals in a species be represented by R, R + 1 ... ; their colours by a, a′ ... ; the rest of those characters which produce series by p, p′ ... ; the individuals themselves will be represented by the following aggregates :

R ;	R + 1 ;	R + 2 ;	R + 3 ...
a	a′	a″	a‴ ...
p	p′	p″	p‴ ...

If we divide here only in the series of the forms, so as to ascribe the members R and R + 1 to the first division or sub-species, R + 2 and R + 3 to the second, the individuals uniting the rest of the characters will fall either in the first or in the second of these divisions. But if the division at the same time extends to the colours, and determines, that besides the mentioned forms, the first sub-species should be of the colours a and a′ ; the second, besides its peculiar forms, of the colours a″ and a‴ ; all the compositions like (R + 1). a″, (R + 2). a′ will not belong to any one of those divisions, although they occur as generally in the species as any one of those contained in the two sub-species. Many divisions are found in mineralogical works, of the description above mentioned. These divisions within the species, in whatever character they may be effected, must always remain entirely arbitrary, and on this account they never will be unanimously received. It is much more useful to suppress them altogether, which is the plan adopted in the course of this work.*

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* The application of Mineralogy to the objects of every day's occurrence, may require to give particular attention to certain varieties, which have been employed in the arts, and provided with particular names. This will be properly attended to in the second volume of this work; but not being of any scientific value, it is foreign to our present consideration.

§. 223. PRINCIPLE OF CLASSIFICATION.

The principle of classification in Natural History is the *Natural-Historical Resemblance.*

Several bodies are similar, or resemble each other, which approximate more or less in their properties; and this resemblance is the greater, the higher we find the degree of approximation.

In Geometry, similarity consists in the equality of the relations among homogeneous quantities, and allows of no variation. The idea of similarity in Natural History is not so simple; it cannot be expressed by a single ratio, because here a great many properties must be taken into consideration. It receives a certain latitude, in which there may occur some variation. This, however, has no prejudicial influence, either upon its evidence, or upon its applicability. On the contrary, if we apply this idea of similarity to nature, we find that *only owing to the greater extent* ascribed to it in a natural-historical consideration, *it is capable of being the principle of classification in Natural History.*

It is not difficult to decide the question, whether or not the natural-historical resemblance should be fixed upon as the principle of classification in Natural History. In every science the classification must rest upon such relations as are objects of the science, and therefore it must represent nature according to the image expressed by these relations. Natural History refers to none but the natural-historical properties; hence the approximation of heterogeneous bodies in these properties, or the natural-historical resemblance, is the only relation expressed among or by means of the productions of nature. For this reason Natural History is *forced*, not only to apply this resemblance as its principle of classification, but also carefully to explain and illustrate it, in order to render the classification a true and sufficient representation of nature. In this respect a classification may be called *natural.* The organic kingdoms of nature have always had the advantage of similar classifications, and those departments of Natural History which refer to them, have long ago made a scientific progress. In the Mineral Kingdom, unfortunately, a different way has been proceeded upon; and as a science, the Natural History of the Mineral Kingdom has not been promoted.

In the Natural History of the Mineral Kingdom, the classifiable objects are not the individuals of this kingdom; but according to the preceding inquiries, the natural-historical species (§. 220.). The natural-historical resemblance has therefore nothing to do with individuals. These, indeed, notwithstanding their homogeneity, bear in many instances so little resemblance to each other, that according to the principle of similarity, they rather should be divided than joined. It becomes necessary indeed to demonstrate their connexion by means of the transitions, that is to say, by considering their series of characters, in order to convince ourselves of their homogeneity. Hence the two ideas of *Resemblance* and of *Homogeneity* are essentially different, and the former is not a higher degree, or a nearer restriction of the latter. We must carefully avoid confounding them with each other. In another science, referring to the Mineral Kingdom, another principle of classification may replace the natural-historical resemblance; but there is not another idea which can be substituted for that of homogeneity.

§. 224. DEGREES OF NATURAL-HISTORICAL RESEMBLANCE.

The degrees of natural-historical resemblance in different species, are not everywhere the same.

If we consider the species as unities to be classified, and compare them with each other in respect to their natural-historical properties; we perceive that some of them are more, some of them less allied to each other in resemblance. Thus hexahedral Iron-pyrites is more similar to prismatic

Iron-pyrites, than to rhombohedral Lime-haloide. The latter species again is more similar to the rest of the Lime-haloides, than to prismatic Feld-spar, or to rhombohedral Corundum, &c.

This relation of similarity is not unavoidably necessary. One of the species, for instance, might present the same degree of resemblance to every one of the others, and *vice versa;* so that among these species there could not be perceived even the consecutive order of series. As there really exist several species, among which this is the case, the same might occur among all. This would not limit the variety of nature, although that variety would then appear under a different form.

On the other hand, the relation of similarity might be found different in every particular case; so that if we would suppose a certain species to bear a certain degree of resemblance to another, a second one could not be found, among which and the first, the same degree of resemblance would prevail. The consequence of this would be the impossibility of producing any other arrangement among the species than that of a series, in which they would follow each other according to their different degrees of resemblance.

A single glance at the species of the Mineral Kingdom, and at the above examples, will suffice to shew, that neither of these two suppositions takes place in nature. There exist different degrees of resemblance, by which a series may be produced, but not a series of the single species.

The different degrees of resemblance lay the foundation for the higher ideas of the Theory of the System; that is to say, for the ideas of classification.

§. 225. GENUS.

An assemblage of species, connected by the highest degree of natural-historical resemblance, is termed a *Genus*.

The genus is the resemblance of different species of bodies. In Botany, this idea is commonly limited to the similar formation of the organs of fructification. In Mineralogy, such a restriction is impossible, because the productions of the Mineral Kingdom do not present any parts, different in the same manner from the rest, as is the case in plants. But even supposing their existence, Mineralogy will not admit of any such restriction, on account of the necessity to preserve the idea of the species in its original generality, which admits of no exceptions, deviations, or ambiguities, necessarily connected with a restriction to single characters. For the rest, the idea of the species in Mineralogy is identical with that in Botany; and there is nothing more required, but to shew that it is equally applicable. Thus we shall find ourselves enabled to employ it with the same security in the Mineral Kingdom, as in the Vegetable one.

The species of hexahedral Iron-pyrites agrees so very closely with that of prismatic Iron-pyrites in every character, except the forms, that but for this difference in their systems of crystallisation, they would join into one and the same species. They possess that degree of resemblance which requires their union into the same genus; a degree of similarity expressed in the present instance by the perfect agreement of all the natural-historical properties, except the crystalline forms. The same degree of resemblance prevails among the two species of Emerald; but here, beside the difference in the systems of crystallisation, there is also a difference in the specific gravity. In other genera, as, for instance, in those of Garnet, of Kouphone-spar, and others, we observe differences in many characters at once; and yet the resemblance is here as great as in the above mentioned examples of Iron-pyrites and Emerald; a resemblance which becomes evident upon ocular inspection, the only method of ascertaining its existence. The examples quoted prove, that there may exist differences sometimes only in a few, sometimes in many characters at a time, without having any influence upon the *de-*

gree of resemblance itself. On this account it becomes impossible to express this resemblance by the agreement in one or a certain number of characters. This does not, however, prevent the application of the idea of the genus to the natural productions altogether; for this application does not pre-suppose the idea to be limited to single characters; but it allows, and even requires, to preserve it in its full generality. The genera being thus founded upon the resemblance of the species, as is already demonstrated in Zoology and Botany, render all the services which Natural History possibly may expect; and the same will be likewise confirmed by their introduction in future in Mineralogy.

The natural-historical idea of the genus is peculiar to Natural History, and is solely intended for the purpose of promoting *that* science. Hence the natural-historical genera must not be compared with ideas of this kind otherwise determined; not even with those of the same denomination, which have been hitherto applied in Mineralogy. For, the establishment of these genera is founded in part upon determinative reasons, foreign to Natural History; so that the inferences drawn from them must be inconsistent with the principles of that science, even though it were demonstrated that in every case they correspond to the natural-historical resemblance. Other sciences must consider the natural-historical genus in the same point of view. In a chemical system of minerals, the genus must have a chemical foundation. It is not necessary that it should agree with the genus in Natural History; though the species, determined according to chemical ideas in the one, and according to natural-historical ideas in the other science, must be identical in both (§. 220.). The method of connecting several points of view into a single one, does not promote the sciences, and gives full scope to all sorts of hypotheses. Different sciences, which refer to the same subject, must follow without deviation the course determined by their peculiar principles, or they will cease to be different sciences. Their way will finally converge in one great aim, which is the discovery of truth, perhaps hidden for ever, if we endeavour to arrive at it according to any other method.

§. 226. MINERAL KINGDOM.

The Mineral Kingdom is represented by *a Series of Natural Historical Genera.*

The object of mineralogy in producing its higher degrees of classification, is first, to acquire a clear idea of the Mineral Kingdom, which consists in a general survey of all its productions; and secondly, to become capable of collecting every one of these productions with facility and security, under the above mentioned ideas.

If we examine the systems of Natural History, we find that for the most part they are founded upon the idea of a series. Yet it is difficult to decide, whether in the mineral systems this is a series of genera or of species: for in these systems both genus and species are founded upon reasons so uncertain, that it becomes utterly impossible to derive any clear idea from the existence of their determinations. A series of species is in direct opposition with that of a series of genera, in fact to the idea of the genus itself, in as much as it supposes that there exist no equal degrees of resemblance among different species. In order to convince ourselves in this respect, the best contrivance will be, to produce a series of species in nature, in which those placed nearest must resemble each other most, and where we may begin at any chosen member. Evidently for this process a single variety cannot represent the species, but we must employ the whole species as completely known as possible. In endeavouring to produce a series of this kind, very soon we shall meet with species which render it doubtful whether the one or the other, or even a third, a fourth, &c. should follow the preceding species; and at last we must either entirely abandon the experiment or we must suppose that two, three, or more species occupy the same place in the series. The *Groupes* of species thus formed are the natural-historical genera (§. 225.). In the real existence of genera in the Mineral Kingdom, we discover the reason why there is no series of single species. A more evident demonstration of the preceding observations may be acquired by considering a few examples, for instance, the genera Schiller-spar, Disthene-spar, Triphane-spar, Dystome-spar, Kouphone-spar, Petaline-spar, Feld-spar, Augite-spar, &c., or the genera of the order Baryte, or of any other somewhat more comprehensive order of the natural-historical system of Mineralogy. The series thus representing the Mineral Kingdom is a series of genera, exactly as in the Animal and Vegetable Kingdoms; and like these it does not contain a series of single species.

The Mineral Kingdom is constituted by a series of natural-historical genera, each of which contains similar species (if it contain more than one); every one of these, again, being the assemblage of homogeneous individuals. Thus the idea of the Mineral Kingdom receives its fullest evidence, and requires in this respect no other notions intermediate between that of the genus and that of the Mineral Kingdom.

The idea of a series requires a beginning and a terminal point. There can be no objection to the received order of the three kingdoms, in which the Animal Kingdom is followed by the Vegetable one, and this again by the Mineral Kingdom; and it deserves, in fact, that general reception which it has always found. Those productions of the Mineral Kingdom, which resemble most some of the Vegetable Kingdom, immediately will follow these, and form one of the terminal points in the series of genera which constitutes the Mineral Kingdom. Thus, always according to the principle of similarity, we obtain the whole series of genera, as it is contained in the natural-historical system of the present work.

It will be useful to add here a few remarks on the reception of what has been called the *Atmospherilia* into the Mineral Kingdom. This evidently depends upon the very idea or definition of a mineral. If we examine this idea as contained in most of the mineralogical works, we find several characters, which are not natural-historical ones, and which therefore by no means can be an object of any natural-historical inquiry. If we omit all these as being foreign to the science, the only character still remaining is that of an *Inorganic Natural Production;* so that the mineral kingdom contains the inorganic natural productions *altogether*, and among these consequently also the atmospherilia.

Hence we cannot exclude Water, the different kinds of Gas, of Acid, and of other productions of inorganic nature, from the mineral system, because upon this supposition it would be impossible to define the idea of a mineral, in conformity with the requisites of Natural History, and containing therefore *solely Natural-Historical Characters.* Moreover, it is impossible to say upon what should be *founded* the difference between the Mineral Kingdom and that of the Atmospherilia; nor would the idea of Natural History itself allow to follow the example of most of the naturalists, to pass with silence over the latter.

With the assistance of all the ideas developed till now, it cannot be difficult to decide, whether or not a natural production belongs to the Mineral Kingdom. It will be attended with as little difficulty to determine the species of a mineral, if we know its genus. It is more difficult, from the idea of the Mineral Kingdom to reach that of the genus, that is to say, to determine *the Genus of a Mineral;* and therefore some preparations are wanted, consisting in several intermediate ideas, by which it becomes more easy to descend, as it were, from the highest to the lowest of these ideas. On account of their employment, they have been called the *Degrees of Classification.* These intermediate ideas must be founded upon the general principle of classification in Natural History, exactly like the idea of the genus itself. Their purpose is not to illustrate, but only to render more easy the application of the general ideas of the genus and of the species in the

Mineral Kingdom. These are the ideas of the Order and of the Class.

§. 227. ORDER.

The *Order* is an assemblage of similar genera.

The order is in respect to the genera, what the genus is in respect to the species. The idea of the order is therefore perfectly evident from the preceding inquiries; and the only object that requires some consideration in the present place, is to shew its application to the Mineral Kingdom.

The genus Iron-pyrites, in the peculiar place it occupies in the general series of genera, is surrounded by several other genera, which exhibit so high a degree of resemblance to each other, that they seem to have been formed after a common type or original. These are, the genera Nickel-pyrites, Cobalt-pyrites, Arsenical-pyrites, and Copper-pyrites. There is not another genus to be found in the whole Mineral Kingdom, as hitherto known, which could be enumerated along with them, without destroying the idea produced by the assemblage of the above-mentioned genera. In a similar manner the genus Iron-ore is connected on one side with the genus Manganese-ore, on the other side with the genera Chrome-ore, Cerium-ore, Uranium-ore, Tantalum-ore, Copper-ore, Scheelium-ore, Tin-ore, Zinc-ore, and Titanium-ore. Thus likewise round the genus Feld-spar are assembled the other genera of Spars under similar circumstances. Every groupe of this kind which is an assemblage of genera similar to each other, is an order.

These orders are as distinct from one another, and as remarkable as the genera in the Mineral Kingdom, but they require also, like these, to be observed in nature. They represent in the Mineral Kingdom the natural families of the Vegetable Kingdom; and their reception and determination in the one and the other, depends upon the same principle.

The orders, like the genera, must not be compared with what formerly has been designated by this name in Mineralogy. The chemical principles of classification, received in these systems, more particularly exercise their influence in their higher divisions. In general, Mineralogy must not be compared with any science except Zoology and Botany, and it has nothing to fear, if it is found to admit of this comparison. For this will demonstrate *that it has applied the general principles of Natural History in conformity with itself to its object, which is its peculiar, but at the same time also its only business.*

§. 228. CLASS.

The *Class* is an assemblage of similar orders.

What the genus is to the species, or the order to the genera, the class is in respect to the orders. *Generum genus est Ordo, ordinum autem genus Classis est.* LINN. Phil. Bot. 204. The idea of the class is so comprehensive, that it becomes difficult to judge of its applicability, without the direct inspection of the objects themselves. This inspection proves, that every one of the three classes of the natural-historical system in Mineralogy does contain orders which are connected by a greater degree of similarity, with each other, than with those of other classes.

The idea of the class likewise depends solely upon natural-historical considerations, and does not admit of any foreign principle. It is analogous to the classes in the organic kingdoms, in as far as these are not artificial, or produced by a mere division. These ideas, however, do not allow of any comparison with the classes hitherto used in Mineralogy; which, not being framed according to the principles of Natural History, are partly founded upon hypotheses, and partly on principles which the present state of Chemistry no longer admits.

The systematical ideas of the species, of the genus, the order, and the class, are all that is necessary for producing what has been called a *System of Nature;* and it appears that they possess the same requisites in all the three kingdoms of nature. To form the classes into Kingdoms, and to comprehend these within the still higher idea of Nature, is a problem belonging to Natural History in general, from which the Natural History of every particular kingdom must borrow these higher ideas.

In order to give a general view of the subject, it will be useful shortly to repeat the whole process of developing these ideas, and the systematical unities themselves.

All material bodies are distinguished, according to their most general differences, into *organic* and *inorganic* natural productions (§. 7.).

Organic nature comprehends two kingdoms, the Animal and the Vegetable (§. 8.): inorganic nature comprehends only one, the Mineral Kingdom (§. 9.).

The Mineral Kingdom is a series of natural-historical genera, the succession of which is determined according to their greater or less agreement or similarity (§. 226.). It contains three classes.

Every class comprehends part of the series of genera collected into several orders. The classes are not of the same extent; and the orders which they contain are joined by an equal degree of similarity (§. 228.).

Every order is an assemblage of several genera in their regular succession; hence it is likewise a portion of the general series of genera. The genera comprised within an order, present equal degrees of similarity (§. 227.).

Every genus is an assemblage of similar species; it is a unity in the series of genera. The species within the genera are connected by equal degrees of similarity (§. 225.).

Every species is an assemblage of homogeneous individuals; the individuals of a species are connected by the series of characters, that is to say, by real natural-historical transitions (§. 221. 222.).

The individual is the simple mineral, produced by nature, either singly (§. 160.), or in various compositions (§. 178...189.). It is the only systematic idea which *immediately* refers to nature, or *to which an object of observation* *corresponds.* In respect to the whole of the species, the individual is called a *variety.*

It must here be observed, that none of these ideas have been obtained, or deduced from the others by means of *a division.* For, in order to arrive at them, we have not begun with the highest, but with the lowest one, which is that of the individual, and then we have first determined the idea of the species according to the idea of homogeneity, those of the genus, the order, &c. according to different degrees of natural-historical resemblance; and the whole of them by *aggregation or assemblage.* Besides the idea of the species, a division would have presupposed also that of the Mineral Kingdom, and it would have required a principle, according to which it might have been effected with consistency. In another place it will appear, that these conditions in fact may be fulfilled; yet, by such a division, it would not have been possible to obtain the same classes, orders, and genera, which have been obtained by the other process; and the degrees of classification thus obtained, would not have been suitable to the purpose of giving a general view of inorganic nature, in agreement with the similarity which exists among its productions; and this nevertheless is the last and highest aim of Natural History. *Methodus Naturalis ultimus finis Botanices est et erit.* LINN. Phil. Bot. 206.

§. 229. MINERAL SYSTEM.

The *Mineral System* is the collection of the natural-historical ideas, conformably to the degree of their generality, and applied to the productions of the Mineral Kingdom.

The mineral system is a representation of the Mineral Kingdom by means of general ideas. These ideas must be clear, precise, complete, and correctly subordinate to one another, so as to become consistent with each other, and applicable to experience.

The nature of these ideas evidently depends upon the preceding observations. If, therefore, in general, and from reasons of Natural History, no objection can be made to the ideas of the species, the genus, the order, or the class; the applicability of the mineral system, arising from the connexion of these ideas, will depend solely upon their consistent application to the present state of knowledge.

To apply the ideas of the species, the genus, &c. to experience, is to collect the individuals occurring in nature, agreeably to their homogeneity, into species; to join these species into genera, according to the highest degree of natural-historical resemblance which occurs among them, &c.; or, in other words, accurately to determine the more particular contents of the system. This supposes an accurate examination and a careful comparison of the individuals among each other; and in that respect the only mode in which the mineral system may arrive at its perfection, is by a thorough knowledge of the objects themselves. This knowledge, however, being an empirical one, must always remain incomplete, and is really so at present in a great measure: a perfect mineral system, therefore, is an object, which, though we may approach, we never can reach.

We must proceed with some caution in thus determining the contents of the mineral system: a few rules in this respect will not be superfluous. The first of these rules requires, that we attach the highest importance to the correct determination of the species, because this is the foundation of all the other ideas, and therefore of the system itself. According to another of these rules, we must endeavour to unite newly discovered individuals, along with other species already determined, till we may convince ourselves by an accurate examination, that this is no longer possible; because it is a general rule in Natural History not to multiply the number of the species, without an evident necessity. In fact, one of the greatest errors of many mineral systems consists in their containing too many species, which on that account cannot be well determined, and which, besides other bad consequences, only serve to render the nomenclature difficult and inapplicable. A third rule recommends us not to enter newly determined species into the system with too much precipitation, but to follow the example of other cautious naturalists, and to expect some more extended knowledge from future observations, in order not to hazard precipitous determinations. Indeed it seems better that something, though it were known for some time, should be wanting in the system; than to allow an ill determined species to injure the connexion of the whole. For the rest, mere hypotheses or the results of other sciences, should never be relied on in these determinations, because it is below the dignity of a science, which admits of the application of mathematics, to build its frame upon hypotheses. Natural History too possesses so many means of assistance peculiar to itself, that if well applied (which has not always been the case in Mineralogy) it may rest solely upon its own determinations.

