Sampling With Unequal Probabilities to Estimate Cubic-foot Volume Growth on Permanent Sample Plots

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Abstract

The combination of unequal probability sampling and the optical dendrometer makes it possible to eliminate tree-volume tables along with their inherent errors and express the results on an area basis.

Repeated computer drawn samples were used to analyze the sampling method. The results of a field trial were comparable to the results of the repeated computer-drawn samples.

An analysis of repeated optical dendrometer measurements of individual trees over time suggests that upperstem diameter changes can be determined after only two growing seasons and that there should be little difficulty in measuring cubic-foot volume growth on permanent research plots at four to five-year intervals.

An analysis was made of data for sample estimates from 50 repeated computer samples representing four measurement times. The estimated growth varied little from the actual growth.

Introduction

Greater emphasis on the management of second-growth timber in the Douglas-fir region has pointed up a need for more comprehensive measurement of the growth of individual trees in stands. It is imperative to have improved research data to facilitate wise management. As a result, large numbers of permanent sample research plots have been established in this region. For example, the School of Forestry, Oregon State University, has over 70 acres of permanent sample research plots.

Present methods which enumerate each tree and employ some type of tree volume table are time consuming and only as accurate as the volume table used.

Repeated measurements of a permanent research plot over a period of time show the amount of growth made by the trees on the plot. Cubic-foot volume generally is considered to be the best unit of growth measurement for research purposes. In the past, standard volume tables have been adjusted to local conditions through the use of felled-tree measurements. However, it is difficult to completely remove the original bias of the standard volume tables since felled trees are seldom truly representative of the standing trees which are to be measured. Even if the volume tables were accurate, they fail to detect changes in the upper stem form.

An improvement over the felled-tree approach is a local volume table developed by measuring representative trees with a high-performance optical dendrometer. It is difficult to arbitrarily select a truly representative group of trees to be observed. An alternative is to observe each tree in the study area with the dendrometer. However, the procedure is cumbersome and expensive.

Preliminary work done by the School of Forestry, Oregon State University, using the Barr and Stroud optical dendrometer to measure plots that have been thinned as compared to unthinned plots indicate that there are upper stem form differences between thinned and control plots.

It becomes apparent that conventional methods of estimating volumes by diameter, height, and indirect form estimates are not sufficiently accurate to measure upper stem form changes. There is a need to eliminate tree volume tables and their inherent errors. High performance optical dendrometers make it possible to accurately measure upper stem diameters and lengths to the extent that tree-volume tables as such are eliminated. The Barr and Stroud optical dendrometer is an instrument with this capability.

Some type of unequal probability sampling is an efficacious method for sampling permanent research plots. The combination of unequal probability sampling and the optical dendrometer makes it possible to eliminate tree-volume tables along with their inherent errors and express the results on an area basis.

PPS Sampling

Probability proportional to size (PPS) sampling was introduced by Hansen and Hurwitz (1949). Sample selection with variable probabilities was an important development in sampling theory since the errors of estimates depend only on the choice of selection probabilities and not on the population variance. In PPS sampling, a sample of n elements is selected from a population of N elements in such a way that the probability of the i^{th} element being in the sample is proportional to its size. Since it is possible for more than one random integer to fall within a given interval, sampling is with replacement.

Madow (1949) is credited by Hartley (1966) with developing a systematic method for selecting a PPS sample of n elements (with a random start) from a population of N elements so that the probability of the i^{th} element being in the sample is proportional to its size and sampling is without replacement. Hartley (1966) has further altered the procedures so that the estimated values are listed in order of their size.

Hartley (1966) compared the ordered systematic PPS sampling with a random start (without replacement) to PPS sampling with replacement and to systematic PPS sampling with random order (without replacement). He found that the ordered systematic PPS sampling method had greater precision than the other two. This latter method is hereafter referred to as Hartley-PPS sampling.

Simulated Sampling

Computer drawn samples from known populations of Douglas-fir (55 years old) were used to analyze the proposed sampling method. The populations for this study consisted of three young-growth populations containing a total of 190 trees. Dendrometer determined cubic-foot volumes of all trees were known. Therefore, sampling results were checked against known parameters. The computer program accepts data in the form of N ordered pairs (y_i, x_i) where y_i represents the measured cubic-foot volume of the i^{th} tree and x_i is the estimated volume of the i^{th} tree. In this study tarif table values (Turnbull, et al., 1963) were used as the x-values.

The estimated total volume $(\hat{T}_y)$ and its variance are calculated as follows:

$$\hat{T}_y = \frac{T_x}{n} \sum_{i=1}^{n} y_i/x_i$$

where $T_x = \sum_{i=1}^{N} x_i$

Table 1 presents statistics for sample size and mean standard error of mean (percent) for 15 computer runs for 100 repeated Hartley-PPS samples. The sampling ratio (n/N) varied from 0.105 for the combined young-growth populations to 0.385 for population A. A comparison of t_x/T_x [1] to n/N for the 15 computer runs shows that this ratio varied from 1.229 to 1.453 and illustrates the effects of unequal probability sampling. A comparison of the average standard error of the mean in percent for the 15 computer runs of the 100 repeated samples for Hartley-PPS from Table 1 suggests that for the populations studied, the Hartley-PPS sampling method makes it feasible to sample permanent research plots.

TABLE 1

STATISTICS FOR SAMPLE SIZE AND STANDARD ERROR OF MEAN (PERCENT) FOR 100 REPEATED COMPUTER HARTLEY-PPS SAMPLES.