A system thus produced is what has been termed the *Natural System*, because it expresses the different degrees of natural-historical resemblance with which nature itself has stamped its productions. This, however, is the only part nature takes in a natural system, and it is therefore not the *System of Nature.* This idea is merely an imaginary one, and no object corresponds to it. Nature produces only different bodies, but no abstract ideas; and the system of nature, mentioned by several natural philosophers, are only words without ideas, or ideas without object. Opposed to the natural system, there are also *Artificial Systems.* The natural system produces its general ideas by the process of aggregation, while in the artificial ones, the assemblages depend upon general ideas, in as much as they are obtained by the process of division.

The expressions of *natural* and *artificial* systems, though generally received, do not convey the exact idea of what they have to express. They have on that account been the source of many differences of opinion, according to the correct or erroneous ideas attached to them by naturalists. For more than one reason, it would therefore be advisable to substitute in their stead, the expressive denominations of *synthetical* and *analytical* systems.

Doubts have been raised against the possibility of artificial systems in Mineralogy. The experiments indeed which had formerly been instituted, seemed only to demonstrate that these doubts were well founded. The systems of a mixed or double principle do not even belong to these, for they are neither analytical nor synthetical, because they do not possess the unity of principle required in every system.

The production of any system requires the previous determination of the species. The species therefore must have already been established, if we intend to build also an artificial or analytic system. Upon this supposition, any analytic system in the Mineral Kingdom has to resolve one single problem of importance, which is to effect the first division, without impairing the unity, or destroying the connexion within the species. There is no great difficulty in this respect in effecting the subsequent lower divisions.

If therefore we fix upon the systems of crystallisation as a principle of the first division; we obtain the base of an analytic system of Mineralogy, in which the species remains entire, and in which the farther subdivisions may be effected without difficulty.

The natural-historical similarity is entirely lost in analytic systems. Although systems of this kind, if well managed, greatly facilitate the determination of individuals occurring in nature, yet they possess so very little of the other requisites of a system, that they do not at present deserve to be considered in greater detail. Analytic systems may be compared to registers, in which the objects follow each other according to certain single properties, like words in a dictionary, without regard to signification; whereas in the synthetic system, the succession of objects is determined by their natural-historical resemblance, no attention being given to single properties.

I shall not at present venture to examine, whether a natural-historical system, partly analytical and partly synthetical, like the Linnean system in Botany, might not unite the advantages of both, whilst it avoids their inconveniences. A system of this kind would be less objectionable indeed, and more useful than those founded at the same time upon two different principles; but it could not unite those properties which the synthetical system presents, if it only approaches perfection to a certain degree.

The double purpose for which the systematic ideas (§. 226.) have been produced, likewise will require our attention in constructing a mineral system. The first demands a general view of the variety of nature, collected within different unities; the other requires a method of recognising individuals occurring in nature, or of assigning to them their peculiar place in the system, and of providing them with the names and denominations connected with these places.

It seems, that attention has been paid only to the first of these purposes in several of the systems hitherto constructed. The natural-historical system of the present work perfectly answers this requisite. It represents nature according to the different degrees of resemblance which, notwithstanding all their variety, exist among its productions. Several mineral systems do not represent the relations among the bodies or natural productions themselves; but among the results of their chemical analyses, which are not objects of Natural History.

A mineral system may be found to suffice as to the first requisite, without answering the second; it may facilitate the general survey of the productions of the Mineral Kingdom, without assisting in the determination of individuals. On the contrary, it may answer to the second, without fulfilling the first; it may assist in the determination, without producing a general view; and this is the case in analytic systems.

If we examine the systems hitherto published in respect to the second point, we find ourselves completely dissatisfied. An unknown plant may be determined by the help of the Linnean system; but there exists no system, by the assistance of which we might determine an unknown mineral. In order to find the class, we must analyse the mineral; and by this analysis, the mineral is destroyed: nothing remains to determine the order, the genus, and the species. The difficulty is not removed by asserting that only a small portion of the whole is to be taken for the examination; because here, our object is not to determine a mineral in a particular case, but to fix the general and scientific method of determination. Thus, in the systems noticed, we must immediately proceed to the species, for there is no methodical way to arrive at it through the intermediate degrees of the class, the order, and the genus. In short, we must content ourselves with acquiring an *empirical* knowledge of minerals, how small soever the scientific value of this knowledge may be, because indeed it is better to know the natural productions in this way, than not to know them at all. It is evident that a description can be of no use for this purpose; and thus it appears, that systems of the kind mentioned above do not possess either of the two necessary requisites. How far the determination of occurring individuals is facilitated by the natural-historical system, will be examined in another part of the present Treatise.

PART III.

NOMENCLATURE.

§. 230. DEFINITION.

The *Systematic Nomenclature* is the assemblage of those denominations which Natural History applies to natural productions, and which refer to a natural-historical system.

There is only one mode in which Natural History may provide the productions of nature with denominations; but this mode is in the closest connexion with the whole being of the science. The whole object of Natural History in this respect, is to resolve that single problem: to comprehend the unities of observation within certain ideas, which may be formed either by assemblage, or by division (§. 229.). We may say that we *know* a natural production, if we are capable to tell to which of these assemblages or divisions it belongs; but we may say the same thing, if we know its denomination. This denomination must therefore be intimately connected with the above-mentioned ideas; it must *express* the relation, in which that natural production, to which it is applied, stands to others, with whom it agrees more or less in respect to their natural-historical resemblance.

According to the preceding observations, the natural or synthetical systems represent this connexion. That only mode of nomenclature, therefore, which expresses this connexion, will deservedly be called the systematic nomenclature.

The systematic nomenclature alone is capable of fulfilling those conditions which Natural History requires from nomenclature in general. For it provides every natural production with a denomination, and represents, by these denominations, the natural-historical resemblance by which these bodies are connected in the system.

In every part of Natural History, nomenclature is the mirror, which reflects an image of the whole science. The image hitherto produced by Mineralogy, has not been an agreeable one, or calculated to invite the zoologist or botanist, or any enlightened naturalist, to bestow particular attention on Mineralogy. A mass of names and denominations, formed arbitrarily or accidentally, and subject to perpetual change, retard the solid progress of the science, and are a great impediment to the acquisition of knowledge in its purity. The want of a well constructed systematic nomenclature, is therefore an essential defect in the Natural History of the Mineral Kingdom; and the present attempt to remove it, how imperfect soever it may be, is founded upon the very idea of Natural History (§. 18.), which cannot exist without it.

§. 231. OBJECT OF THE SYSTEMATIC DENOMINATION.

The object to which the systematic denomination must refer, is to express the correctly determined natural-historical species.

The natural-historical species is the *foundation*, the systematic nomenclature the *verbal expression* of the system. The species, therefore, is the object to which the systematic denomination refers.

Nomenclature requires that the species be previously correctly determined, according to the principles of Natural History. For the necessary connexion among several of these unities, which is to be represented by a verbal expression, cannot take place upon any other supposition; and the systematic nomenclature is degraded into a mere jumble of words, to which no object corresponds. In this state, it cannot any longer be useful to Natural History;

but it really becomes an impediment, to remove which, the trivial nomenclature (§. 241.), has been resorted to.

In most of the mineral systems, the species has as yet not been correctly determined. *In these systems, therefore, neither a name nor a denomination appertains to a well determined species.*

Such systems contain names and denominations of single varieties of a species; as, for instance, Rock-crystal, Flint, Chrysoprase, &c.; Common Quartz, Conchoidal Hornstone, &c.; but neither a name nor a denomination exists for the whole species of rhombohedral Quartz, to which all these varieties belong.

If it be necessary to denominate a newly discovered species, it is still more necessary to provide a species with a new denomination, which has been *corrected*, because none of the old ones will express it, and because it is impossible to apply all of them at once. The same mode of reasoning must be applied, if a species has not been correctly determined, and contains varieties of several natural-historical species.

In a scientific treatment of Mineralogy, this becomes again a new and urging reason for altering the nomenclature; and it could not be forgiven, if, under these circumstances, we should not at the same time endeavour to give the new nomenclature a systematical arrangement.

The object of nomenclature in general, and more particularly that of the systematic nomenclature, is to express *by words*, or to *denominate* those things or bodies (the species), of which other sciences, Natural Philosophy, Chemistry, Geology, &c., afford more detailed and particular information. This is effected by substituting names and denominations, instead of the characters and the general descriptions of the species. These names and denominations, therefore, must possess such properties as will enable us to find them out, or to recognise them, whenever the characters or natural-historical properties of a natural production are given. This is effected by means of the Characteristic, to be explained hereafter. If they be meant to *excite*, or to *produce*, an image of the natural-historical quality of the objects to which they refer, or to remind us of those which are more or less similar to them, they must indicate the place which the species occupy in the general assemblage of the natural productions belonging to the kingdom.

This is the point of view from which we must consider and develope the arrangement of the systematic nomenclature in general, and the properties of the systematic denominations in particular.

§. 232. PROPERTIES OF THE SYSTEMATIC DENOMINATION.

The systematic denomination must be composed of several words, the order of which expresses the connexion between the denominated object and several others, to which it is more or less similar.

In order to recognise a given individual, or to determine the place which it occupies in the system, it is necessary to proceed with it through all the general ideas of this system, from the highest degree to the lowest one. For this is the means by which we learn the connexion in which it is with others.

If we have to express this connexion by words, we must construct the denomination in such a manner that it may tell the unities of all the above-mentioned ideas, in as far as it is necessary (§. 234.); it must therefore consist of several words; and since the ideas, according to their contents, are subordinate to each other (§. 228.), these words must follow each other according to the same order. That word which expresses the highest idea must precede, and that which expresses the lowest idea must follow, in the order of these expressions. This distribution of words should agree with the general spirit of the language; and the *Latin* language, therefore, would be preferable in the mineralogical nomenclature, as well as in the zoological, or in the botanical one. This language, however, has been so very little employed in Mineralogy, particularly of late years, when the science itself has been more particularly cultivated and enriched, that it would be very difficult to produce a Latin systematic nomenclature, without introducing almost endless innovations. This has been the reason why, in the present first attempt at constructing a systematic nomenclature, the German language has been made use of, to the spirit of which the English language in this respect exactly corresponds. In these two languages, however, the successive order of the words is exactly contrary to that required in the Latin language. In the English nomenclature, therefore, the highest idea will be expressed by the last word, while the lowest idea is indicated in the first word.

That word, with which we designate a single object, or a single species, without regard to its genus, or a single genus independently from its connexion with others in one and the same order, &c., is termed its *Name*. If a name be restricted by means of an adjective, it is transformed into a *Denomination*. A name consisting of a single word, is a *simple* name ; one consisting of two words, is a *compound* name. Simple names can never express the connexion of those bodies to which they are given ; this may, however, be effected by compound names, or by denominations. Hence the mere simple names are of no use in a systematic nomenclature ; but it will require compound names or denominations, or even both of them. The simple name designates the highest idea occurring in the nomenclature, and inversely, this idea must always be expressed by a simple name. The compound name designates a lower degree, the denomination the lowest degree of the ideas expressed by the systematic nomenclature. Agreeably to the observation in §. 234., it may be considered as a rule of the systematic nomenclature in general, that compound names, in the event of their being applied, should never contain more than two words, and that a denomination should never admit of more than one adjective.

§. 233. OBJECT OF THE NAMES.

The ideas expressed by the names are the higher unities of classification, immediately preceding that of the species.

In order to enable us to add the proper restrictions in the denomination of the species, we must apply the name to the genus, or to the order, in general to one of the higher unities of classification. The name, therefore, is not fixed upon a single natural production, or upon an individual, not even upon a single species; but it is applied to an assemblage of a greater extent, and is allowed to be transferred upon the species or upon the individual, only in so far as the one and the other of these belong to the above-mentioned more extensive assemblage, in virtue of their natural-historical properties. This is a peculiar property of every systematic nomenclature, and it is a character by which in particular it differs from the trivial nomenclature. In the latter, the species, or in general the lowest systematic idea bears the name, without any regard to the genus, or to the order in which it is contained. The trivial nomenclature therefore bestows its names upon the objects themselves, without indicating the connexion which exists between them and other bodies, in one and the same system. This definition of the trivial nomenclature explains its farther arrangement, as shewn in §. 241.

The trivial nomenclature allows of an unlimited arbitrariness in providing the species with names; which is limited in a systematic nomenclature. This alone would suffice to shew the usefulness of its general reception, even though it were not recommendable on account of its other qualities. A newly discovered mineral which does not belong to any known species, may perhaps belong to a known genus, and assume its name; or it may belong at least to a known order, and as such receive the name of that order. And even if neither of these be the case, the systematic nomenclature contains the rules, after which it may be provided with a correct denomination.

§. 234. NAME OF THE ORDER.

In the Natural History of the Mineral Kingdom, the *Order* is the highest idea expressed in the systematic nomenclature. The order consequently will bear the simple name.

It depends entirely upon the kind of the system and of the classified objects themselves, whether the genus or the order should be that higher idea above the species, which bears the simple name. If the order be produced by the analytical process of division, as in the Linnean system in Botany, or if the system itself, to which the nomenclature refers, be an analytical system (§. 229.), and if moreover the orders contain a vast number of genera, and these genera again a vast number of species: then it will be more advisable to fix the name upon the genus, and to be contented with the advantages thus arising to the mode of denomination. By placing the name upon the order, the nomenclature would be rendered more difficult, without becoming more useful. For in an analytical or artificial system, there exists no similarity among the classified objects (§. 229.); and a nomenclature expressing the order would not serve to simplify or illustrate the general survey of the objects; it would rather serve to produce confusion. On the other hand, it could not but be useful, even in Botany, to fix the name upon the order, supposing the system to which the nomenclature refers to be the natural system. *Si classes naturales essent inventæ omnes, maxime arrideret terminatio nominum conformis in affinibus, ni nimia mutatio prohiberet.* Linn. Crit. Bot. In the Natural History of the Mineral Kingdom, however, the system itself, and consequently also the order, is a natural or synthetical one; hence there exists a connexion among the classified objects, which may, and even must be expressed by means of the systematic nomenclature. Thus it becomes *necessary* to fix the name upon the order. Only a trifling advantage, indeed, would be gained, if in Mineralogy the genus should be fixed upon as the unity of classification which has to bear the name, since most of the genera contain comparatively only very few species. The systematic nomenclature would thus not find room to shew its usefulness, and to yield the services required by Natural History.

The higher the degree of classification is, upon which we may fix the name, the more we shall be enabled to express, in the denomination of the species, the extent of the connexion among the classified objects. This, however, must have its limits, and should not be carried on so far that it ceases to be either useful or convenient. It is very useful to express the order in the Natural History of the Mineral Kingdom; and this may be effected without any inconvenience. But if we would endeavour also to express the class, this would be both, of little utility, in as much as there are only three classes in the natural-historical system of Mineralogy; and very inconvenient, because, in order to denominate a species, this would require four different words. Moreover, the last of these words being in most cases the same, the constant repetition would produce at least a very disagreeable monotony.

It has been considered as a rule in Natural History, that the genus should bear the name. This rule, as we have seen above, is not a general one. We may consider it, however, as a general rule, that the systematic nomenclature should unite as much as possible all the advantages, and render all those services which Natural History reasonably can expect. The name, therefore, should be so placed, as best to fulfil these demands. From the reasons developed, and under the circumstances above mentioned, this really takes place, if in Botany it is the genus, if in Mineralogy it is the order which bears the name. For the sake of a greater conformity, however, with the method received in Botany, a compound name is fixed in Mineralogy upon the genus, or the name of the genus receives such an arrangement, that it expresses at the same time the order; a contrivance which cannot be considered as objectionable, according to the principles of Zoology or Botany.

§. 235. SELECTION OF THE NAMES OF THE ORDERS.

The simple names are the foundation of the whole nomenclature. In their selection, therefore, we must proceed with the necessary precaution and attention.

I have endeavoured to introduce as few new names as possible, and I have derived therefore as many from ancient Mineralogy, as I found to answer the purpose. *Nomina veterum plantis imposita laudo, ad conspectum vero recentiorum plurium horreo. Hæc enim maximam partem sunt nihil nisi Chaos confusionis, cujus Mater barbaries, Pater authoritas, Nutrix præjudicium.* LINN. Crit. Bot. Several of these ancient names have been of late severely censured, others have been entirely rejected: yet I have thought it right to select some of them which possessed the necessary qualities, and to introduce them into the systematic nomenclature of the present work. These names are, *Gas*, *Water*, *Acid*, *Salt*, *Baryte*, *Malachite*, *Mica*, *Spar*, *Gem*, *Ore*, *Metal*, *Pyrites*, *Glance*, *Blende*, *Sulphur*, *Resin*, and *Coal.* These names are sufficient for all the orders hitherto known, except two; and it seems that a spirit of innovation cannot be with propriety reproached to a new nomenclature, if it does not contain more than two new names, particularly if we recollect that it refers to a new system, or rather to a system in which a new application of principles has been made, which, though not new in themselves, have yet been neglected in Mineralogy. These new names are *Haloide* and *Kerate.* They would not have been made use of, had any others of a similar signification occurred in the older mineralogy.

§. 236. SIGNIFICATION OF THE NAMES OF THE ORDERS.

The simple names receive their signification in agreement with the ideas of the orders. They must always be used in these, never in any other significations.

It may be useful in this place to add a few remarks on the meaning attached to several of these names as applied to the orders. The name of *Ore* has been hitherto applied to a great many minerals, and it is one of those, whose signification has been particularly vague and uncertain. Even its technical signification had disappeared; for though in this respect we understand very well what is meant by Iron-ore, Copper-ore, Manganese-ore, and Chrome-ore; yet it will be very difficult to say what we should understand by Liver-ore, Tinder-ore, Horn-ore, Tile ore, Oliven-ore, Pea-ore, Pitch-ore, &c. Names of that kind have indeed no signification at all, as long as there does not exist an order *Ore*, to the genera of which they may be referred.

If, on the contrary, we attempt to comprise in one single expression all the minerals which have hitherto been designated by the name of Ore, and add such as are naturally allied to them in their properties and resemblance, in order to produce the natural-historical idea of the name of Ore; the idea conveyed by this term would necessarily become so complex and varied as to be utterly devoid of clearness and consistency with itself. It would moreover include so many species, that the greater part of the minerals would become ores, and thus the whole Mineral Kingdom would not admit of another idea of that kind.

If therefore the name of Ore shall not lose both signification and applicability in the Natural History of the Mineral Kingdom; it must be confined to a few only of those minerals which hitherto have been called ores. Among these are Red Copper-ore (octahedral Copper-ore), Tin-ore (pyramidal Tin-ore), the Iron-ores (rhombohedral, octahedral and prismatic Iron-ore), Grey Manganese-ore (prismatoidal Manganese-ore), &c. The same name of Ore must likewise be applied to other species, which are connected with the preceding ones by equal degrees of natural-historical similarity. Such are Rutile (peritomous Titanium-ore), Anatase (pyramidal Titanium-ore), Wolfram (prismatic Scheelium-ore), Uranite (uncleavable Uranium-ore), &c. All these species form together one natural order, which bears the name. The name acquires, therefore, its signification and application, from the nature of the genera and species which the order contains; and is transferred upon them in so far as they belong to that order. Hence every thing must also bear the name of Ore, which belongs to the order Ore, or, in other words, it will be said *to be an Ore;* and a mineral is called an Ore, only because it belongs to this order. Thus the idea of the name of Ore becomes exactly determined in conformity to the idea of the order. A newly discovered mineral, which, on account of its natural-historical properties, belongs to the order Ore, assumes this name, or comes as it were with this name to the notice of the world, and is thus secured from a burden which arbitrariness very soon would load upon it, as long as there does not exist a systematical nomenclature.

The signification of the name *Pyrites* has been better maintained in its purity. Most of the minerals hitherto called Pyrites, really belong to that natural order, upon which the name of Pyrites has been fixed. Such are Copper-pyrites (pyramidal Copper-pyrites), Iron-pyrites (hexahedral and prismatic Iron-pyrites), Arsenical pyrites (axotomous and prismatic Arsenical-pyrites), &c. Other species, that nevertheless belong to the same order, have hitherto not been called pyrites; as Cobalt-glance (hexahedral Cobalt-pyrites), white Cobalt-ore (octahedral Cobalt-pyrites), Kupfernickel (prismatic Nickel-pyrites). Cobalt and Nickel are metals, and minerals which bear these names must belong to the order Metal. But if Iron-pyrites be a Pyrites, that is to say, if it belong to a certain natural order, all the rest likewise must belong to that order, and consequently assume its name. They are therefore compelled as it were, by their natural-historical properties, to part with their former names.

No other name has been subject to more ambiguities, and to abuses, than the name of *Spar*. The word Spar is generally said to signify a certain structure which has been called the *spathose* one. We should never derive a name—a word, which designates a very general idea,—from this property; for no property is less constant in a determined signification, as its constancy does not extend beyond the species, if we except the tessular system. In a more comprehensive or indeterminate signification, again, it is so general, that it occurs not only in all the systems of crystallisation, but also in many different orders.

The name of *Felspar* or *Feld-spar* is almost generally received in many languages, and may be useful on that account in deriving the name of an order. Hence only those minerals are said to be *Spars*, *which belong with Feld-spar to one and the same natural order*, and not those species only which shew the so called spathose structure.

Mica signifies a mineral, which may be cleaved with facility into thin shining laminæ. Common Mica (rhombohedral Talc-mica), Uran-mica (pyramidal Euchlore-mica), Copper-mica (rhombohedral Euchlore-mica), are of this kind. Although this relation of structure alone is not sufficient for determining a natural order; yet the order Mica contains only such species as present it in a high degree of perfection, and it bears therefore deservedly this name.

A mineral which may with propriety bear the name of a *Metal*, must really be a metal, or it must present the properties peculiar to metals. Hence this name must not be given to species belonging to the orders Pyrites, Glance, Blende or Ore, because they do not possess the properties of metals, although metals actually may be extracted from them. The classification of metallic substances in the greater part of the systems hitherto published, regard much more the *reguli* obtained by chemical aid, than the productions of nature; and thus they betray that they are not meant to be systems of Natural History, but something which indeed cannot always be defined or reduced to clear ideas.

The rest of the names need no further remarks. Some of them belong entirely to the English idiom; others have long ago been introduced by mineralogical authors. The use of these names is similar to that of the words explained before; the ideas which they express being dependent upon the contents of the orders to which they have been applied.

The only two names newly introduced are *Haloide* and *Kerate*. The first of these has been used by several chemists for certain compositions of Muriatic acid with other bodies. The name, however, does not indicate this acid to be contained in the mixture. It means only a substance *resembling salt*, like octahedral Fluor-haloide, prismatoidal Gypsum-haloide, rhombohedral Lime-haloide, &c.

The above mentioned examples shew that there exist mineral productions in nature, so very similar to the salts, that is to say, to *the minerals contained in the fourth order of the first class*, in respect to their natural-historical properties, that without a nearer examination, they might be easily mistaken for one another; and which, therefore, indeed possess that *resemblance to salt* in a remarkably high degree. These substances form the first order of the second class, and have received the name of Haloides.

Kerate is a translation of Horn-ore (to which Quicksilver Horn-ore likewise belongs), suppressing only the word ore, a name to which these substances are not entitled. In Mineralogy, we find very often comparisons with horn, sometimes not in a proper place, as in hornstone, hornblende, hornslate, &c. It may be tolerated in the Kerates, though its application is not the consequence of its particular merits.

§. 237. NAME OF THE GENUS.

In the genus, the name of the order is more restricted by connecting another word with the name of the order; and thus the product is a compound name, which is *the Name of the Genus, or the Generic Name.*

The generic name should refer to the natural-historical properties of the genus; and it should therefore express, as much as possible, some striking feature of its resemblance with other bodies. Such is the name *Garnet-blende.* The genus, designated by this name, belongs to the order Blende; the individuals which it contains very often look like garnet. This garnet-like appearance, however, is not a similarity of that kind, upon which the classification of the genus might depend; besides, this classification must precede both nomenclature (§. 240.) and characteristic (§. 247.). There can be no doubt, that for the nomenclature of a synthetical system, no better mode of constructing the generic names could be invented. *Nomina generica, quæ characterem essentialem vel habitum (plantæ) exhibent, optima sunt. Habitus indicat similitudinem, qua excitatur idea, et ex idea nomen.* LINN. Phil. Bot. 240. This mode of nomenclature, however, could not be applied to every genus, without introducing a great many new words, which is attended with all kinds of disadvantages, and must therefore be carefully avoided. On this account, two different methods have been resorted to. In many instances common trivial names have been employed in forming generic names; in others these names have been derived from a property which is not a natural-historical one. According to the first process were obtained the names of Feld-spar, Augite-spar, Triphane-spar, Disthene-spar, Azure-spar; according to the second process, Iron-pyrites, Cobalt-pyrites, Lead-glance, Molybdena-glance, Scheelium-baryte, Lead-baryte, and others. The latter of these names really have a chemical *sound:* but they have in the present place no chemical meaning; and it is only the meaning which must here be taken into consideration. The genera Iron-pyrites, Nickel-pyrites, Cobalt-pyrites, the genera Uranium-ore, Chrome-ore, Manganese-ore, &c. are not assembled because they contain the metals of which they bear the names; but because the first are Pyrites, the second Ores. Many minerals indeed contain the said metals, and yet are not joined to these species within the same orders. Thus the above mentioned names not only become innocuous, but they are even useful, in as much as they convey the idea, though not a natural-historical one, of the quality of the contents to which these bodies owe their application in the arts. Yet they would not have been employed, but for the above mentioned rule in the construction of a new nomenclature, not to introduce too many new words.

In the names of two of the orders, Gem and Metal, an exception seems to have been made from the general process of applying compound names to the genera, which at the same time should express the orders. The name of the order is, however, only suppressed in these, because to a certain extent it is understood from itself. Every body knows Gold, Silver, Bismuth, Tellurium, &c. to be metals, and the names Gold-metal, Silver-metal, &c. would certainly not meet with general approbation. It is the same with Diamond, Topaz, Zircon, &c. Nobody would approve of the names Diamond-gem, Topaz-gem, Zircon-gem, &c. The name of a metal, and the name of a gem, therefore, signify by themselves the order to which the one and the other belongs. The only genus of the order Sulphur has as yet no generic name.

§. 238. DENOMINATION OF THE SPECIES.

The nearer restriction of the generic name to the species is effected by an adjective. Thus *the Denomination of the Species* is produced.

The adjective, with which the species is designated within its genus, must be taken from its natural-historical properties, and if possible so selected, that it refers to one of those properties of the species which are most useful in distinguishing it from other species of the same genus. To this end, the most desirable are the systems of crystallisation and the relations of cleavage. Examples are, hexahedral, prismatic, rhombohedral Iron-pyrites ; rhombohedral, octahedral, dodecahedral, prismatic Corundum ; rhombohedral, octahedral, prismatic Iron-ore, and many others. If two or three species of a genus belong to one system of crystallisation, one of them retains the adjective expressing the system ; the others receive a denomination taken from a general property of crystallisation, or of cleavage, as in peritomous and pyramidal Titanium-ore, in prismatic, prismatoidal, hemi-prismatic, and paratomous Augite-spar, in rhombohedral and peritomous Ruby-blende, in rhombohedral, macrotypous, brachytypous and paratomous Lime-haloide, and several others. In the same manner have also been employed the adjectives axotomous, diatomous, diprismatic, and prismatoidal, agreeably to the explanations given above. The adjectives *uncleavable* and *native* should be retained, only till the cleavage or the form of those species to which they are now applied, will be ascertained. Only a few, for want of better, have been taken from colours, none from localities or persons, nor has, in any instance, the adjective *common* been employed, which is indeed the worst of all. The natural quality of the species contained in the three first orders of the first class, and in the second order of the third class, has produced the necessity of deriving adjectives from other relations, which would else not have been applied.

§. 239. REPRESENTATION OF THE SPECIES THROUGH ITS DENOMINATION.

The systematic denomination produces a representation of the species. This nevertheless cannot be substituted in the place of the character in the Characteristic, nor of the general description in the Physiography.

It is not a small advantage of that systematic nomenclature, which refers to a natural or synthetical system, to create an image of the object to which it refers, by means of its denominations, which never can be erroneous, though it may be insufficient. The trivial nomenclature can never render this service, how perfect soever it may be in respect to its peculiar properties. If we hear the name *Rutile*, and do not know the species itself, to which it belongs, we never can imagine any thing like a representation of the object, though for the rest, our knowledge of Mineralogy may be very extensive. If, on the contrary, we hear *peritomous Titanium-ore*, and have only an idea of the order Ore, this at once will produce a general image of the species, which will be still more restricted, if we have some idea of the genus *Titanium-ore*. But *we may indeed suppose that every person possesses an image of the orders and genera in his mind, who is in some degree acquainted with the Natural History of the Mineral Kingdom.* If, besides, we attend to the cleavage, which is peritomous, this will suffice, by the mere denomination, for distinguishing the varieties of peritomous Titanium-ore from those of the prismatic or the pyramidal Titanium-ore. The denominations may be still more useful, if they express the cleavage more minutely, or if they refer to the system of crystallisation. By the denomination hexahedral Iron-pyrites we learn, that in the order Pyrites, the species belongs to the genus Iron-pyrites, and the adjective hexahedral signifies that its forms belong to the tessular system, and that cleavage takes place in the direction of the faces of the hexahedron. An image, thus produced of the species by the mere denomination, is indeed very useful. Yet it cannot serve as a substitute to the character in respect to the process of determining an individual, by the assistance of the Characteristic, nor can it be employed instead of the General Description, for representing the varieties of the species. For commonly it contains only one character, in most cases a very general one, and refers besides to images or representations, which can only be acquired by ocular inspection, and never can receive the precision of a character, nor that exact idea of the species, which is produced by the general description. The characters contain every thing required for a correct distinction within their sphere: the general descriptions contain every thing required to a perfect natural-historical knowledge of the species itself.

It will not remain unnoticed by those who consider an empirical knowledge of the productions of the Mineral Kingdom as a valuable acquirement, that the methodical way is the only one which leads to this end with the greatest facility; and, what is still more important, with the greatest exactness and security, while every other attempt must remain fruitless.

§. 240. SYSTEMATIC NOMENCLATURE HOW TO BE JUDGED OF.

The systematic nomenclature presupposes a system, to which it refers; and upon the due consideration of this system must necessarily depend a complete judgment of the nomenclature. The system requires a systematic nomenclature, in order to be applicable to the objects of experience.

Although the first part of this proposition immediately follows from the very idea of a systematic nomenclature (§. 230.), yet it seems necessary to add here a few remarks.

The reason why as yet there has not existed a systematic nomenclature in Mineralogy, was the want of a system, capable of serving as basis to a systematic nomenclature, a system which for that purpose would have required to contain correctly determined species, arranged according to the general principles of Natural History. Several attempts have been made to construct a Latin systematic nomenclature; they have not succeeded, because the systems to which they referred, did not possess the necessary properties. The celebrated ABBÉ HAÜY acknowledges indeed the great value of a systematic nomenclature, yet only a part of that nomenclature, which he applies himself, is systematic, the rest is trivial nomenclature. This mixed nomenclature is a consequence of a mixed system; and proves that the nomenclature cannot acquire uniformity, unless the system rest upon simple principles. Mineralogy, as hitherto treated, has amply demonstrated this observation. Zoology and Botany prove the reverse. These parts of Natural History have always proceeded according to one and the same principle, and on this account they have long ago possessed the advantage of a systematic nomenclature.

It appears evidently from the preceding observations, that the systematic nomenclature must be judged of according to the system to which it refers. If this be founded upon relations appertaining to the science, and if its different parts be consistent with each other; that is to say, if it fulfil the above mentioned demands of Natural History; the whole business of nomenclature will be to express this system by words, so that it becomes possible from the denomination of a species to infer the connexion in which it stands with others. The nomenclature, moreover, should not be contrary to the spirit of the language; its expressions should be concise and intelligible, and the denominations, if possible, should be expressive of the objects themselves. These are the chief properties of nomenclature, upon which its applicability depends. It will thus allow of all those improvements and refinements, or, in general, of all those changes, which are rendered necessary by the continual advancement of our knowledge regarding the productions of the Mineral Kingdom.

The object of the systematic nomenclature is to promote and facilitate the application of the system to nature, or to the data of observation. For while we attribute certain names or denominations to the natural productions, we arrange them at the same time under the general ideas of the system; the system, however, is constructed for the purpose of collecting the variety of nature within its general ideas, and thus reducing it to a unity, in order to enable us to survey and to understand it, and to acquire some more exact knowledge of it, than that which consists in a mere work of memory. The system would lose its application if there was no nomenclature, and both, System and Nomenclature, appear therefore equally important in respect to the idea of Natural History, and they are connected with each other by means of the Characteristic.

§. 241. TRIVIAL NOMENCLATURE.

In the trivial nomenclature the name is fixed upon the species.

The trivial nomenclature does not express the connexion among those bodies, which it provides with names. Any name, not destined to express this connexion, is termed a *Trivial Name*, which rests accordingly upon the lowest idea of the system, that is to say, upon the species. The meaning attached here to a trivial name is somewhat different from the trivial name as defined by Linnæus. The latter consists of a mere adjective, substituted for the character of the species: it is not properly a name (§. 232.), and can therefore never be used by itself, but only in connexion with a name, as a denomination.

It is a custom generally received, and in no way objectionable, to provide the natural productions, and, above all, those which are used in the arts of life, with particular names, which, on account of their conciseness and simplicity, are more convenient for use than the long and compound systematic denominations. Moreover, it is supposed here, that we are already acquainted with the object thus named, or the name at least is not meant to express some farther information, and so they are destined as it were, for a less strict or scientific employment. But it would be blameable indeed, if, for the trivial nomenclature, we should neglect the systematic one, and thus betray an indifference towards the science itself, which could not but produce evil consequences.

Together with the knowledge of the bodies themselves, the trivial nomenclature likewise supposes that of the connexion in which they are with others. Hence it supposes the systematical nomenclature; and it is evident, therefore, that although it may exist beside it, yet it never can, instead of it, fulfil the demands of Natural History.

The properties requisite in trivial names, may be very easily inferred from the preceding observations. Their chief recommendation consists in their simplicity; they must be *simple* names (§. 232.). For a compound name expresses a connexion or a relation with other objects, with which trivial nomenclature has nothing to do, and refers to a system, which does not exist, at least not in respect to the trivial nomenclature. The names, *Spinel*, for dodecahedral Corundum; *Euclas*, for prismatic Emerald; *Rutile*, for peritomous Titanium-ore, are excellent trivial names. The name *Hornblende*, if applied to a species, supposes a *genus* Blende, which does not exist in any of the systems in which that name has been used; if it be supposed to refer to one or to several varieties, it will suppose the existence of a *species* Blende; to which, however, in these systems, hornblende does not belong. Examples of this kind, of which a great many more might be quoted, are calculated to shew, that there exist rules even in respect to the trivial nomenclature, which it is indispensable to observe, if we intend not to confound those ideas, which it is the purpose of the system to explain, and the purpose of nomenclature to preserve in their purity. Compound trivial names, moreover, without either the necessity or the advantage, produce all the difficulties of a systematic nomenclature. And yet, the only motive of introducing a trivial nomenclature is to avoid these difficulties; hence it appears that compound trivial names are entirely to be rejected.

It requires but a limited knowledge of the object to understand, that the difficulties, connected with the construction of a good trivial nomenclature, possessing the required properties, by far surpass those which attend the construction of a systematic nomenclature. Many names contained in several of the systems of Mineralogy hitherto published, might be of use in the construction of a good trivial nomenclature; they should be collected and properly completed. The difficulty of introducing a nomenclature of this kind, even though it were acknowledged as useful, will be probably found greater, than if it had been systematic. On account of the great difficulties in establishing certain and general rules in the selection of trivial names, there will always remain some arbitrariness, which is unavoidable, and will form the principal impediment of a general agreement in this respect.

The natural-historical determination of natural productions, does not go beyond the species (§. 222.). The systematic nomenclature, therefore, must stop at the denomination, the trivial nomenclature at the name of the species. The disadvantage arising to the systematic nomenclature, from a want of attention to this rule, consists in the circumstance, that the denominations become composed of too many words, because they require at least two adjectives. If the trivial nomenclature applies names to particular varieties, as Amethyst, Prase, Adularia, Amiantus, Anhydrite, &c. the idea of the species becomes too much dismembered. If it produces denominations by adding adjectives to the names of the species, it assumes the appearance of a systematic nomenclature, neither of which properties would serve to its recommendation.

PART IV.

CHARACTERISTIC.

§. 242. DEFINITION.

The *Characteristic* is the assemblage of certain natural-historical properties, arranged according to a certain system, for the purpose of *distinguishing* the unities contained in this system.

Any single natural-historical property, or a collection of several of them, if it be subservient to the distinction of several species of a genus, or of several genera of an order, or of several orders of a class, &c. is termed *a Character*, and the single properties it contains are its *Characteristic Terms* or *Marks*. If a character contains only one characteristic mark, this mark itself represents the character.

Without a system, there cannot exist a character; and therefore likewise no Characteristic, because the distinction of several bodies, by means of characters, takes place only within the unities of the system. Thus it becomes possible that a character may contain, or be limited to a single one, or a small number of characteristic marks. A character calculated for distinguishing one single species from all other species, to whatever genera or orders they might belong; that is to say, a general character, would require the enumeration of *all* the natural-historical properties of the species, as its characteristic marks. But this enumeration of all the properties of a natural production, is its *Description* (§. 27.), which is beyond the limits of the Characteristic, and enters those of Physiography.

The term *natural* character is applied by LINNÆUS to the description. Among its properties, he mentions that it contains all the characteristic marks, of the genus in Botany, to which every thing LINNÆUS says of characters, more particularly refers; *Naturalis character notas omnes genericas possibiles allegat; adeoque Essentialem et Factitium includit.* LINN. Phil. Bot. 189. He likewise says, that it is invariable and not dependent upon the system; and that it may serve for every system; *inservit omni systemati; Basin sternit novis systematibus, immutatus persistit, licet infinita genera nova detegerentur.* LINN. Phil. Bot. ibid. These properties do not belong to a Character properly so called; but they are essential to the General Description. The preceding observations contain the reasons why I have thought necessary to abandon the denominations used by LINNÆUS.

§. 243. NATURAL AND ARTIFICIAL CHARACTERS.

If the system to which the character refers is the natural or synthetical system, also the character is said to be a *natural* one; if the system is an artificial or analytical one, it likewise contains *artificial* characters.

This is the correct idea of a natural and of an artificial character, which gives no occasion to ambiguities. *Essentialis character unica idea distinguit Genus a congeneribus suis sub eodem ordine naturali.* LINN. Phil. Bot. 187. *Factitius character Genus ab aliis, ejusdem tantum ordinis artificialis, distinguit.* LINN. Phil. Bot. 188. This natural character, therefore, must not be considered as something produced by nature, for nature does not institute comparisons between its productions, from which the natural character might be derived. It seems not to be exactly in harmony with the idea of this character, to call it an *essential* one, since it depends upon the properties of the objects (orders, genera, species) compared, and is a result of their comparison; so that by the discovery of new genera in an order, or of new species in a genus, it may be subject to alterations, which will never cease, and therefore always hinder the character from being infallible, till every thing is known which nature has produced within that kingdom. *Nullus character infallibilis est, antequam secundum omnes suas species directus est.* LINN. Phil. Bot. 193. It would be possible, on the contrary, to call the artificial character an essential one, at least in respect to a *certain* system, since, as it will appear afterwards, it is the foundation of the divisions in that artificial system. The division is effected according to general properties, and every individual necessarily belongs to one or to the other of these divisions, in as far as it contains the characteristic property. Thus the artificial character is not dependent upon the enlargement of our knowledge by experience. Yet even here it is better to avoid this expression, because commonly it gives rise to accessory considerations, which may lead to erroneous ideas.

The denomination of the characters corresponds with their object, so that we have *Characters of the Orders*, *Generic Characters*, &c.

§. 244. PROPERTIES OF THE CHARACTERS.

The characters must be sufficient to a precise distinction within their respective sphere, and as short as the necessary degree of evidence in the determination of the species will allow.

The first requisite is an immediate consequence of the very idea of a character. Characters are entirely useless, if they serve for the distinction only of some of the species contained within their genus, or of some of the genera contained within their order. If a single characteristic mark suffice for a general distinction, this mark will represent the character itself: if not, several of them must be applied in connexion, and thus form the character. A compound character of this kind can only belong to a natural system, because only in this it is possible, that one single characteristic term should not be found sufficient for a general distinction. This natural character it is, which requires the greatest possible *Conciseness* and *Uniformity*. *Character essentialis, quo brevior, eo etiam præstantior est.* LINN. Phil. Bot. 187. The shorter the character is, the more facility and certainty it will afford to the distinction; and this facility and certainty it is, which we demand in a character. The more uniformly we arrange the characters of the same kind, the less likely we are to omit any characteristic marks. Hence the characters should not contain any thing, but what is unavoidably required for the distinction and the evidence in the determination of the species; and every superfluous word, every word of an ambiguous signification, is reprehensible; so is every restriction in regard to time, or other relations, and, above all, every verbal exception, quite contrary to the idea of a character. Even now, at least in respect to the Natural History of the Mineral Kingdom, it is not superfluous to add: *Oratorio stylo in charactere, nil magis abominabile.* LINN. Phil. Bot. 199.