Population	x	n	n/N	t_x/T_x	$S_R\%$
A 1/	Tarif	20	.385	.481	1.72
B 2/	Tarif	20	.322	.449	1.76
C 3/	Tarif	20	.263	.342	1.84
A,B,C	Tarif	20	.105	.141	2.36
A,B,C,	Tarif	35	.184	.246	1.66
A,B,C	Tarif	50	.263	.352	1.27

1/ Population A contains 52 trees representing unthinned stands.
2/ Population B contains 62 trees representing lightly thinned stands.
3/ Population C contains 76 trees representing heavily thinned stands.

Field Trial

A field trial was made on a one-acre plot in the Black Rock Forest Management Research Area. The plot contained 146 Douglas-fir trees which had been thinned several times. The sample trees were measured with the Barr and Stroud optical dendrometer.

TABLE 2

RESULTS OF FIELD TRIAL

n	T_y	$\frac{\hat{V}(\hat{T}_y)}{\hat{T}^2}$	$S_{\bar{R}}$ %	C	n/N	t_x/T_x
40	5,973	.000139	1.18	9.9	.27	.41

A comparison of the standard error of mean in percent in Table 2 and in Table 1 shows that the field trial results are well within the range suggested by the computer trials. These results substantiate the feasibility of estimating cubic-foot volume on permanent sample plots with the Hartley-PPS sampling method. The magnitude of the standard error of the mean ratio in percent $S_{\bar{R}}$ % particularly indicates the potential of sampling permanent research plots.

Determination of Cubic-Foot Volume Growth

The cubic-foot volume growth for a given plot is determined as the difference between the estimated volume at the beginning of the growth period and the estimated volume at the end of the growth period.

The estimator of total volume at time one is:

$$\hat{T}_{y_{a2}} = T_{x1}/n_1 \sum_{i=1}^{n_1} y_{i1}/x_{i1}$$

and the estimator of total volume at time two is:

$$\hat{T}_{y_{a2}} = T_{x2}/n_2 \sum_{i=1}^{n_2} y_{i2}/x_{i2}$$

where subscripts 1 and 2 refer to times one and two respectively and y_i is the dendrometer-determined cubic-foot volume of the i^{th} tree.

The estimated cubic-foot volume growth for a given period is

$$\hat{T}_{y_{a2}} - \hat{T}_{y_{a1}} \text{ or } G.$$

Table 3 presents data for sample estimates for 50 repeated computer Hartley-PPS samples over time. The data are from a one-acre plot in the Black Rock Forest Management Research Area. The plot was established in 1953 and thinnings were made in 1957, 1963 and 1965. Tarif table volumes were used for the x-values and Black Rock local volume table volumes were used for the y-values.[1] The total number of trees was reduced from 454 in 1953 to 184 in 1965. The sample was selected at the time of the initial measurement. At subsequent measurement times, T_x and the sample y_i/x_i ratios were updated to reflect the trees removed during the measurement period and the changes in those x and y-values. Table 3 shows that there is little difference between the total estimated cubic-foot volume ($\hat{T}_y$) for each measurement period for the 50 repeated computer trials. For the Hartley-PPS samples, the mean sample size varies from 60.0 to 30.98 while t_x/T_x varies only from 0.253 to 0.261.

The simulated relative variances of the estimated cubic-foot volume growth for the periods 1957-1963 and 1963-1965 are less than the simulated relative variances of the estimated total volume for 1957, 1963 and 1965 for the 50 repeated computer trials. The simulated expected estimated relative variance of the estimated total volume was highest in 1953 and lowest in 1963. The anomaly that the lowest relative variance occurred after repeated thinnings appears to be explained by the trend in population variance of the y_i/x_i ratios

$$(\sigma^2_{y_i/x_i})$$

which varies from 0.173 to 0.074 for 1953 to 1965, respectively. This implies that as the stand was thinned the population of y_i/x_i ratios became more uniform.

1/ These measurements were made prior to the availability of the Barr and Stroud optical dendrometer.

TABLE 3

SAMPLE ESTIMATES FOR 50 REPEATED COMPUTER HARTLEY-PPS SAMPLES OVER TIME

	1953	(57-53)	1957	(63-57)	1963	(65-63)	1965
T_x	6695.2		5842.3		6745.8		6166.9
T_y	6042.4		5376.7		6708.3		6299.9
$\hat{T}_y$ 1/	6030.2		5383.0		6727.0		6319.3
$\overline{G}$		-647.2		1,344.0		-407.7	
t_x/T_x 1/	0.257		0.253		0.259		0.261
$\frac{E_e[\hat{V}(\hat{T}_y)]}{\hat{T}_y^2}$ 2/	0.000292		0.000170		0.000133		0.000169
$\frac{V_e(\hat{T}_y)}{\hat{T}_y^2}$ 3/	0.000078		0.000252		0.000237		0.000244
$\frac{V_e(\overline{G})}{\overline{G}^2}$		0.000324		0.000051		0.000205	
N	454		259	218		184	
$\overline{n}$	60.00		41.62		36.68		30.98
$\sigma^2 y_i/x_i$	0.173		0.104		0.082		0.074

1/ Mean for the 50 computer trials.

2/ Simulated expected estimated relative variance of the estimated total volume.

3/ Simulated relative variance of the estimated total volume.

It is recommended that a new sample be selected should additional thinnings reduce the sample size to the point where the estimated variances are unacceptable.

In actual practice each sample tree remaining at the time of remeasurement would have its upper stem diameters and lengths remeasured with a high performance optical dendrometer.

Conclusion

This study has shown that it is highly efficient to estimate cubic-foot volume growth on long-term research plots by using a combination of repeated unequal probability sampling and repeated upperstem diameter and length measurements made with a high performance optical dendrometer. These direct tree measurements make it possible to eliminate tree volume tables and their inherent errors.

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