The higher the degree of classification, within the sphere of which the distinction is to take place, the more it is necessary to bestow all possible attention upon the above mentioned properties of the characters. For if the first distinction be not evident and correct, the subsequent ones will be still less worthy our consideration. It is obvious that the characters will acquire the properties of conciseness and uniformity to a comparatively higher degree, the more the system corresponds to nature, in proportion likewise to the correctness and consistency within itself, with which it expresses the relations of the natural-historical resemblance. The characters of the classes and of the genera possess these properties to a considerable extent. Only the characters of some of the orders in the second class are longer, they contain more characteristic marks, than it is desirable they should contain. Though in this degree of classification the variety in the connexion among the

"Solid : taste;" where solidity is the condition under which the property of exciting some taste *must* necessarily take place. The characters, and every single mark which they contain, must be taken literally; and they admit of no explanation or other accessory significations, but that which is really expressed by the words. In the instance just mentioned, it would be erroneous to infer, that if a mineral which shall belong to the first class is not solid, it must be insipid. The character does not express this; and it is therefore quite indifferent whether, if not solid, the mineral has any taste or not. Sometimes the conditioning part, sometimes the conditioned part of the characteristic mark, at other times both of them, are compound. Yet the employment of the conditioned characteristic mark is not different from that of the absolute ones, as explained above.

§. 246. ARRANGEMENT OF THE CHARACTERS OF THE SPECIES.

The arrangement of the characters of the species must be such, that, by their assistance, the determination of the individuals receives the greatest evidence which the science can possibly produce.

The only thing we may reasonably demand from the characters of the classes, the orders and genera, is, that they should exclude every individual which does not belong to them, and that they should not exclude those which these unities comprehend. It is quite indifferent by what properties, and in what manner this is effected, provided the properties be sufficient for a general distinction within their sphere, and the method agreeable to the principles of Natural History. The character of the species requires something more. For here the object of our inquiry is not only to know that a given individual is not excluded from a certain species, but we wish to find out, and to convince ourselves, that it really does belong to that species. For this reason the character of the species should contain marks, which, if not always, at least very often may be superfluous in respect to the mere process of distinction. The genus Emerald contains two species, prismatic and rhombohedral Emerald. For the sake of mere distinction, the character of each of these species need not to contain any thing besides the system of crystallisation, or the limits of specific gravity, for these likewise would suffice for distinguishing the two species. A third species, however, might exist, besides the two species above mentioned; a species which, on account of its natural-historical properties, did belong to the genus Emerald, and that species might agree with one or the other of these in the above mentioned characteristic marks. In order to assure ourselves that an individual belonging to the genus Emerald enters either within the species of the prismatic or of the rhombohedral Emerald, their characters are made to contain a greater number of marks, whose properties leave no doubt, upon the supposition of the individual being an Emerald, whether and to which of the two species the individual belongs. This arrangement, moreover, produces an uniformity in the specific characters, which, according to what has been stated above, is one of their principal qualities.

The specific characters, therefore, consist chiefly of three marks of this kind, which, wherever the quality of the species would allow, have been given in all instances. These are the crystalline forms (including cleavage), the degrees of hardness, and the specific gravity. The first characteristic mark in the specific character is the system of crystallisation. Then follows, together with its angles (if these be known), the fundamental form, from which all the other simple and compound forms of the species may be derived. Of rhombohedrons the terminal edge is given; for instance, in rhombohedral Lime-haloide, $R = 105° 5'$; of an isosceles four-sided pyramid, first the terminal edge, then the lateral edge; as in pyramidal Zircon, $P = 123° 19', 84° 20'$; of a scalene four-sided pyramid the obtuse terminal edge, the acute terminal edge, and the lateral edge in succession; thus, in prismatic Topaz, P = 141° 7′, 101° 52′, 90° 55′, &c. This is observed in every species, where the fundamental form commonly occurs in nature, and therefore may be often observed; or where the combinations do not present any peculiar character, as the hemi- and tetarto-prismatic, or the di-rhombohedral ones, &c. If the fundamental form is seldom observable, its angles are not indicated; in their stead, however, the characters contain the angles of such derived forms, as commonly are to be met with in nature, as in prismatic Hal-baryte $\breve{P}r = 105° 6'$; $(\breve{P}r + \infty)^3 = 77° 27'$. In every instance regarding horizontal prisms, that angle is given which is contiguous to the axis of the fundamental form, in vertical prisms the angle corresponding to the obtuse terminal edge of the fundamental form. If the combinations of a species possess a particular character, the Characteristic indicates only those dimensions of the fundamental form, which correspond to that character, and therefore are immediately observable, as in paratomous Augite-spar, $\frac{\breve{P}}{2} = 120° 0'$; in a like manner in rhombohedral Fluor-haloide, 2 (R) = 131° 14′, 111° 20′, &c.

In those forms which assume a hemi-prismatic character, it is very often the case, that the axis of the fundamental form is not perpendicular upon the base (§. 98.). The inclination of the axis takes place in a plane through the axis, and one of the diagonals of the base. This plane bisects the angle produced by those faces of the simple forms, which appear in the combination, and which therefore is indicated in the designation of the form. Together with the magnitude of this terminal edge, the angle of inclination itself is likewise contained in the character. Thus in prismatic Azure-malachite " $\frac{\bar{P}}{2} = 117° 37'$, *Inclination* (of the axis), = 2° 21′" signifies, that those faces of the fundamental form, which necessarily appear together in the combinations, are contiguous to the short diagonal, and meet under angles of 117° 37′; and that the axis of P includes an angle of 2° 21′ in the plane of the short diagonal with the perpendicular line erected in the centre of the base. "*Inclination* = 0" means, either that there exists no inclination at all, or that this inclination has not been as yet attended to in the present determination of the crystalline forms.

The angle indicated in horizontal prisms that assume a hemi-prismatic character, is the one contained between the occurring face and the axis, together with the situation of the face by means of the signs + and —, agreeably to §. 153. For instance, in paratomous Augite-spar, $\frac{\breve{P}r}{2} =$ 73° 54′, where the sign + is understood. For designating vertical prisms in hemi-prismatic combinations, the lateral angle is given, which corresponds to that terminal edge of the fundamental form, which occurs in the combinations.

With respect to cleavage, the expression "*Cleavage*, R," for instance, in rhombohedral Lime-haloide, means, that this mineral has its cleavage parallel to the faces of a rhombohedron, similar to the fundamental form of the species; "*Cleavage*, $P - \infty$. $P + \infty$. $[P + \infty]$" in pyramidal Garnet, means, that this mineral has its cleavage parallel to the faces of two rectangular prisms, and at the same time perpendicular to their axis; "*Cleavage*, $\bar{P}r + \infty$" in prismatic Chrysolite, indicates, that the cleavage of this mineral is parallel to a plane passing at the same time through the axis and the short diagonal of the prism $P + \infty$; "*Cleavage*, $(\breve{P}r + \infty)^3 = 87° 42'$. $\breve{P}r + \infty$. $\bar{P}r + \infty$" in paratomous Augite-spar, expresses that the individuals of this species may be cleaved, first, parallel to the faces of an oblique-angular four-sided prism, of the given angles; and secondly, parallel to planes, which pass through the axis and both diagonals of the prism $P + \infty$; or, what comes to the same, parallel to the faces of a rectangular prism.

Cleavage is sometimes found to be *hemi-prismatic* or *tetarto-prismatic*. Paratomous Augite-spar shews an example of the first. The hemi-prismatic faces of P, expressed above by $\frac{\breve{P}}{2} = 120° \ 0'$, under which angle they meet, appear in several varieties as faces of cleavage. This relation, and all others of the same kind, are expressed by signs analogous to those used in the forms of crystallisation. In general, every face of cleavage is expressed by the sign of the face of crystallisation to which it corresponds. The degree of perfection of the faces of cleavage has likewise been indicated, and needs no farther explanation. The student, however, who intends to employ the Characteristic for the determination of occurring individuals, has particularly to attend to those faces of cleavage, which are most apparent.

The degrees of hardness, which are in general expressed by H., and those of specific gravity, expressed by G., are given in the Characteristic with their limits, or those points between which the hardness and the specific gravity of the varieties are found to be contained; observation will very seldom yield these limits themselves; and only in such cases, where it cannot have any prejudicial influence upon the determinative process itself. Evidently this must apply in a still higher degree to the characters of the orders and the genera. Those who make use of the Characteristic, must *not* therefore *compare one character with another*, but they have to compare occurring individuals of the species contained in it with these characters.

Besides these three characteristic terms, the characters of some species contain also the indication of several occurrences of colour, more particularly the streak; also the lustre, or the general aspect; sometimes also the state of aggregation, taste, &c. In most cases they would have been superfluous, had a more accurate knowledge of the forms existed. In this particular, we have reason to expect a great deal from future and accurate observations, which will enable us to keep the characters free from all such marks, as do not allow of a perfectly strict definition.

The specific characters of *fluid minerals* require another process, because in these bodies two of the most valuable marks in the characters of solid minerals, form and hardness, are wanting. They have not as yet been brought to any degree of perfection; and indeed there is little to be done in this respect, our knowledge of their natural-historical properties being still very defective.

The specific characters should not contain either conditioned or exclusive characteristic marks, though this may be, and indeed is, the case with the characters of the orders and the genera. In the latter characters there occur sometimes terms mutually excluding each other, as in the genus Corundum, "*Tessular*, *rhombohedral*, *prismatic*," or in the genus Iron-ore, "*Streak red*, *brown*, *black*." This evidently means, that the forms of an individual belonging to the genus Corundum, must be either tessular, or rhombohedral, or prismatic; and that an individual of the genus Iron-ore must yield a streak either red, or brown, or black; because in one and the same individual, two different kinds of the same characteristic property are impossible. In another place, an example will be given to shew, that characters thus arranged do really convey all possible security; which example will at the same time serve to illustrate the use of the Characteristic in this respect.

§. 247. NO CHARACTERISTIC BEFORE THE SYSTEM.

The Characteristic presupposes in its full extent the system, to which it refers.

In the natural or synthetic system, or that whose foundation is the natural-historical similarity, those objects are placed nearest, which are connected by the highest degrees of resemblance, or which are most similar to each other. In arranging them, no attention is paid to single properties, and perhaps least of all to such as might be useful in the distinctive characters. Indeed, the conformity of the different parts of the system would be very soon lost, should we allow such accessory views to be introduced. First of all, the system, as far as experience allows, must have been completed; *then* only it becomes possible to *compare* the different homologous unities which it contains, with each other, classes with classes, orders with orders, genera with genera, species with species, in order to discover the characteristic marks in which they differ, and from which their characters must be formed.

Hence our mode of considering the Characteristic in the natural system will only then be correct, if we keep in mind that the order, the genus, &c. is not produced and determined by the character, but that the character depends upon the order, the genus, &c. *Scias characterem non constituere Genus, sed Genus characterem. Characterem non esse ut Genus fiat, sed ut Genus noscatur.* LINN. Phil. Bot. 169. We must not, therefore, look for the reasons for which the unities of the system have been adopted and determined, in these characters, from which they never can be deduced, because they consist solely in the relations of natural-historical similarity, by which the objects either approach to, or recede from each other, a matter brought to full evidence in the preceding paragraphs. The only object of the Characteristic is to collect with facility the individuals occurring in nature, under the ideas of the system. This is effected without regard to any thing, except the distinctive characters. The idea of the species, or of any higher unity, does not come into consideration, since in general the Characteristic has nothing to do with the developement or establishment of general ideas, which belongs to the Theory of the System. Here we do not ask, which properties are peculiar to the bodies, but only what are the properties in which they differ. The characters of a species, or of any other of the systematic unities, must not be considered as defective or erroneous, if they should not contain so many characteristic marks as are necessary for exciting the idea of the species; for this is not its object, and belongs to the General Description. Every character is perfect, which affords a general distinction within its sphere, and thus attains its object. It would be an error to collect in it superfluous marks, which are of no use in the process of distinguishing individuals.

Thus it appears that the Characteristic presupposes the existence of the system to its full extent, its only object being to distinguish minerals occurring in nature, while that of the Theory of the System is to produce the systematic ideas agreeably to the principle of natural-historical resemblance with consistency. Both of them must keep strictly in their peculiar course, and will then become the more useful as departments of the science; by these properties, the Characteristic will become the link between the systematic ideas and the systematic names and denominations, while both the Characteristic and Theory of the System will produce the connexion between the natural-historical properties and the same systematic names and denominations.

§. 248. BASE OF A PERFECT CHARACTERISTIC.

The perfection of the Characteristic depends upon the perfection and accuracy of our natural-historical knowledge of natural productions.

The truth of this proposition, in regard to the Characteristic of both natural and artificial systems, is so very evident, that it would be superfluous to add any explanatory remarks. But as a consequence of this truth, we may mention, that the most useful, or rather the only means of bringing the Characteristic nearer perfection, consists in the continued study, and in the accurate investigation of nature. The more we inquire into the nature of bodies, and the more our knowledge becomes accurate and extensive, the more the ideas of the system will advance towards purity and correctness, and afford in proportion a higher degree of facility and certainty to the process of distinguishing them from one another by means of the Characteristic. The want of an accurate knowledge is still perceptible in many of the indications of forms, which in many species are either not determined at all, or at least not to a sufficient degree of accuracy. From such imperfect information, the greater part of the difficulties derive, which we have to encounter in the construction and application of the Characteristic. This has no doubt been one of the reasons which has deterred naturalists from following that path in Mineralogy which has been found the right one in Botany and Zoology, and they have considered accordingly as impracticable, every attempt towards the construction of a Characteristic. It is too soon as yet to expect it to be perfect; yet it is not too soon to make the first step, and the science itself requires that it should be done, in order to obtain in its regular scientific form the third, and not least important part of Natural History, that of the Mineral Kingdom. The imperfections of the Characteristic appear more strikingly in the first class than in the second, or even in the third; but in that class we know so very little of the natural-historical properties of the bodies which it contains, that it has been introduced, almost entirely for the sake of *exhibiting* with some degree of completeness, what the Characteristic should contain. Thus also, the systematic nomenclature, which is always in proportion to our knowledge of the objects themselves, is here more imperfect than in any other part of the system. It is to be expected, however, from the progress already made, that these difficulties will entirely disappear; and this will take place the sooner, the more we convince ourselves that in order to remove those which are still remaining, we are not compelled to recur to foreign assistance, a process by which the purity of the principles of Natural History would be entirely sacrificed; and yet this purity has been the only source from which flows even that little, which has as yet been effected.

§. 249. USE OF THE CHARACTERISTIC.

The use of the Characteristic is the same in Mineralogy as in Zoology and Botany.

It will be useful to give a short explanation of the process used in the determination of minerals.

If a mineral is to be determined, first its *Form*, if this be regular, must be ascertained, at least as far as to know the system to which it belongs. Then *Hardness* and *Specific Gravity* must be tried with proper accuracy, and expressed in numbers. It is sufficient, however, to know the latter to one or two decimals. The specific character *requires* these data; they are also of use in the characters of the classes, orders, and genera. After this examination, the Characteristic may be applied, and it will at the same time point out what other characters are still wanting; so that a mere inspection of the mineral, or a very easy experiment, as, for instance, to try the streak upon a file, or still better, upon a plate of porcelain biscuit, will very often be sufficient. The given individual is now carried through the subordinate characters of the classes, orders, genera, and species, one after the other, comparing its properties with the characteristic marks contained in the characters of these systematic unities. From their agreement with some, and their difference from other characters, we infer, that the individual belongs to one of the classes, to one of the orders, to one of the genera, and to one of the species. Having advanced in this manner to the character of the species, it will in some instances be necessary, and in all cases advisable, for the sake of certainty (§. 246.), to have recourse to the dimensions of the forms. This is particularly necessary, if the genus, to which the mineral belongs, contain several species having forms of the same system, as is the case in the genus Augite-spar. The common goniometer in most cases will suffice for determining the dimensions of the forms, the differences in the angles being in general so great, that they cannot easily be missed, even by the application of this instrument. If the differences be small, and their distinction require on that account a higher degree of accuracy, it will be necessary to recur to the reflective goniometer.

It will seldom be necessary to read over the whole of any character of a class, order, genus, or species, excepting those which comprise the individual; one term that does not agree sufficing for its exclusion. Thus even the characters of the orders, though the longest, will not be found troublesome.

The application of the Characteristic has been facilitated in a great measure by separating the absolute characteristic marks from the conditioned ones. It becomes still more easy and expedititious, by taking particular notice of some characters, which might be termed *prominent.* Such are a metallic appearance; a high degree of specific gravity, particularly if the appearance be not metallic; and a high degree of hardness. The observation of these will immediately decide whether an individual can belong to any particular class, order, genus, or species. It is understood, that if it be not thereby excluded, the other characters must next be examined, till either an excluding one be found, or if not, the individual may be considered as belonging to that class, order, &c. with which it has been compared and found to agree.

§. 250. DETERMINATION OF INDIVIDUALS, BY MEANS OF THE CHARACTERISTIC. EXAMPLE.

An individual, which has been carried through the characters of the classes, orders, genera and species, and whose systematic denomination has thus been found, is said to have been *determined.* The determination is *complete*, if the individual has been traced to a species; it is *incomplete*, if it has only been brought under a certain order or genus.

It is not difficult to arrive in this way at the determination of an individual, provided those properties can be ascertained, which the complete determination requires. The determination will be defective only in consequence of the impossibility of observing at all, or at least with sufficient accuracy, one or more of the characteristic marks in the mineral.

In illustration of this, let us take the following example. Let the form of the mineral which is to be determined, be a combination of a scalene eight-sided pyramid, of an isosceles four-sided pyramid, and of a rectangular four-sided prism; the cleavage parallel to the faces of two rectangular four-sided prisms, in diagonal position to each other; form and cleavage therefore *pyramidal*, or belonging to the pyramidal system. Let *Hardness* be = 6·5; *Specific Gravity* = 6·9.

In this case, both hardness and specific gravity are prominent characters, and exclude the individual at once from the first and third, but not from the second class: with the characters of this class, its other properties also perfectly agree. Hence the individual belongs to the second class.

Comparing the properties of the individual with the characters of the orders in the second class; hardness and specific gravity will be found too great for the order Haloide; hardness too great for the orders Baryte and Kerate; both of them too great for the orders Malachite and Mica; and specific gravity too great for the orders Spar and Gem. But in the character of the order Ore, both hardness and specific gravity fall between the fixed limits, and cannot exclude the individual from this order. The other parts of this character are now to be taken into consideration. If the appearance of the individual be metallic, its colour must be black, otherwise it cannot belong to the order Ore. But the appearance is not metallic; therefore the colour of the individual is quite indifferent; that is, this conditional characteristic mark does not affect the individual, and consequently cannot decide. Since the appearance is not metallic, the individual must exhibit adamantine or imperfect metallic lustre. The first will be found, particularly in the fracture. The following characteristic marks refer to minerals of a red, yellow, brown, or black streak; and as the individual gives none of these, its streak being uncoloured, these characteristic marks do not come into consideration. The next mark requires, that if hardness be = 4·5 and less, the streak should be yellow, red or black; but hardness is = 6·5, therefore the colour of the streak indifferent. If hardness be = 6·5 and more, and streak uncoloured; then specific gravity must be = 6·5 and more. Now this condition takes place; hardness is = 6·5; streak is uncoloured. But also the conditioned character takes place, specific gravity being = 6·9, which is greater than 6·5.

In regard to the individual, which is to be determined, all the characteristic marks constituting the Character of the order Ore, may be divided into two parts. The first part contains those which refer to the individual; the second those which do not; the last evidently cannot be decisive. But with the first, all the properties of the individual concur. These properties agree consequently with the whole character of the order, as far as it is applicable to the individual, and determine it to belong to the order Ore, or, in shorter terms, to be an *Ore*.

It will be advisable to beginners, who do not yet possess a sufficient practice in the use of the Characteristic, also to compare the characters of the remaining orders, which will enable them to find out any error they might have committed in the comparison of the individual with the characters of the preceding orders. In the present case, the non-metallic appearance excludes the individual from the orders Metal, Pyrites and Glance; hardness from the order Blende; and both hardness and specific gravity from the order Sulphur. This fully confirms the above determination, and we must now return to the order Ore for comparing the properties of the individual with the generic characters which the order contains.

Considering again hardness and specific gravity as prominent, the individual will be immediately excluded from the genera Titanium-ore, Zinc-ore, and Copper-ore, but not from the genus *Tin-ore.* The form of the pyramidal system, and the uncoloured streak, shew that it belongs to this genus. If we compare the individual with the remaining generic characters, we find that it is excluded from the genus Scheelium-ore by its too great hardness, and too little specific gravity ; from the genera Tantalum-ore, Uranium-ore, Cerium-ore, Chrome-ore, Iron-ore, and Manganese-ore, by hardness and specific gravity, both of them being too great; as also by its uncoloured streak, which only agrees with that genus from which the individual differs most by its hardness and specific gravity. From all this we infer that the individual cannot belong to any other than to the fourth genus, and that we are therefore entitled to give it the name of *Tin-ore.*

This genus contains but one species. The conclusion that the individual must belong to this species, might nevertheless be erroneous. There could exist a second species of this genus. Hence we must accurately consider the dimensions of the forms. If these coincide with the angles given in the character, the highest degree of certainty, that the individual belongs to or is *pyramidal Tin-ore*, will be obtained.

The perfect determination of an individual depends, as the above example has shewn, upon the possibility of correctly ascertaining those three properties : viz. form, including cleavage; hardness; and specific gravity. If one or the other of these characteristic marks be wanting, the determination will remain incomplete. It does not, however, become prejudicial to the method, that minerals of this kind cannot thoroughly be determined by its assistance. It is exactly the same in the other parts of Natural History, in Zoology and Botany. The characteristic properties must be completely observable, otherwise a complete determination will be impossible. In Mineralogy the Characteristic affords sometimes more : it leads to a correct determination, even if the knowledge of the forms remains imperfect, or if it is entirely wanting. But such a determination wants evidence (§. 246.); and for this reason it will be a useful rule for beginners to occupy themselves at first with the determination of such individuals as present properties, which may be easily and fully investigated. The rest will come of itself, when their knowledge of the Mineral Kingdom, and particularly of the properties of minerals, increases, and when they have, by experience, acquired the skill to judge properly of form and cleavage, at least so far as is necessary for the determination of the system of crystallisation, even in those cases where form and cleavage are somewhat difficult to be observed. This exercise is particularly recommendable to every person who intends to acquire a satisfactory knowledge of minerals, with the help of the Characteristic.

§. 251. IMMEDIATE AND MEDIATE DETERMINATION. EXAMPLE.

If a mineral can be determined without the help or intervention of one or of several other minerals, the determination is said to be an *immediate* one. If, on the contrary, we must employ one or several other minerals for this purpose, we only obtain a *mediate* determination.

The immediate determination has been explained and illustrated by an example in §. 250. An example will likewise be useful in the mediate determination.

The variety of hemi-prismatic Augite-spar, which has received the name of Amiantus, occurs in such very delicate crystals, that even supposing they should be regular, their form could not be observed, even through the most powerful magnifying instruments: it is the same with cleavage. The crystals are flexible, like fibres of flax, their hardness accordingly not to be ascertained. Their surface is so large compared to their bulk, that wherever they may be placed in water, or in another liquid, they neither sink nor rise: although their specific gravity is not inconsiderable; but we have no means to ascertain it. However, there are varieties, for the rest exactly agreeing with Amiantus, in which the crystalline filaments are somewhat coarser. They are no longer flexible, but still too weak to stand the experiment of determining their hardness. Others are still thicker: we may discern traces of their regular structure; yet on account of their minuteness, we cannot apply the goniometer for taking their dimensions. They sink in water, scratch prismatoidal Gypsum-haloide, but they lose their coherence, if we try to pass them over a face of rhombohedral Lime-haloide. At last we meet with varieties, whose form and cleavage are more apparent and observable, whose specific gravity is about three times that of water, and the hardness between 5·0 and 6·0. These allow of an immediate determination, and will be placed by that process within the species of hemi-prismatic Augite-spar. The mode of reasoning applied here will be the following. The variety preceding the last is the same as that which has been determined; those immediately preceding are again the same as the one immediately preceding the last; and thus we finally arrive at the Amiantus itself. The determination of this mineral is effected by the assistance of a greater or less number of varieties, interposed between one that is immediately determined, and another which cannot be determined immediately; the method employed is therefore that of the *Mediate Determination*. The more general our knowledge of the productions of the Mineral Kingdom, the greater facility we shall experience in the mediate determination. Through this means, a great number of minerals may be determined, and reduced to their respective species, which could never have been ascertained by immediate determination. The mediate determination has indeed been hitherto very often applied, though it was not clearly reduced or brought in connexion with the immediate determination, upon which nevertheless both the correctness and certainty of the mediate determination depends. The mediate determination is peculiar to, and intimately connected with, the natural-historical method of Mineralogy; hence we may infer, that nothing can escape this, which may be determined by any other method.

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§. 252. BASE OF THE MEDIATE DETERMINATION.

The mediate determination entirely depends upon the transitions in the series of characters (§. 221.).

The mediate determination is effected by a series or concatenation of varieties, whose terminal member on one side is immediately determinable. This series of varieties is produced by the gradation in the differences of their properties, which likewise represent members of connected series, as it has been amply demonstrated above. But in these series we observe the transitions; and thus they appear as the base upon which the mediate determination is founded.

The transitions must always be employed with the necessary precautions, as mentioned in §. 221. But upon this supposition, the mediate determination is effected with a security by no means inferior to that of the immediate determination, with which it is in the closest connexion. Mineralogy is not the only part of Natural History which makes use of the mediate determination. It is necessary also in Zoology and Botany, in both of which it is employed; yet it does not occur so frequently in these sciences, because the individuals of the organic kingdoms do not constitute compound masses; the only case excepted if the individual to be determined has not yet arrived at the state of greatest perfection. In that case the botanist compares a plant, which is not in flower, with another which presents the perfect flower, and with other individuals, representing intermediate stages of efflorescence, between the perfect immediately determinable plant, and that which he wants to determine, in perfect agreement with the rules developed above; and he knows by experience how far he may extend this comparison, in order to obtain results, upon the accuracy of which he may rely.

CHARACTERS

OF THE

CLASSES, ORDERS, GENERA, AND SPECIES.

CHARACTERS OF THE CLASSES.

CLASS I.

G. under 3·8.
No bituminous odour.
Solid : taste.

CLASS II.

G. above 1·8.
Tasteless.

CLASS III.

G. under 1·8.
Fluid : bituminous odour.
Solid : no taste.

CHARACTERS OF THE ORDERS.

CHARACTERS OF THE ORDERS OF CLASS I.

I. ORDER. GAS.

G. = 0·0001 ... 0·0014.
Expansible.
Not acid.

II. ORDER. WATER.

G. = 1·0.
Liquid.
Without odour or taste.

III. ORDER. ACID.

G. = 0·0015 ... 3·7.
Acid.

IV. ORDER. SALT.

G. = 1·2 ... 2·9.
Solid.
Not acid.

CHARACTERS OF THE ORDERS OF CLASS II.

I. ORDER. HALOÏDE.

Non-metallic.
Streak uncoloured.
H. = 1·5 ... 5·0.
G. = 2·2 ... 3·3.

Pyramidal or prismatic : H. = 4·0 and less, cleavage imperfect, in oblique directions.

Tessular : H. = 4·0.

Cleavage monotomous, eminent : G. = 2·4 and less.

H. under 2·5 : G. = 2·4 and less.

G. = 2·4 and less : H. under 2·5, no resinous lustre.

II. Order. BARYTE.

Non-metallic.
Streak uncoloured or orange-yellow.
H. = 2·5 ... 5·0.
G. = 3·3 ... 7·3.

Cleavage monotomous : G. = 4·0 and less; or = 5·0 and more.
Lustre adamantine or imperfect metallic : G. = 5·0 and more.
Streak orange-yellow : G. = 6·0 and more.
H. = 5·0 : G. under 4·5.
G. under 4·0; and H. = 5·0 : cleavage diprismatic.

III. Order. KERATE.

Non-metallic.
Streak uncoloured.
Cleavage not monotomous, not perfect peritomous.
H. = 1·0 ... 2·0.
G. above 5·5.

IV. Order. MALACHITE.

Non-metallic.
Colour blue, green, brown.
Cleavage not monotomous.
H. = 2·0 ... 5·0.
G. = 2·0 ... 4·6.

Colour or streak brown : H. = 3·0 and less, G. above 2·5.
Streak blue : H. = 4·0 and less.
Streak uncoloured : G. = 2·2 and less, H. under 3·0.

V. Order. MICA.

Cleavage monotomous, eminent.
H. = 1·0 ... 4·5.
G. = 1·8 ... 3·2.

Metallic : G. under 2·2.
Non-metallic : G. above 2·2.
H. = 3·0 and more : rhombohedral.
G. under 2·5 : metallic.

VI. Order. SPAR.

Non-metallic.
Streak uncoloured ... brown, blue.
H. = 3·5 ... 7·0.
G. = 2·0 ... 3·7.

Tessular : G. = 3·0 and less.

Rhombohedral : G. = 2·2 and less; or H. = 6·0.

H. = 4·0 and less : cleavage monotomous, eminent.

H. above 6·0 : pearly lustre; G. under 2·5 or above 2·8.

G. above 3·3 : forms hemi- or tetarto-prismatic; or H. = 6·0; no adamantine lustre.

G. = 2·4 and less : not without traces of form and cleavage.

VII. Order. GEM.

Non-metallic.
No metallic adamantine lustre.
Streak uncoloured.
H. = 5·5 ... 10·0.
G. = 1·9 ... 4·7.

H. = 6·0 and less : tessular, G. = 3·1 and more; or G. = 2·4 and less, and no traces of form and cleavage.

G. under 3·8 : no pearly lustre upon faces of cleavage.

VIII. Order. ORE.

No green streak.
H. = 2·5 ... 7·0.
G. = 3·4 ... 7·4.

Metallic : colour black.

Non-metallic : lustre adamantine or imperfect metallic.

Streak yellow or red : H = 3·5 and more, G. = 4·8 and more.

Streak brown or black : H. = 5·0 and more; or cleavage monotomous.

H. = 4·5 and less : streak yellow, red or black.

H. = 6·5 and more, and streak uncoloured : G. = 6·5 and more.

IX. Order. METAL.

Metallic.
Colour not black.
H. = 0·0 … 5·0.
G. = 5·7 … 20·0.

Colour grey : malleable, G. = 7·4 and more.

H. above 4·0 : malleable.

X. Order. PYRITES.

Metallic.
H. = 3·0 … 6·5.
G. = 4·1 … 7·7.

H. = 4·5 and less : G. under 5·3.

G. = 5·3 and less : colour yellow or red.

XI. Order. GLANCE.

Metallic.
Colour grey, black.
H. = 1·0 … 4·0.
G. = 4·2 … 7·6.

Cleavage monotomous ; G. being under 5·0 : colour lead-grey.

G. above 7·4 : colour lead-grey.

XII. Order. BLENDE.

Streak green, red, brown, uncoloured.
H. = 1·0 ... 4·0.
G. = 3·9 ... 8·2.

Metallic : colour black.
Non-metallic : lustre adamantine.
Streak green : colour black.
Streak brown ... uncoloured : G. between 4·0 and 4·2, form tessular.
Streak red : H. = 2·5 and less.
G. = 4·3 and more : streak red.

XIII. Order. SULPHUR.

Non-metallic.
Colour yellow, red, brown.
Prismatic.
H. = 1·0 ... 2·5.
G. = 1·9 ... 3·6.

Cleavage monotomous : G. = 3·4 and more.
G. above 2·1 : streak yellow or red.

CHARACTERS OF THE ORDERS OF CLASS III.

I. ORDER. RESIN.

H. = 0·0 ... 2·5.
G. = 0·7 ... 1·6.
G. = 1·2 and more : streak uncoloured.

II. ORDER. COAL.

Streak brown, black.
H. = 1·0 ... 2·5.
G. = 1·2 ... 1·5.

CHARACTERS

OF THE

GENERA AND SPECIES

OF THE

ORDERS OF CLASS I.

I. ORDER. GAS.

I. HYDROGEN-GAS. Odour.
G. = 0·0001 ... 0·0014.

1. PURE. Odour of hydrogen.
G. = 0·00012.
Pure Hydrogen-Gas. JAMESON. Vol. ii. p. 17.

2. EMPYREUMATIC. Empyreumatic odour.
G. = 0·0008.
Empyreumatic Hydrogen-Gas. J. ii. 18.

3. SULPHUROUS. Odour of putrid eggs.
G. = 0·00135.
Sulphuretted Hydrogen-Gas. J. ii. 19.

4. PHOSPHOROUS. Odour of putrid fish.
G. unknown.
Phosphuretted Hydrogen-Gas. J. ii. 19.

II. ATMOSPHERIC-GAS. Without odour or sapidity.
G. = 0·001 ... 0·0015.

1. PURE. As above.
Pure Atmospheric-Air. J. ii. 20.

II. ORDER. WATER.

I. ATMOSPHERIC-WATER. Without odour or sapidity.

1. PURE. As above.
Water. ii. 21.

III. ORDER. ACID.

I. CARBONIC-ACID. Taste slightly acid.
G. = 0·0018.

1. GASEOUS. Expansible.
Taste acidulous, pungent.
Aëriform Carbonic Acid. J. ii. 22.

II. MURIATIC-ACID. Odour pungent.
Taste strongly acid.
G. = 0·0023.

1. GASEOUS. Expansible.
Odour pungent.
Aëriform Muriatic Acid. J. ii. 23.

III. SULPHURIC-ACID.
G. = 0·0025 ... 1·9.

Expansible: odour sulphurous.
Liquid: taste strongly acid.

1. GASEOUS. Expansible.
G. = 0·0028.
Aëriform Sulphuric Acid. J. ii. 23.

2. LIQUID. Liquid.
G. = 1·8 ... 1·9.
Liquid Sulphuric Acid. J. ii. 24.

IV. BORACIC-ACID. Solid.
G. = 1·4 ... 1·5.

1. PRISMATIC. Prismatic.
Taste acidulous, afterwards bitter and cooling, lastly sweetish.
G. = 1·4 ... 1·5.
Sassoline. ii. 25.

V. ARSENIC-ACID. Solid.
G. above 3·0.

1. OCTAHEDRAL. Tessular.
Cleavage, octahedron.
Taste sweetish astringent.
H. = 1·5.
G. = 3·6 ... 3·7.
Octahedral Arsenic-Acid. J. ii. 26.

IV. ORDER. SALT.

I. NATRON-SALT. Prismatic.
Taste pungent, alcaline.
H. = 1·0 ... 1·5.
G. = 1·4 ... 1·6.

1. Hemi-Prismatic. Hemi-prismatic. $\frac{\breve{P}}{2} = 79° 41'$. Inclination $= 3° 0'$.

Cleavage, $\frac{\breve{P}r}{2} = 58° 52'$. Less distinct, $\bar{P}r + \infty$ and $(\breve{P}r + \infty)^3 = 76° 28'$.
H. = 1·0 ... 1·5.
G. = 1·4 ... 1·5.
Prismatic Natron. J. ii. 27.

2. Prismatic. Prismatic. $\breve{P}r = 83° 50'$, $(\breve{P}r + \infty)^3 = 107° 50'$.
Cleavage, $\breve{P}r + \infty$, very indistinct.
H. = 1·5.
G. = 1·5 ... 1·6.
Prismatic Natron. J. ii. 29.

II. Glauber-Salt. Prismatic.

Taste cool, then saline and bitter, weak.
H. = 1·5 ... 2·0.
G. = 1·4 ... 1·5.

1. Prismatic. Hemi-prismatic. $\frac{\breve{P}}{2} = 93° 12'$. Inclination $= 14° 41'$.
Cleavage, $\breve{P}r + \infty$, perfect. Traces of $-\frac{\breve{P}r}{2} = 72° 15'$. and of $\bar{P}r + \infty$.
Prismatic Glauber-Salt. J. ii. 31.

III. Nitre-Salt. Prismatic.

Taste saline, cool.
H. = 2·0.
G. = 1·9 ... 2·0.

1. Prismatic. Prismatic. P = 132° 22′, 91° 15′, 107° 43′.

Cleavage, $P + \infty = 120°$. Somewhat more distinct $\breve{P}r + \infty$.

Nitre. ii. 34.

IV. Rock-Salt. Tessular.

Taste saline.
H. = 2·0.
G. = 2·2 ... 2·3.

1. **Hexahedral.** Tessular.
Cleavage, hexahedron.
Hexahedral Rock-Salt. J. ii. 36.

V. Ammoniac-Salt. Tessular.

Taste saline, pungent.
H. = 1·5 ... 2·0.
G. = 1·5 ... 1·6.

1. **Octahedral.** Tessular.
Cleavage, octahedron.
Octahedral Sal Ammoniac. J. ii. 39.

VI. Vitriol-Salt. Prismatic.

Taste astringent.
H. = 2·0 ... 2·5.
G. = 1·8 ... 2·3.

1. **Hemi-Prismatic.** Hemi-prismatic. $\frac{\breve{P}}{2} = 101° 35'$.
Inclination = 14° 20′.
Cleavage, $P - \infty$. Somewhat less distinct, $P + \infty = 82° 21'$. Inclination of $P - \infty$ on $P + \infty = 99° 23'$.
Colour green.
H. = 2·0.
G. = 1·8 ... 1·9.
Green Vitriol. J. ii. 41

2. Tetarto-Prismatic. Tetarto-prismatic.
Cleavage, two faces, one of them more apparent; both indistinct. Inclination = 124° 2′.
Colour blue.
H. = 2·5.
G. = 2·2 ... 2·3.
Blue Vitriol. J. ii. 44.

3. Prismatic. Prismatic. $P + \infty = 90° 42'$.
Cleavage, $\breve{P}r + \infty$, perfect.
Colour white.
H. = 2·0 ... 2·5.
G. = 2·0 ... 2·1.
White Vitriol. J. ii. 46.

VII. Epsom-Salt. Prismatic.

Taste saline, bitter.
H. = 2·0 ... 2·5.
G. = 1·7 ... 1·8.

1. Prismatic. Prismatic. $P + \infty = 90° 38'$.
Cleavage, $\breve{P}r + \infty$, perfect.
Prismatic Epsom Salt. J. ii. 48.

VIII. Alum-Salt. Tessular.

Taste sweetish, astringent.
H. = 2·0 ... 2·5.
G. = 1·7 ... 1·8.

1. Octahedral. Tessular.
Cleavage, octahedron.
Alum. ii. 50.

IX. Borax-Salt. Prismatic.

Taste sweetish, feebly alcaline.
H. = 2·0 ... 2·5.
G. = 1·7 ... 1·8.

1. PRISMATIC. Hemi-prismatic. $\frac{'\check{P}}{2} = 120° 23'$. Inclination $= 0$.

Cleavage, $(\check{P}r + \infty)^3 = 80° 9'$. Somewhat more distinct, $\check{P}r + \infty$.

Borax. ii. 52.

X. BRITHYNE-SALT. Prismatic.

Taste saline, feebly astringent.

H. = 2·5 ... 3·0.

G. = 2·75 ... 2·85.

1. PRISMATIC. Hemi-prismatic. $\frac{\check{P}'}{2} = 120° 22'$. Inclination $= 22° 49'$.

Cleavage, $P - \infty$, perfect. Traces of $P + \infty = 80° 6'$. Inclination of $P - \infty$ on $P + \infty = 104° 28'$.

Glauberite. ii. p. 54.

Bloedite.	iii. 79.
Mascagnine.	iii. 125.
Nitrate of Soda, of Chemists.	iii. 132.
Polyhalite.	iii. 141.
Sulphate of Cobalt. Chem.	iii. 145.
Sulphate of Potash. Chem.	iii. 159.
Trona.	iii. 164.

CHARACTERS

OF THE

GENERA AND SPECIES

OF THE

ORDERS OF CLASS II.

I. ORDER. HALOIDE.

I. GYPSUM-HALOIDE. Prismatic.

H. = 1·5 ... 3·5.
G. = 2·2 ... 3·0.

G. above 2·5: cleavage in three directions, perpendicular to each other, one of them being less distinct.

1. PRISMATOIDAL. Hemi-prismatic. $\frac{P}{2} = 143° 52'$. Inclination $= 9° 11'$.

Cleavage, $\breve{P}r + \infty$, perfect and eminent. $- \frac{\bar{P}r}{2} = 66° 52'$. $\bar{P}r + \infty$.

H. = 1·5 ... 2·0.
G. = 2·2 ... 2·4.

Gypsum. ii. 57.

2. PRISMATIC. Prismatic. P = 121° 32′, 108° 35′, 99° 7′.
Cleavage, P̆r + ∞. P̄r + ∞. Less distinct, P — ∞.
Traces of P + ∞ = 100° 8′.
H. = 3·0 ... 3·5.
G. = 2·7 ... 3·0.
Anhydrite. ii. 62.

II. CRYONE-HALOIDE. Prismatic.

Cleavage in three directions, perpendicular to each other, one of them being more distinct.
H. = 2·5 ... 3·0.
G. = 2·9 ... 3·0.

1. PRISMATIC. Prismatic.
Cleavage, P — ∞. Less distinct, P̆r + ∞. P̄r + ∞
Traces of P.
Cryolite. ii. 66.

III. ALUM-HALOIDE. Rhombohedral.

H. = 5·0.
G. = 2·5 ... 2·8.

1. RHOMBOHEDRAL. Rhombohedral. R = 92° 50′.
Cleavage, R — ∞. Less distinct, R.
Rhomboidal Alumstone. J. ii. 67.

IV. FLUOR-HALOIDE. Tessular, rhombohedral.

H. = 4·0 ... 5·0.
G. = 3·0 ... 3·3.
Rhombohedral: cleavage peritomous.

1. OCTAHEDRAL. Tessular.
Cleavage, octahedron.
H. = 4·0.
G. = 3·0 ... 3·3.
Fluor. ii. 69.

2. Rhombohedral. Di-rhombohedral. P = 142° 20′, 80° 25′. $(P + n)^m$ hemi-di-rhombohedral with parallel faces.
Cleavage, R − ∞. P + ∞.
H. = 5·0.
G. = 3·0 ... 3·3.
Apatite. ii. 73.

V. Lime-Haloide. Rhombohedral, prismatic.

Cleavage, rhombohedral and paratomous, or prismatoidal.
H. = 3·0 ... 4·5.
G. = 2·5 ... 3·2.
H. above 4·0 : G. = 2·8 and more.
G. = 2·9 and more : H. = 3·5 and more.

1. Prismatic. Prismatic. P = 112° 39′, 93° 33′, 123° 34′.
Cleavage, $\bar{P}r - 1$ = 108° 8′. $(\breve{P}r + \infty)^3$ = 63° 44′. More distinct $\bar{P}r + \infty$.
H. = 3·5 ... 4·0.
G. = 2·6 ... 3·0.
Arragonite. ii. 79.

2. Rhombohedral. Rhombohedral. R = 105° 5′.
Cleavage, R.
H. = 3·0.
G. = 2·5 ... 2·8.
Rhomboidal Limestone. J. ii. 83.

3. Macrotypous. Rhombohedral. R = 106° 15′.
Cleavage, R.
H. = 3·5 ... 4·0.
G. = 2·8 ... 2·95.
Dolomite. ii. 93.

4. Brachytypous. Rhombohedral. R = 107° 22′.
Cleavage, R.

H. = 4·0 ... 4·5.
G. = 3·0 ... 3·2.
Breunnerite. ii. 98.

4. PARATOMOUS. Rhombohedral. R = 106° 12′.
Cleavage, R.
H. = 3·5 ... 4·0.
G. = 2·95 ... 3·1.
Ankerite. ii. 100.

Childrenite? iii. 85.
Fluellite. iii. 101.
Hopeite. iii. 109.
Magnesite. iii. 121.
Pharmacolite? iii. 135.
Roselite. iii. 147.
Wavellite. iii. 169.

II. ORDER. BARYTE.

I. PARACHROSE-BARYTE. Rhombohedral.

Cleavage paratomous.
H. = 3·5 ... 4·5.
G. = 3·3 ... 3·9.

1. BRACHYTYPOUS. Rhombohedral. R = 107° 0′.
Cleavage, R.
H. = 3·5 ... 4·5.
G. = 3·6 ... 3·9.
Rhomboidal Sparry Iron. J. ii. 101.

2. MACROTYPOUS. Rhombohedral. R = 106° 51′.
Cleavage, R.
H. = 3·5.
G. = 3·3 ... 3·6.
Rhomboidal Red Manganese. J. ii. 106.

II. Zinc-Baryte. Rhombohedral, prismatic.

H. = 5·0.

G. = 3·3 ... 4·5.

Rhombohedral: G. above 4·0.

1. Prismatic. Prismatic. $\bar{P}r - 1 = 128° \, 27'$.
   Cleavage, $\breve{P}r = 116° \, 40'$. Somewhat more distinct, $(\bar{P}r + \infty)^3 = 76° \, 7'$.
   H. = 5·0.
   G. = 3·3 ... 3·6.
   Prismatic Calamine. J. ii. 108.

2. Rhombohedral. Rhombohedral. $R = 107° \, 40'$.
   Cleavage, R.
   H. = 5·0.
   G. = 4·2 ... 4·5.
   Rhomboidal Calamine. J. ii. 111.

III. Scheelium-Baryte. Pyramidal.

H. = 4·0 ... 4·5.

G. = 6·0 ... 6·1.

1. Pyramidal. Pyramidal. $P = 107° \, 27', \; 113° \, 35'$.
   Combinations hemi-pyramidal with parallel faces.
   Cleavage, $P + 1 = 100° \, 8', \; 130° \, 20'$. P. $P - \infty$.
   Pyramidal Tungsten. J. ii. 113.

IV. Hal-Baryte. Prismatic.

H. = 3·0 ... 3·5.

G. = 3·6 ... 4·7.

1. Peritomous. Prismatic.
   Cleavage, $P + \infty = 117° \, 19'$. Less distinct, $\breve{P}r$. Traces of $\breve{P}r + \infty$.
   H. = 3·5.
   G. = 3·6 ... 3·8.
   Strontianite. ii. 116.

2. Di-Prismatic. Prismatic.
Cleavage, P + ∞ = 118° 30′. P̆r + ∞. P̆r.
H. = 3·0 ... 3·5.
G. = 4·2 ... 4·4.
Witherite. ii. 119.

3. Prismatic. Prismatic. P̆r = 105° 6′; (P̆r + ∞)³ = 77° 27′.
Cleavage, P̄r = 78° 18′. Somewhat easier, P̆r + ∞. Traces of P — ∞.
H. = 3·0 ... 3·5.
G. = 4·1 ... 4·7.
Heavy-Spar. J. ii. 121.

4. Prismatoidal. Prismatic. P̆r = 103° 58′. (P̆r + ∞)³ = 78° 35′.
Cleavage, P̄r = 76° 2′. More apparent, P̆r + ∞. Less distinct, P — ∞.
H. = 3·0 ... 3·5.
G. = 3·6 ... 4·0.
Celestine. ii. 126.

V. Lead-Baryte. Rhombohedral, pyramidal, prismatic.

H. = 2·5 ... 4·0.
G. = 6·0 ... 7·3.
H. above 3·5: G. = 6·5 and more.

1. Di-Prismatic. Prismatic. P = 130° 0′, 108° 28, 92° 19′.
Cleavage, P̆r = 117° 13′. (P̆r + ∞)³ = 69° 20′.
H. = 3·0 ... 3·5.
G. = 6·3 ... 6·6.
Di-Prismatic Lead-Spar. J. ii. 130.

2. Rhombohedral. Di-rhombohedral. P = 142° 12′, 80° 44′.
Cleavage, P. P + ∞. Both very indistinct.

H. = 3·5 ... 4·0.
G. = 6·9 ... 7·3.
Rhomboidal Lead-Spar. J. ii. 133.

3. Hemi-Prismatic. Hemi-prismatic. $\frac{P}{2}$ = 119° 0′.
Inclination = 12° 30′.
Cleavage, P + ∞ = 93° 40′. $\breve{P}r + \infty$. $\bar{P}r + \infty$.
Streak orange yellow.
H. = 2·5.
G. = 6·0 ... 6·1.
Red Lead-Spar. J. ii. 137.

4. Pyramidal. Pyramidal. P = 99° 40′, 131° 35′.
Cleavage, P. Less distinct P — ∞.
H. = 3·0.
G. = 6·5 ... 6·9.
Yellow Lead-Spar. J. ii. 140.

5. Prismatic. Prismatic. $\breve{P}r$ = 104° 55′; $(\breve{P}r + \infty)^3$ = 78° 45′.
Cleavage, $\bar{P}r$ = 76° 11′. $\breve{P}r + \infty$.
H. = 3·0.
G. = 6·2 ... 6·3.
Sulphate of Lead. J. ii. 142.

6. Axotomous. Hemi-prismatic. $\frac{P}{2}$ = 72° 36′. Inclination = 0° 29′.
Cleavage, P — ∞, perfect and eminent.
H. = 2·5.
G. = 6·2 ... 6·4.
Sulphato-tri-Carbonate of Lead. Brooke. ii. 144.

VI. Antimony-Baryte. Prismatic.

H. = 2·5 ... 3·0.
G. = 5·5 ... 5·6.

1. Prismatic. Prismatic. $\bar{P}r$ = 70° 32′.
Cleavage, $(\bar{P}r + \infty)^3$ = 136° 58′, highly perfect. $\breve{P}r + \infty$.

Prismatic White Antimony. J. ii. 151.

Corneous Lead. J. ii. 150.
Cupreous Sulphate of Lead. BROOKE. ii. 149.
Cupreous Sulphato-Carbonate of Lead. BROOKE. ii. 149.
Fluate of Cerium? Chem. iii. 100.
Hemi-prismatic Hal-baryte. iii. 76.
Peritomous Lead-baryte. ii. 151.
Plombgomme? iii. 140.
Stromnite? iii. 159.
Sulphato-Carbonate of Lead. BROOKE. ii. 148.
Tungstate of Lead. Chem. iii. 165.
Yttro-Cerite? J. iii. 172.

III. ORDER. KERATE.

I. PEARL-KERATE. Tessular, pyramidal.

H. = 1·0 ... 2·0.
G. = 5·5 ... 6·5.

1. HEXAHEDRAL. Tessular.
Cleavage, none.
Malleable.
H. = 1·0 ... 1·5.
G. = 5·5 ... 5·6.
Hexahedral Corneous Silver. J. ii. 154.

2. PYRAMIDAL. Pyramidal. P = 126° 31′, 79° 3′.
Cleavage, $P + \infty$, imperfect.
Sectile.
H. = 1·0 ... 2·0.
G. = 6·4 ... 6·5.
Calomel. ii. 156.

IV. Order. MALACHITE.

I. Staphyline-Malachite. Amorphous.

H. = 2·0 ... 3·0.
G. = 2·0 ... 2·2.

Uncleavable. Cleavage none.
Fracture conchoïdal.
Chrysocolla. ii. 158.

II. Lirocone-Malachite. Tessular, prismatic.

H. = 2·0 ... 2·5.
G. = 2·8 ... 3·0.

1. Prismatic. Prismatic.
Cleavage, P̆r = 71° 59′. P + ∞ = 119° 45′ imperfect.
Streak pale verdigris-green ... sky-blue.
H. = 2·0 ... 2·5.
G. = 2·8 ... 3·0.
Prismatic Liriconite. J. ii. 160.

2. Hexahedral. Semi-tessular with inclined faces.
Cleavage, hexahedron, imperfect.
Streak pale olive-green ... brown.
H. = 2·5.
G. = 2·9 ... 3·0.
Hexahedral Liriconite. J. ii. 162.

III. Olive-Malachite. Prismatic.

Colour or streak neither blue nor bright green.

H. = 3·0 ... 4·0.
G. = 3·6 ... 4·6.

1. Prismatic. Prismatic.
Cleavage, P̆r = 110° 50′. P + ∞ = 92° 30′. Both very indistinct.
Streak olive-green ... brown.

H. = 3·0.
G. = 4·2 ... 4·6.
Prismatic or Acicular Olivenite. J. ii. 164.

2. Di-Prismatic. Prismatic. $\breve{P}r$ = 111° 58′. P + ∞ = 95° 2′.
Cleavage, $\breve{P}r$ + ∞. $\bar{P}r$ + ∞. Both very indistinct.
Streak olive-green.
H. = 4·0.
G. = 3·6 ... 3·8.
Di-prismatic Olivenite. J. ii. 166.

IV. Azure-Malachite. Prismatic.

Colour blue.
H. = 3·5 ... 4·0.
G. = 3·7 ... 3·9.

1. Prismatic. Hemi-prismatic. $\frac{P}{2}$ = 117° 37′. Inclination = 2° 21′.
Cleavage, $(\bar{P}r + \infty)^3$ = 59° 14′. Less distinct, P — ∞. Traces of $\bar{P}r$ = 99° 32′.
Streak blue.
Prismatic Blue Malachite. J. ii. 167.

V. Emerald-Malachite. Rhombohedral.

H. = 5·0.
G. = 3·2 ... 3·4.

1. Rhombohedral. Rhombohedral. R + 1 = 95° 48′.
Cleavage, R = 126° 17′.
Streak green.
Dioptase. ii. 171.

VI. Habroneme-Malachite. Prismatic.

Colour or streak bright green.

H. = 3·5 ... 5·0.
G. = 3·6 ... 4·3.

1. PRISMATIC. Hemi-prismatic. $(\bar{P}r + \infty)^3 = 38° 56'$.
Cleavage, traces of $-\frac{\breve{P}r - 1}{2}$ and $\breve{P}r + \infty$.
Streak emerald-green.
H. = 5·0.
G. = 4·0 ... 4·3.
Prismatic Green Malachite. J. ii. 173.

2. HEMI-PRISMATIC. Hemi-prismatic. $\frac{\bar{P}}{2} = 139° 17'$.
Inclination = 0. $P + \infty = 103° 42'$.
Cleavage, $-\frac{\bar{P}r}{2} = 61° 49'$, and $\breve{P}r + \infty$ highly perfect.
Streak grass-green, apple-green.
H. = 3·5 ... 4·0.
G. = 3·6 ... 4·05.
Common Malachite. J. ii. 175.

Atacamite.	iii. 74.
Brochantite.	iii. 81.
Euchroite.	iii. 94.
Green Iron-Earth ? WERNER.	iii. 106.
Radiated Acicular Olivenite. J.	iii. 144.
Scorodite ?	iii. 149.
Vauquelinite ?	iii. 167.
Velvet-Blue Copper ? J.	iii. 168.

V. ORDER. MICA.

I. EUCHLORE-MICA. Rhombohedral, pyramidal, prismatic.

Streak green ... yellow.
H. = 1·0 ... 2·5.
G. = 2·5 ... 3·2.

Streak green : G. = 2·6 and less; or = 3·0 and more.

1. RHOMBOHEDRAL. Rhombohedral. R = 68° 45′.
Cleavage, R — ∞.
Streak emerald-green, apple-green.
H. = 2·0.
G. = 2·5 ... 2·8.
Hemi-prismatic Copper-Mica. J. ii. 178.

2. PRISMATIC. Prismatic.
Cleavage, P — ∞.
Laminæ flexible.
Streak pale, apple-green.
H. = 1·0 ... 1·5.
G. = 3·0 ... 3·2.
Kupferschaum. WERNER. ii. 180.

3. PYRAMIDAL. Pyramidal. P = 95° 46′, 143° 2′.
Cleavage, P — ∞.
Laminæ not flexible.
Streak green ... yellow.
H. = 2·0 ... 2·5.
G. = 3·0 ... 3·2.
Uranite. ii. 182.

II. COBALT-MICA. Hemi-prismatic.

Cleavage parallel to the plane of inclination.
H. = 2·5.
G. = 2·9 ... 3·1.

1. PRISMATIC. Hemi-prismatic. $\frac{P}{2}$ = 118° 23′. Inclination = 9° 47′. $\frac{\check{P}r}{2}$ = 55° 9′.

Cleavage, $\breve{P}r + \infty$.
Streak red ... green.
Prismatic Red Cobalt. J. ii. 184.

III. Iron-Mica. Prismatic.

Streak uncoloured ... blue.
H. = 2·0.
G. = 2·6 ... 2·7.

1. Prismatic. Hemi-prismatic $\frac{P}{2} = 119° \ 4'$. Inclination $= 10° \ 53'$. $\frac{\breve{P}r}{2} = 54° \ 13'$.
Cleavage, $\breve{P}r + \infty$.
Vivianite. ii. 188.

IV. Graphite-Mica. Rhombohedral.

H. = 1·0 ... 2·0.
G. = 1·8 ... 2·1.

1. Rhombohedral. Di-rhombohedral.
Cleavage, $R - \infty$.
Metallic.
Streak black.
Plumbago. ii. 191.

V. Talc-Mica. Rhombohedral, prismatic.

Streak uncoloured ... green.
H. = 1·0 ... 2·5.
G. = 2·7 ... 3·0.

Streak green : G. = 2·8 and less.
Hemi-prismatic: cleavage perpendicular to the plane of inclination.

1. Prismatic. Prismatic. $P + \infty = 120°$ (nearly).
Cleavage, $P - \infty$.

Laminæ flexible.
Streak uncoloured ... green.
H. = 1·0 ... 1·5.
G. = 2·7 ... 2·8.
Prismatic Talc-Mica. J. ii. 193.

2. Rhombohedral. Di-rhombohedral.
Cleavage, R — ∞.
Laminæ elastic.
Streak uncoloured.
H. = 2·0 ... 2·5.
G. = 2·8 ... 3·0.
Rhomboidal Talc-Mica. J. ii. 198.

VI. Pearl-Mica. Rhombohedral.

H. = 3·5 ... 4·5.
G. = 3·0 ... 3·1.

1. Rhombohedral. Di-rhombohedral.
Cleavage, R — ∞.
Streak uncoloured.
Margarite. ii. 204.

Cronstedtite? iii. 90.
Hydrate of Magnesia. Chem. iii. 112.
Pyrosmalite. iii. 143.

VI. Order. SPAR.

I. Schiller-Spar. Prismatic.

Cleavage monotomous, eminent.
H. = 3·5 ... 6·0.
G. = 2·6 ... 3·4.

1. DIATOMOUS. Prismatic.
Cleavage prismatoidal.
Lustre metallic-pearly.
H. = 3·5 ... 4·0.
G. = 2·6 ... 2·8.
Diatomous Schiller-Spar. J. ii. 206.

2. HEMI-PRISMATIC. Hemi-prismatic. $\frac{P}{2}$.
Cleavage, $\breve{P}r+\infty$. Less distinct, $\frac{\breve{P}r}{2}=72°$ and $P+\infty = 94°$. Traces of $\bar{P}r+\infty$.
Lustre metallic-pearly.
H. = 4·0 ... 5·0.
G. = 3·0 ... 3·3.
Bronzite. ii. 207.

3. PRISMATOIDAL. Prismatic.
Cleavage, $\breve{P}r+\infty$. Less distinct, $P+\infty = 93°$ (nearly). $\bar{P}r+\infty$.
Lustre metallic-pearly.
H. = 6·0.
G. = 3·3 ... 3·4.
Hypersthene. ii. 210.

4. PRISMATIC. Prismatic.
Cleavage, $\bar{P}r+\infty$. Somewhat less distinct, $P+\infty = 124° 30'$. $\breve{P}r+\infty$.
Lustre almost metallic-pearly.
H. = 5·0 ... 5·5.
G. = 3·0 ... 3·3.
Anthophyllite. ii. 211.

II. DISTHENE-SPAR. Prismatic.
H. = 5·0 ... 7·0.
G. = 6·0 ... 3·7.

1. PRISMATIC. Tetarto-prismatic.
Cleavage, two faces, one of them more distinct, perfect and eminent. Inclination = 100° 50′.
Kyanite. ii. 213.

III. TRIPHANE-SPAR. Prismatic.

Cleavage, somewhat more distinct in one direction.

Colour not blue.

H. = 6·0 ... 7·0.

G. = 2·8 ... 3·1.

1. PRISMATIC. Prismatic.

Cleavage, $P + \infty = 93°$. Somewhat more distinct, $\check{P}r + \infty$.

H. = 6·5 ... 7·0.

G. = 3·0 ... 3·1.

Spodumene. ii. 216.

2. AXOTOMOUS. Prismatic.

Cleavage, $P + \infty = 99° 30'$. More distinct, $P - \infty$.

H. = 6·0 ... 7·0.

G. = 2·8 ... 3·0.

Prehnite. ii. 217.

IV. DYSTOME-SPAR. Prismatic.

Cleavage difficult; lustre of the fracture resinous.

Colour not blue.

H. = 5·0 ... 5·5.

G. = 2·9 ... 3·0.

1. PRISMATIC. Hemi-prismatic. $\frac{\check{P}}{2} = 122° 0'$. Inclination $= 1° 41' 30''$.

Cleavage, $P + \infty = 71° 30'$, very indistinct; a little more distinct, $\check{P}r + \infty$.

Datolite. ii. 220.

V. KOUPHONE-SPAR. Tessular, rhombohedral, pyramidal, prismatic.

H. = 3·5 ... 6·0.
G. = 2·0 ... 2·5.

Pyramidal: cleavage axotomous, eminent.
H. = 6·0: tessular.

1. TRAPEZOIDAL. Tessular.
Cleavage, hexahedron, dodecahedron, imperfect.
H. = 5·5 ... 6·0.
G. = 2·4 ... 2·5.
Leucite. ii. 224.

2. DODECAHEDRAL. Tessular.
Cleavage, dodecahedron distinct.
H. = 5·5 ... 6·0.
G. = 2·25 ... 2·35.
Sodalite. ii. 225.

3. HEXAHEDRAL. Tessular.
Cleavage, hexahedron, imperfect.
H. = 5·5.
G. = 2·0 ... 2·2.
Analcime. ii. 227.

4. PARATOMOUS. Prismatic.
Cleavage, P. $\breve{P}r + \infty$. Somewhat easier $\bar{P}r + \infty$. Imperfect.
H. = 4·5.
G. = 2·3 ... 2·4.
Harmotome. ii. 229.

5. RHOMBOHEDRAL. Rhombohedral. R = 94° 46′.
Cleavage, R.
H. = 4·0 ... 4·5.
G. = 2·0 ... 2·1.
Chabasite. ii. 232.

6. DIATOMOUS. Hemi-prismatic. $P + \infty = 86° 15'$.
Cleavage, $\breve{P}r + \infty$. Traces of $\bar{P}r + \infty$.
H. unknown.
G. = 2·3 ... 2·4.
Laumonite. ii. 234.

7. PRISMATIC. Prismatic. P = 143° 20′, 142° 40′, 53° 20′.
Cleavage, P + ∞ = 91° 0′.
H. = 5·0 ... 5·5.
G. = 2·2 ... 2·3.
Mesotype. ii. 236.

8. PRISMATOIDAL. Prismatic. P = 119° 15′, 114° 0′, 96° 0′.
Cleavage, P̆r + ∞, eminent.
H. = 3·5 ... 4·0.
G. = 2·0 ... 2·2.
Stilbite. ii. 239.

9. HEMI-PRISMATIC. Hemi-prismatic. Irregular six-sided prism of 129° 40′, 116° 20′, and 114° 0′.
Cleavage, P̆r + ∞, very eminent.
H. = 3·5 ... 4·0.
G. = 2·0 ... 2·2.
Heulandite. ii. 242.

10. PYRAMIDAL. Pyramidal. P = 104° 2′, 121° 0′.
Cleavage, P — ∞, eminent. [P + ∞] imperfect.
H. = 4·5 ... 5·0.
G. = 2·2 ... 2·5.
Apophyllite. ii. 244.

11. AXOTOMOUS. Prismatic. P = 106° 52′, 101° 37′, 120° 34′.
Cleavage, P — ∞, eminent. Less distinct, P̆r + ∞. P̄r + ∞.
H. = 4·5 ... 5·0.
G. = 2·2 ... 2·5.
Apophyllite. ii. 246.

Brewsterite.	iii. 0.
Comptonite.	iii. 89.
Gmelinite.	iii. 104.
Levyne.	iii. 120.
Mesole.	iii. 126.
Mesoline.	iii. 126.
Sarcolite.	iii. 147.
Thomsonite.	iii. 162.

VI. Petaline-Spar. Prismatic.

H. = 6·0 ... 6·5.

G. = 2·4 ... 2·5.

1. Prismatic. Prismatic.
   Cleavage, a prism of 95° (nearly). More distinct, $\bar{P}r + \infty$.
   Petalite. ii. 248.

VII. Feld-Spar. Rhombohedral, pyramidal, prismatic.

H. = 5·0 ... 6·0.

G. = 2·5 ... 2·8.

H. = 5·5 and less: form pyramidal, cleavage not axotomous.

1. Rhombohedral. Di-rhombohedral. P = 139° 19′, 88° 6′.
   Cleavage, $R - \infty$. $R + \infty$.
   H. = 6·0.
   G. = 2·5 ... 2·6.
   Nepheline. ii. 250.
2. Prismatic. Hemi-prismatic. $\frac{\breve{P}}{2} = 126° 12'$. Inclination = 0.
   Cleavage, $-\frac{\breve{P}r}{2} = 64° 34'$, perfect. $\bar{P}r + \infty$ perfect, but often interrupted. $(\breve{P}r + \infty)^3 = 118° 52'$, imperfect.
   H. = 6·0.
   G. = 2·5 ... 2·6.
   Prismatic Felspar. J. ii. 251.
3. Pyramidal. Pyramidal. P = 136° 7′, 63° 48′.
   Cleavage, $P + \infty$. $[P + \infty]$. Traces of $P - \infty$.
   H. = 5·0 ... 5·5.
   G. = 2·5 ... 2·8.

Wernerite. ii. 264.

Albite. ii. 255.
Anorthite. iii. 71.
Elaolite. ii. 93.
Felspar from Baveno. Vulg. ii. 258.
Felspar from the Saualpe. Vulg. ii. 257.
Labradorite. ii. 257.
Latrobite. iii. 118.
Nuttallite. iii. 133.

VIII. Augite-Spar. Prismatic.

Lustre not metallic-pearly.
H. = 4·5 ... 7·0.
G. = 2·7 ... 3·5.
H. above 6·0: G. = 3·2 and more.
G. under 3 2: cleavage oblique-angular peritomous, perfect.

1. Paratomous. Hemi-prismatic. $\frac{\check{P}}{2} = 120° 0'$. $\frac{\check{P}r}{2} = 73° 54'$. Inclination = 0.
Cleavage, $(\check{P}r + \infty)^3 = 87° 5'$. $\check{P}r + \infty$. $\bar{P}r + \infty$. Sometimes $\frac{\check{P}}{2}$.
H. = 5·0 ... 6·0.
G. = 3·2 ... 3·5.
Pyroxene. ii. 268.

2. Hemi-Prismatic. Hemi-prismatic. $\frac{\check{P}}{2} = 148° 39'$. $\frac{\check{P}r}{2} = 75° 2'$. Inclination = 0.
Cleavage, $(\check{P}r + \infty)^3 = 124° 34'$. Less distinct, $\check{P}r + \infty$. $\bar{P}r + \infty$.

H. = 5·0 ... 6·0.
G. = 2·8 ... 3·2.
Amphibole. ii. 274.

3. PRISMATOIDAL. Hemi-prismatic. $\frac{\check{P}}{2} = 70° 33'$. $\frac{\check{P}r}{2} = 63° 43'$. Inclination = 0° 33'.
Cleavage, $-\frac{\check{P}r}{2} = 64° 36'$. More distinct, $\check{P}r + \infty$.
H. = 6·0 ... 7·0.
G. = 3·2 ... 3·5.
Epidote. ii. 282.

4. PRISMATIC. Prismatic.
Cleavage, perfect in two directions, one of them being more easily obtained. Inclination = 95° 25'.
H. = 4·5 ... 5·0.
G. = 2·7 ... 2·9.
Wollastonite. ii. 286.

Acmite. iii. 67.
Arfvedsonite. iii. 73.
Babingtonite. iii. 75.
Indianite. iii. 113.
Jeffersonite. iii. 115.
Manganese-Spar? J. iii. 122.
Withamite. iii. 170.

IV. AZURE-SPAR. Tessular, prismatic.

Colour blue.
H. = 5·0 ... 6·0.
G. = 2·9 ... 3·1.

1. DODECAHEDRAL. Tessular.
Cleavage, imperfect.
Colour bright.
Streak blue.

H. = 5·5 ... 6·0.
G. = 2·9 ... 3·0.
Azurestone or Lapis Lazuli. J. ii. 288.

2. Prismatic. Prismatic.
Cleavage, P + ∞ = 91° 30′, imperfect.
Colour bright.
Streak uncoloured.
H. = 5·0 ... 5·5.
G. = 3·0 ... 3·1.
Lazulite. ii. 290.

3. Prismatoidal. Prismatic.
Cleavage, prismatoidal, imperfect.
Colour pale.
Streak uncoloured.
H. = 5·5 ... 6·0.
G. = 3·0 ... 3·1.
Prismatoidal Azure-Spar or Blue-Spar. J. ii. 292.

Amblygonite.	iii. 70.
Bergmannite.	iii. 77.
Bucklandite?	iii. 83.
Calaite.	iii. 83.
Chiastolite.	iii. 84.
Diaspore.	iii. 92.
Eudialyte.	iii. 96.
Gehlenite.	iii. 102.
Haüyne.	iii. 107.
Karpholite?	iii. 116.
Nephrite.	iii. 131.
Saussurite.	iii. 148.
Sillimanite.	iii. 153.
Somervillite.	iii. 154.
Thulite.	iii. 162.

VII. Order. GEM.

I. Andalusite. Prismatic.

Cleavage not prismatoidal.

H. = 7·5.

G. = 3·0 ... 3·2.

1. Prismatic. Prismatic.

Cleavage, P + ∞ = 91° 33′. P̆r + ∞. P̄r + ∞.

Andalusite. ii. 293.

II. Corundum. Tessular, rhombohedral, prismatic.

H. = 8·0 ... 9·0.

G. = 3·5 ... 4·3.

Prismatic: G. = 3·65 and more; H. = 8·5.

Colour red or brown; G. = 3·7 and more: H. = 9·0.

1. Dodecahedral. Tessular.

Cleavage, octahedron, difficult.

H. = 8·0.

G. = 3·5 ... 3·8.

Spinelle. ii. 295.

2. Octahedral. Tessular.

Cleavage, octahedron, perfect.

H. = 8·0.

G. = 4·1 ... 4·3.

Gahnite. ii. 298.

3. Rhombohedral. Rhombohedral. R = 86° 6′.

Cleavage, R. Sometimes, R — ∞.

H. = 9·0.

G. = 3·9 ... 4·05.

Corundum. ii. 299.

4. PRISMATIC. Prismatic. $\breve{P}r = 119° 46'$. $(\breve{P} + \infty)^3 = 70° 41'$.
Cleavage, $\breve{P}r + \infty$. Less distinct, $\bar{P}r + \infty$.
H. = 8·5.
G. = 3·65 ... 3·8.
Chrysoberyl. ii. 304.

III. DIAMOND. Tessular.
H. = 10·0.
G. = 3·4 ... 3·6.

1. OCTAHEDRAL. Tessular.
Cleavage, octahedron, perfect.
Diamond. ii. 306.

IV. TOPAZ. Prismatic.
Cleavage, axotomous.
H. = 8·0.
G. = 3·4 ... 3·6.

1. PRISMATIC. Prismatic. $P = 141° 7', 101° 52', 90° 55'$. $P + \infty = 124° 19'$.
Combinations sometimes different in the opposite ends of the crystals.
Cleavage, $P - \infty$, highly perfect.
Topaz. ii. 308.

V. EMERALD. Rhombohedral, prismatic.
Cleavage, rhombohedral axotomous and peritomous, or prismatoidal of a high degree of perfection.
H. = 7·5 ... 8·0.
G. = 2·6 ... 3·2.

1. PRISMATIC. Hemi-prismatic.

Cleavage, $\breve{P}r + \infty$, of a high degree of perfection.

$\frac{\breve{P}r}{2} = 49° 52'$.

H. = 7·5.

G. = 2·9 ... 3·2.

Euclase. ii. 313.

2. RHOMBOHEDRAL. Di-rhombohedral. P = 151° 9′, 59° 47′.

Cleavage, $R - \infty$. Less distinct, $P + \infty$.

H. = 7·5 ... 8·0.

G. = 2·6 ... 2·8.

Emerald. ii. 316.

VI. QUARTZ. Rhombohedral, prismatic.

Cleavage, not axotomous.

H. = 5·5 ... 7·5.

G. = 1·9 ... 2·7.

1. PRISMATIC. Prismatic.

Cleavage, $P + \infty = 120°$ (nearly). $\breve{P}r + \infty$.

Dichroism, parallel and perpendicular to the axis.

H. = 7·0 ... 7·5.

G. = 2·5 ... 2·6.

Cordierite. ii. 319.

2. RHOMBOHEDRAL. Rhombohedral. R = 75° 55′.

Combinations, hemi-rhombohedral and hemi-di-rhombohedral; $R + n$ and $(P + n')^m$ with inclined faces, $P + n''$ with parallel faces. $\frac{P}{2} = 94° 15'$.

Cleavage, P = 133° 44′, 103° 35′. Commonly $\frac{P}{2}$ somewhat less distinct. $P + \infty$.

H. = 7·0.

G. = 2·5 ... 2·7.

Quartz. ii. 321.

3. UNCLEAVABLE. Reniform ... massive.

Cleavage, none.

H. = 5·5 ... 6·5.
G. = 1·9 ... 2·2.
Opal. ii. 332.

4. EMPYRODOX. Grains ... massive.
Cleavage, none.
H. = 6·0 ... 7·0.
G. = 2·2 ... 2·4.
Fusible Quartz. J. ii. 337.

VII. AXINITE. Prismatic.

Lustre pure vitreous.
H. = 6·5 ... 7·0.
G. = 3·0 ... 3·3.

1. PRISMATIC. Tetarto-prismatic.
Cleavage, two faces, one of them more distinct. Inclination = 101° 30′.
Axinite. ii. 341.

VIII. CHRYSOLITE. Prismatic.

Lustre pure vitreous.
H. = 6·5 ... 7·0.
G. = 3·3 ... 3·5.

1. PRISMATIC. Prismatic. $\breve{P}r = 80° 53'$. $(\bar{P}r + \infty)^3 = 130° 2'$.
Cleavage, $\breve{P}r + \infty$. Traces of $\bar{P}r + \infty$.
Chrysolite. ii. 345.

IX. BORACITE. Tessular.

H. = 7·0.
G. = 2·8 ... 3·0.

1. TETRAHEDRAL. Semi-tessular, with inclined faces.
Cleavage, octahedron, imperfect.
Boracite. ii. 347.

X. TOURMALINE. Rhombohedral.

H. = 7·0 ... 7·5.
G. = 3·0 ... 3·2.

1. RHOMBOHEDRAL. Rhombohedral. R = 133° 26′.
   Combinations, the opposite ends of the crystals containing different faces.
   Cleavage, R. P + ∞. Imperfect.
   Tourmaline. ii. 349.

XI. GARNET. Tessular, pyramidal, prismatic.

Lustre, not pure vitreous.
H. = 6·0 ... 7·5.
G. = 3·1 ... 4·3.
Colour black : G. = 3·9 and less.
H. = 7·5 : colour red or brown.
G. under 3·3 : form tessular.

1. PYRAMIDAL. Pyramidal. P = 129° 29′, 74° 14′.
   Cleavage, P — ∞. P + ∞. [P + ∞].
   H. = 6·5.
   G. = 3·3 ... 3·4.
   Idocrase. ii. 354.

2. TETRAHEDRAL. Semi-tessular, with inclined faces.
   Cleavage, octahedron, imperfect.
   H. = 6·0 ... 6·5.
   G. = 3·1 ... 3·3.
   Helvine. ii. 357.

3. DODECAHEDRAL. Tessular.
   Cleavage, dodecahedron, imperfect.
   H. = 6·5 ... 7·5.
   G. = 3·5 ... 4·3.
   Garnet. ii. 359.

4. PRISMATOIDAL. Prismatic. P̆r = 70° 32′. $(\breve{P}r + \infty)^3$ = 129° 31′.

Cleavage, $\breve{P}r = \infty$, perfect.
H. = 7·0 ... 7·5.
G. = 3·3 ... 3·9.
Staurolite. ii. 366.

XII. Zircon. Pyramidal.

H. = 7·5.
G. = 4·5 ... 4·7.

1. Pyramidal. Pyramidal. $P = 123° 19', 84° 20'$.
Cleavage, P. $P + \infty$.
Zircon. ii. 368.

XIII. Gadolinite. Prismatic.

Colour black.
H. = 6·5 ... 7·0.
G. = 4·0 ... 4·3.

1. Prismatic. Hemi-prismatic.
Cleavage almost none.
Fracture conchoidal.
Gadolinite. ii. 371.

Aplome.	ii. 364.
Chondrodite.	iii. 87.
Essonite.	ii. 364.
Fibrolite ?	ii. 99.
Forsterite.	iii. 102.
Hyalosiderite ?	iii. 111.
Knebelite ?	iii. 118.
Ligurite ?	iii. 121.
Mellilite ?	iii. 125.
Sphærulite ?	iii. 155.
Spinellane ?	iii. 156.
Zeagonite.	iii. 174.

VIII. Order. ORE.

I. Titanium-Ore. Pyramidal, prismatic.
Streak uncoloured ... very pale brown.
H. = 5·0 ... 6·5.
G. = 3·4 ... 4·4.
G. under 4·2 : streak uncoloured.

1. Prismatic. Hemi-prismatic. $\frac{\bar{P}}{2} = 113° 20'$. Inclination $= 8° 18'$. $(\bar{P}r + \infty)^3 = 136° 8'$.
Cleavage, $\frac{P}{2}$. $\frac{\bar{P}r}{2} = 28° 7'$, difficult.
Streak uncoloured.
H. = 5·0 ... 5·5.
G. = 3·4 ... 3·6.
Sphene. ii. 373.

2. Peritomous. Pyramidal. P = 117° 2′, 95° 13′.
Cleavage, $P + \infty$. $[P + \infty]$.
Streak pale brown.
H. = 6·0 ... 6·5.
G. = 4·2 ... 4·4.
Rutile. ii. 376.

3. Pyramidal. Pyramidal. P = 97° 56′, 136° 22′.
Cleavage, $P - \infty$. P.
Streak uncoloured.
H. = 5·5 ... 6·0.
G. = 3·8 ... 3·9.
Anatase. ii. 379.

II. Zinc-Ore. Prismatic.
Streak orange-yellow.
H. = 4·0 ... 4·5.
G. = 5·4 ... 5·5.

1. PRISMATIC. Prismatic.
Cleavage, $P + \infty = 125°$ (nearly). Traces of $\breve{P}r + \infty$.
Prismatic Zinc-Ore. J. ii. 380.

III. COPPER-ORE. Tessular.

Streak brownish-red.
$H. = 2{\cdot}5 \ldots 4{\cdot}0.$
$G. = 5{\cdot}6 \ldots 6{\cdot}0.$

1. OCTAHEDRAL. Tessular.
Cleavage, octahedron.
Octahedral Red Copper-Ore. J. ii. 381.

IV. TIN-ORE. Pyramidal.

Streak not black.
$H. = 6{\cdot}0 \ldots 7{\cdot}0.$
$G. = 6{\cdot}3 \ldots 7{\cdot}1.$

1. PYRAMIDAL. Pyramidal. $P = 133° \; 26', 67° \; 59'$.
Cleavage, $P + \infty$. $[P + \infty]$.
Streak uncoloured ... pale brown.
Pyramidal Tin-Ore. J. ii. 384.

V. SCHEELIUM-ORE. Prismatic.

Streak reddish-brown, dark.
$H. = 5{\cdot}0 \ldots 5{\cdot}5.$
$G. = 7{\cdot}1 \ldots 7{\cdot}4.$

1. PRISMATIC. Hemi-prismatic. $\frac{\bar{P}r - 1}{2} = 62° \; 40'$. Inclination $= 0$. $P + \infty = 101° \; 5'$.
Cleavage, $\breve{P}r + \infty$, perfect.
Prismatic Wolfram. J. ii. 387.

VI. TANTALUM-ORE. Prismatic.

Streak brownish-black.

H. = 6·0.
G. = 6·0 ... 6·3.

1. PRISMATIC. Prismatic.
Cleavage, prismatoidal.
Tantalite. ii. 390.

VII. URANIUM-ORE. Form not determinable.
Streak black.
H. = 5·5.
G. = 6·4 ... 6·6.

1. UNCLEAVABLE. Reniform, massive.
Cleavage, none.
Uncleavable Uranium-Ore. J. ii. 393.

VIII. CERIUM-ORE. Form not determinable.
Streak uncoloured.
H. = 5·5.
G. = 4·9 ... 5·0.

1. UNCLEAVABLE. Massive.
Cleavage, none.
Cerite. ii. 394.

IX. CHROME-ORE. Tessular.
Streak brown.
H. = 5·5.
G. = 4·4 ... 4·5.

1. OCTAHEDRAL. Tessular.
Cleavage, octahedron.
Prismatic Chrome-Ore or Chromate of Iron. J. ii. 396.

X. IRON-ORE. Tessular, rhombohedral, prismatic.
Streak red, brown, black.

H. = 5·0 ... 6·5.
G. = 3·8 ... 5·3.

Streak brown: G. = 4·2 and less, or 4·8 and more.

G. under 4·3; the colour being black: streak without lustre.

1. AXOTOMOUS. Rhombohedral. R = 85° 59′.
Combinations hemi-rhombohedral, with parallel faces.
$\frac{P+1}{2} = 91° 20′$.
Cleavage, R — ∞, perfect. Traces of R.
Streak black.
Weak action upon the magnetic needle.
H. = 5·0 ... 5·5.
G. = 4·4 ... 4·8.
Titanitic Iron. Vulg. ii. 397.

2. OCTAHEDRAL. Tessular.
Cleavage, octahedron.
Streak black.
Strong action upon the magnetic needle.
H. = 5·5 ... 6·5.
G. = 4·8 ... 5·2.
Octahedral Iron-Ore. J. ii. 399.

3. DODECAHEDRAL. Tessular.
Cleavage, octahedron, very indistinct.
Streak brown.
Weak action upon the magnetic needle.
H. = 6·0 ... 6·5.
G. = 5·0 ... 5·1.
Franklinite. ii. 403.

4. RHOMBOHEDRAL. Rhombohedral. R = 85° 58′.
Cleavage, R. Sometimes, R — ∞.
Streak red ... reddish-brown.
Sometimes a weak action upon the magnetic needle.

H. = 5·5 ... 6·5.
G. = 4·8 ... 5·3.
Rhomboidal Iron-Ore. J. ii. 404.

5. Prismatic. Prismatic.
Cleavage, $P + \infty$.
Streak yellowish-brown.
No action upon the magnetic needle.
H. = 5·0 ... 5·5.
G. = 3·8 ... 4·2.
Prismatic Iron-Ore. J. ii. 410.

6. Di-Prismatic. Prismatic. P = 139° 37′, 117° 38, 77° 16′.
Cleavage, $\bar{P}r$ = 113° 2′. $P + \infty$ = 112° 37′. Somewhat more distinct, $P - \infty$. $\breve{P}r + \infty$. Altogether imperfect.
Streak black, sometimes greenish or brownish.
No action upon the magnetic needle.
H. = 5·5 ... 6·0.
G. = 3·8 ... 4·1.
Lievrite. ii. 414.

XI. Manganese-Ore. Pyramidal, prismatic.

Streak dark brown, black.
No action upon the magnetic needle.
H. = 2·5 ... 6·0.
G. = 4·0 ... 4·8.

Streak brown: G. = 4·7 and more, H. = 4·0 and more.

H. above 4·0; the streak being black: lustre in the streak.

Pyramidal. Pyramidal. P = 105° 25′, 117° 54′.
Cleavage, $P - \infty$. Traces of $P - 1$ = 114° 51′, 99° 11′, and of P.
Streak brown.

H. = 5·0 ... 5·5.
G. = 4·7 ... 4·8.
Black Manganese. J. ii. 416.

UNCLEAVABLE. Reniform, botryoidal, massive.
Cleavage, none.
Streak brownish-black, shining.
H. = 5·0 ... 6·0.
G. = 4·0 ... 4·1.
Black Manganese. J. ii. 418.

3. PRISMATOIDAL. Prismatic.
Cleavage, $\breve{P}r + \infty$ perfect, less distinct $P + \infty = 99° 40'$.
Streak black.
H. = 2·5 ... 3·5.
G. = 4·4 ... 4·8.
Grey Manganese. J. ii. 419.

Allanite.	iii. 68.
Brookite.	iii. 82.
Fergusonite.	iii. 98.
Orthite.	iii. 133.
Phosphate of Manganese. Chem.	iii. 136.
Stilpnosiderite.	iii. 158.
Yttro-Tantalite. J.	iii. 173.

IX. ORDER. METAL.

I. ARSENIC. Form unknown.
Colour tin-white.
H. = 3·5.
G. = 5·7 ... 5·8.

1. NATIVE. Reniform, massive.
Arsenic. ii. 423.

II. Tellurium. Form unknown.

Colour tin-white.

H. = 2·0 ... 2·5.

G. = 6·1 ... 6·2.

1. Native. Massive.

Tellurium. ii. 424.

III. Antimony. Rhombohedral, prismatic.

Not malleable.

Colour white, not inclining to red.

H. = 3·0 ... 3·5.

G. = 6·5 ... 10·0.

1. Rhombohedral. Rhombohedral. R = 117° 15′.

Cleavage, R — ∞ perfect. R. Traces of R — 2 and P + ∞.

H. = 3·0 ... 3·5.

G. = 6·5 ... 6·8.

Antimony. ii. 426.

2. Prismatic. Prismatic.

Cleavage, P — ∞. P̆r. Less distinct, P + ∞.

H. = 3·5.

G. = 8·9 ... 10·0.

Prismatic Antimony or Antimonial Silver. J. ii. 427.

IV. Bismuth. Tessular.

Colour silver-white, inclining to red.

H. = 2·0 ... 2·5.

G. = 9·6 ... 9·8.

1. Octahedral. Semi-tessular with inclined faces.

Cleavage, octahedron, perfect.

Bismuth. ii. 430.

V. MERCURY. Tessular, fluid.
Not malleable.
Colour white.
H. = 0·0 ... 3·0.
G. = 10·5 ... 15·0.

1. DODECAHEDRAL. Tessular.
Cleavage, none.
Colour silver-white.
H. = 1·0 ... 3·0.
G. = 10·5 ... 12·5.
Dodecahedral Mercury or Native Amalgam. J. ii. 431.

2. FLUID. Fluid.
Colour tin-white.
H. = 0·0.
G. = 12·0 ... 15·0.
Mercury. ii. 432.

VI. SILVER. Tessular.
Ductile.
Colour silver-white.
H. = 2·5 ... 3·0.
G. = 10·0 ... 10·5.

1. HEXAHEDRAL. Tessular.
Cleavage, none.
Silver. ii. 433.

VII. GOLD. Tessular.
Colour gold-yellow.
H. = 2·5 ... 3·0.
G. = 12·0 ... 20·0.

1. HEXAHEDRAL. Tessular.
Cleavage, none.
Gold. ii. 436.

VIII. Platina. Form unknown.
Colour steel-grey.
H. = 4·0 ... 4·5.
G. = 16·0 ... 20·0.

1. Native. Massive.
Cleavage, none.
Platina. ii. 441.

IX. Iron. Tessular.
Colour pale steel-grey.
H. = 4·5.
G. = 7·4 ... 7·8.

1. Octahedral. Tessular.
Cleavage, none.
Iron. ii. 442.

X. Copper. Tessular.
Colour copper-red.
H. = 2·5 ... 3·0.
G. = 8·4 ... 8·9.

1. Octahedral. Tessular.
Cleavage, none.
Copper. ii. 444.

Iridium. iii. 114.
Lead. iii. 129.
Palladium. iii. 134.

X. ORDER. PYRITES.

I. NICKEL-PYRITES. Prismatic.

Colour copper-red.

H. = 5·0 ... 5·5.

G. = 7·5 ... 7·7.

1. PRISMATIC. Prismatic.
Cleavage, indistinct.
Prismatic Nickel Pyrites. J. ii. 446.

II. ARSENICAL-PYRITES. Prismatic.

Colour not inclining to red.

H. = 5·0 ... 6·0.

G. = 5·7 ... 7·4.

Colour white or grey : G. under 6·3 or above 7·0.

1. AXOTOMOUS. Prismatic. $\bar{P}r = 51° 20'$. $P + \infty = 122° 26'$.
Cleavage, $P - \infty$. Less distinct, $\breve{P}r = 86° 10'$. Traces of $P + \infty$.
H. = 5·0 ... 5·5.
G. = 7·1 ... 7·4.
Axotomous Arsenical Pyrites. J. ii. 448.

2. PRISMATIC. Prismatic.
Cleavage, $P - \infty$. $(\breve{P}r + \infty)^3 = 111° 53'$.
H. = 5·5 ... 6·0.
G. = 5·7 ... 6·2.
Prismatic Arsenical Pyrites. J. ii. 449.

III. COBALT-PYRITES. Tessular.

Colour white, inclining to steel-grey or red.

H. = 5·5.

G. = 6·1 ... 6·6.

1. Octahedral. Tessular.
   Cleavage, hexahedron, octahedron, dodecahedron, indistinct.
   Colour white, inclining to grey.
   H. = 5·5.
   G. = 6·4 ... 6·6.
   Tin-White Cobalt. J. iii. 452.

2. Hexahedral. Semi-tessular with parallel faces.
   Colour white, inclining to red.
   H. = 5·5.
   G. = 6·1 ... 6·35.
   Silver-White Cobalt. J. ii. 455.

IV. Iron-Pyrites. Tessular, rhombohedral, prismatic.

Colour yellow, sometimes inclining to copper-red.
H. = 3·5 ... 6·5.
G. = 4·4 ... 5·05.

1. Hexahedral. Semi-tessular with parallel faces.
   Cleavage, hexahedron, octahedron.
   Colour bronze-yellow.
   H. = 6·0 ... 6·5.
   G. = 4·9 ... 5·05.
   Hexahedral Iron-Pyrites. J. ii. 457.

2. Prismatic. Prismatic. $\breve{P}r = 114° 19'$.
   Cleavage, $\bar{P}r = 106° 36'$, distinct. Traces of $P + \infty = 98° 13'$.
   Colour bronze-yellow.
   H. = 6·0 ... 6·5.
   G. = 4·65 ... 4·9.
   Prismatic Iron-Pyrites. J. ii. 461.

3. Rhombohedral. Di-rhombohedral.
   Cleavage, $R - \infty$. Less distinct, $P + \infty$.
   Colour bronze-yellow, inclining to copper-red.

H. = 3·5 ... 4·5.
G. = 4·4 ... 4·7.
Rhomboidal Iron-Pyrites or Magnetic Pyrites. J. ii. 465.

V. Copper-Pyrites. Tessular, pyramidal.
Colour brass-yellow, copper-red.
H. = 3·0 ... 4·0.
G. = 4·1 ... 5·1.

1. Octahedral. Tessular.
Cleavage, octahedron, very indistinct.
Colour copper-red.
H. = 3·0.
G. = 4·9 ... 5·1.
Variegated Copper. J. ii. 467.

2. Pyramidal. Hemi-pyramidal with inclined faces. P = 109° 53′, 108° 40′.
Cleavage, P + 1 = 101° 49′, 126° 11′.
Colour brass-yellow.
H. = 3·5 ... 4·0.
G. = 4·1 ... 4·3.
Pyramidal Copper-Pyrites. J. ii. 469.

Cobalt Kies? J. iii. 88.
Nickeliferous Grey Antimony. J. iii. 131.

XI. Order. GLANCE.

I. Copper-Glance. Tessular, prismatic.
Colour blackish lead-grey, steel-grey, black.
Cleavage, indistinct, not axotomous.
H. = 2·5 ... 4·0.
G. = 4·4 ... 5·8.

1. Tetrahedral. Semi-tessular with inclined faces.
Cleavage, octahedron.
Colour steel-grey ... iron-black.
H. = 3·0 ... 4·0.
G. = 4·4 ... 5·2.
Tetrahedral Copper-Glance. J. iii. 1.

2. Prismatoidal. Prismatic.
Cleavage, $\breve{P}r + \infty$.
Colour blackish lead-grey.
Brittle.
H. = 3·0.
G. = 5·7 ... 5·8.
Prismatoidal Copper-Glance. J. iii. 4.

3. Di-Prismatic. Prismatic. $\bar{P}r - 1 = 87° 8'$; $(\bar{P}r + \infty)^3 = 96° 31'$.
Cleavage, $\breve{P}r + \infty$. $\bar{P}r + \infty$. The former rather more distinct.
Colour steel-grey, inclining to lead-grey or iron-black.
Brittle.
H. = 2·5 ... 3·0.
G. = 5·7 ... 5·8.
Bournonite. iii. 5.

4. Prismatic. Prismatic. $\breve{P}r = 119° 35'$; $(\breve{P}r + \infty)^3 = 63° 48'$.
Cleavage, $\breve{P}r$, very imperfect.
Colour blackish lead-grey.
Very sectile.
H. = 2·5 ... 3·0.
G. = 5·5 ... 5·8.
Prismatic Copper-Glance or Vitreous Copper. J. iii. 8.

II. Silver-Glance. Tessular.
Colour blackish lead-grey.
H. = 2·0 ... 2·5.
G. = 6·9 ... 7·2.

1. HEXAHEDRAL. Tessular.
Cleavage, traces of the dodecahedron.
Malleable.
Hexahedral Silver-Glance. J. iii. 11.

III. LEAD-GLANCE. Tessular.
Colour pure lead-grey.
H. = 2·5.
G. = 7·4 ... 7·6.

1. HEXAHEDRAL. Tessular.
Cleavage, hexahedron, perfect.
Hexahedral Galena or Lead-Glance. J. iii. 13.

IV. TELLURIUM-GLANCE. Prismatic.
Colour blackish lead-grey.
Cleavage monotomous, perfect.
H. = 1·0 ... 1·5.
G. = 7·0 ... 7·1.

1. PRISMATIC. Prismatic.
Cleavage, axotomous or prismatoidal.
Prismatic Tellurium-Glance. J. iii. 16.

V. MOLYBDENA-GLANCE. Rhombohedral.
Colour pure lead-grey.
Thin laminæ very flexible.
H. = 1·0 ... 1·5.
G. = 4·4 ... 4·6.

1. RHOMBOHEDRAL. Di-rhombohedral.
Cleavage, R — ∞, perfect.
Rhomboidal Molybdena. J. iii. 18.

VI. BISMUTH-GLANCE. Prismatic.
Colour pure lead-grey.

H. = 2·0 ... 2·5.
G. = 6·1 ... 6·4.

1. Prismatic. Prismatic.
Cleavage, P + ∞ = 90°, nearly; also P̆r + ∞ and P̄r + ∞, one of them highly perfect.
Prismatic Bismuth-Glance. J. iii. 19.

VII. Antimony-Glance. Prismatic.
Colour lead-grey, not blackish, steel-grey.
Cleavage, perfect.
H. = 1·5 ... 2·5.
G. = 4·2 ... 5·8.
G. under 5·3 : H. = 2·0 : thin laminæ not very flexible.
G. above 5·3 : colour steel-grey.

1. Prismatic. Prismatic.
Cleavage, P̆r + ∞, perfect in a high degree. Less apparent, P̄r + ∞.
Colour pure steel-grey.
H. = 1·5 ... 2·0.
G. = 5·7 ... 5·8.
Prismatic Antimony Glance. J. iii. 21.

2. Prismatoidal. Prismatic. P = 109° 16′, 108° 10′, 110° 59′.
Cleavage, P̆r + ∞, highly perfect. Less apparent, P — ∞. P + ∞ = 90° 45′. P̄r + ∞.
Colour lead-grey.
H. = 2·0.
G. = 4·2 ... 4·7.
Grey Antimony. J. iii. 23.

3. Axotomous. Prismatic.
Cleavage, P — ∞, perfect. P + ∞ = 101° 20′; P̆r + ∞.
Colour steel-grey.

H. = 2·0 ... 2·5.
G. = 5·5 ... 5·8.
Jamesonite. iii. 26.

VIII. Melane-Glance. Prismatic.

Colour iron-black.
H. = 2·0 ... 2·5.
G. = 5·9 ... 6·4.

1. Prismatic. Prismatic. $\breve{P}r = 115° 39'$.
Cleavage, $(\breve{P}r + \infty)^3 = 72° 13'$. $\breve{P}r + \infty$. Indistinct.
Prismatic Melane-Glance. J. iii. 27.

Argentiferous Copper-Glance. J.	iii. 73.
Bismuthic Silver? J.	iii. 78.
Cobaltic Galena. J.	iii. 88.
Cupreous Bismuth? J.	iii. 91.
Eucairite.	iii. 94.
Flexible Sulphuret of Silver. Phill.	iii. 30.
Molybdena-Silver. J.	iii. 127.
Native Nickel. J.	iii. 129.
Needle-Ore. J.	iii. 130.
Seleniuret of Copper. Phill.	iii. 150.
Sulphuret of Silver and Antimony. Phill.	iii. 30.
Tennantite.	iii. 161.
Tin-Pyrites. J.	iii. 163.
Yellow Tellurium? J.	iii. 171.

XII. Order. BLENDE.

I. Glance-Blende. Tessular.

Streak green.
H. = 3·5 ... 4·0.
G. = 3·9 ... 4·05.

1. Hexahedral. Tessular.
Cleavage, hexahedron, perfect.
Prismatic Manganese-Blende. J. iii. 31.

II. Garnet-Blende. Tessular.

Streak uncoloured ... reddish-brown.
H. = 3·5 ... 4·0.
G. = 4·0 ... 4·2.

1. Dodecahedral. Semi-tessular with inclined faces.
Cleavage, dodecahedron, highly perfect.
Dodecahedral Zinc-Blende. J. iii. 32.

III. Purple-Blende. Prismatic.

Streak cherry-red.
H. = 1·0 ... 1·5.
G. = 4·5 ... 4·6.

1. Prismatic. Hemi-prismatic.
Cleavage prismatoidal.
Red Antimony. J. iii. 36.

IV. Ruby-Blende. Rhombohedral, prismatic.

Streak red.
H. = 2·0 ... 2·5.
G. = 5·2 ... 8·2.

1. Rhombohedral. Rhombohedral. R = 108° 18′.
Combinations sometimes different on the opposite ends of the crystals.
Cleavage, R.
Streak, cochineal-red.
H. = 2·5.
G. = 5·4 ... 5·9.
Rhomboidal Ruby-Blende or Red Silver. J. iii. 38.

2. HEMI-PRISMATIC. Hemi-prismatic. $P + \infty = 86° 4'$.
Inclination of $P - \infty$ on the acute edge $= 101° 6'$.
Cleavage, $\frac{\frac{3}{4} \check{P}r + 2}{2}$ and $\check{P}r + \infty$, imperfect.
Streak dark cherry-red.
H. = 2·0 ... 2·5.
G. = 5·2 ... 5·4.
Var. of Dark Red Silver. Vulg. iii. 42.

3. PERITOMOUS. Rhombohedral. $R = 71° 47'$.
Cleavage, $R + \infty$, highly perfect.
Streak scarlet-red.
H. = 2·0 ... 2·5.
G. = 6·7 ... 8·2.
Cinnabar. iii. 44.

XIII. ORDER. SULPHUR.

I. SULPHUR. Prismatic.
H. = 1·5 ... 2·5.
G. = 1·9 ... 3·6.

1. PRISMATOIDAL. Prismatic. $\bar{P}r = 83° 37'$. $P + \infty = 117° 49'$.
Cleavage, $\check{P}r + \infty$, eminent.
Streak lemon-yellow.
H. = 1·5 ... 2·0.
G. = 3·4 ... 3·6.
Orpiment. iii. 47.

2. HEMI-PRISMATIC. Hemi-prismatic. $\frac{\check{P}}{2} = 130° 0$.
Inclination = 4° 1′.
Cleavage, $-\frac{\check{P}r}{2} = 66° 44'$. $\check{P}r + \infty$. Less distinct, $P + \infty = 74° 30'$. $\bar{P}r + \infty$. Imperfect.

Streak orange-yellow ... aurora-red.
H. = 1·5 ... 2·0.
G. = 3·5 ... 3·6.
Realgar. iii. 49.

3. Prismatic. Prismatic. P = 106° 38′, 84° 58′, 143° 17′.
Cleavage, P. P + ∞ = 101° 59′. Imperfect.
Streak uncoloured ..., sulphur-yellow.
H. = 1·5 ... 2·5.
G. = 1·9 ... 2·1.
Sulphur. iii. 52.

CHARACTERS

OF THE

GENERA AND SPECIES

OF THE

ORDERS OF CLASS III.

I. ORDER. RESIN.

I. MELICHRONE-RESIN. Pyramidal.

H. = 2·0 ... 2·5.
G. = 1·4 ... 1·6.

1. PYRAMIDAL. Pyramidal. P = 118° 4′, 93° 22′.
Cleavage, P, imperfect.
Mellite. iii. 56.

II. MINERAL-RESIN. Amorphous.

H. = 0·0 ... 2·5.
G. = 0·8 ... 1·2.

1. YELLOW. Solid.
Colour yellow ... white.
Streak uncoloured.

H. = 2·0 ... 2·5.
G. = 1·0 ... 1·1.
Amber. iii. 57.

2. Black. Solid ... fluid.
Colour black, brown, red, grey.
Streak black, brown, yellow, grey.
H. = 0·0 ... 2·0.
G. = 0·8 ... 1·2.
Black Mineral-Resin. J. iii. 59.

Retinite. iii. 146.

II. Order. COAL.

I. Mineral-Coal. Amorphous.
H. = 1·0 ... 2·5.
G. = 1·2 ... 1·5.

1. Bituminous. Colour brown, black.
Lustre resinous.
Odour bituminous.
H. = 1·0 ... 2·5.
G. = 1·2 ... 1·5.
Bituminous Mineral Coal. J. iii. 61.

2. Non-Bituminous. Colour black.
Lustre imperfect metallic.
Odour not bituminous.
H. = 2·0 ... 2·5.
G. = 1·3 ... 1·5.
Anthracite. iii. 64.

Among the remaining minerals contained in the Appendix, Professor Mohs proposes to form two new Orders, which will comprehend nearly the following species :

i. *Order, to be inserted between Kerate and Malachite.*

Black Cobalt-Ochre. J.	iii. 78.
Kupferindig. BREITHAUPT.	iii. 118.
Cupreous Manganese. J.	iii. 92.
Black Wad. J.	ii. 421.
Plumbago *.	ii. 191.
Hisingerite.	iii. 108.
Pyrorthite.	iii. 142.
Chloropal.	iii. 85.
Chlorophæite.	iii. 86.
Iron-sinter. J.	iii. 115.
Chrysocolla *.	ii. 158.
Allophane.	iii. 69.

ii. *Order, to be inserted between Mica and Spar.*

Steatite.	iii. 157.
Agalmatolite.	iii. 100.
Serpentine.	iii. 151.
Fahlunite.	iii. 97.
Gieseckite.	iii. 104.
Pinite.	iii. 139.
Killinite.	iii. 117.
Gibbsite ?	iii. 103.
Marmolite ?	iii. 124.
Picrolite.	iii. 136.
Picrosmine.	iii. 137.
Pyrallolite.	iii. 141.

Many of these substances have been too imperfectly described to enable us to receive them as yet in the system. This is still more the case with the following minerals, some of which, moreover, possess properties apparently so discrepant from any of the orders comprised in the system,

* Hitherto contained in V. and IV. Orders.

that no place can yet be assigned to them with any degree of probability.

Aluminite.	iii. 70.
Aphrite.	iii. 72.
Arsenical Bismuth. WERN.	iii. 74.
Breislakite.	iii. 80.
Hatchetine.	iii. 106.
Humboldtine.	iii. 110.
Leelite.	iii. 119.
Sordawalite.	iii. 155.
Torrelite.	iii. 164.
Turnerite.	iii. 166.
Wagnerite.	iii. 169.
Zurlite.	iii. 176.

END OF THE FIRST VOLUME.