TEACHING GROWTH MODEL CONSTRUCTION

A CASE STUDY

by

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ABSTRACT

Types of forest growth models including distance independent, distance dependent, and whole stand are reviewed.

Various methods of growth model construction and validation are illustrated in a case study approach based on the author's experience in teaching elements of growth modelling to senior undergraduate forestry students at the University of British Columbia. Particular emphasis is directed to attaching biological significance to equation coefficients using as a working example the Chapman-Richards Generalization of von Bertalanffy's growth model.

Examples of student work are presented and discussed.

Keywords: growth models, teaching, case study, undergraduate.

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I N T R O D U C T I O N

Growth models originally based on graphical descriptions and interpretations have been used by foresters since late in the 19th Century. More recently however, mathematical growth models have been used. With the advent of modern computers and developments in regression analysis, multivariate analysis, and other statistical methods, growth modelling has become a separate discipline in the general area of growth and yield. This meeting of Section S.4.01-00 in Vienna and recent meetings elsewhere are evidence of the importance of the subject.

As more scientists became involved in growth modelling, a variety of models began to appear. Many of these were developed in response to needs ranging from operations to research to long-term planning. They often incorporated the biases and idiosyncracies of their authors. Indeed, I have heard it said that models and modellers are like people and their dogs: you need only see one in order to recognize the other!

Now that their usefulness has been proven, there has begun to develop a demand for specialized or localized growth models which are useful in day to day forest management. It was a natural development therefore that many forestry schools would encourage their senior students to become familiar with growth modelling and to offer one or more courses in their development. The purpose of this paper is to illustrate the essential elements and concepts of growth model design that I believe should be incorporated in any elementary university growth modelling course. These are illustrated in this paper by way of a case study based on my more than 20 years experience in teaching growth model design and construction to students in the Faculty of Forestry at the University of British Columbia, Vancouver,B.C. Canada.

STUDENT PREPARATION AND BACKGROUND

Students taking this course are in their final year of a five-year university level undergraduate programme which leads to the degree of Bachelor of Science in Forestry. Prior to enrollment they have had university courses in calculus, including derivatives and antiderivatives of elementary functions, derivative applications, graphing, maximum-minimum problems, and growth decay problems; anti-differentiation, integration techniques, definite integrals and applications, series and Taylor expansions for elementary functions. They have also had numerous courses common to most forestry curricula, such as dendrology, surveying, economics, climatology, ecology, wood anatomy, soils, silviculture, forest mensuration, etc. In addition they have an excellent grounding in elementary statistical techniques and are proficient in at least one high level computer language and have considerable experience with real time computing through conversational terminals.

Almost all the students have had at least two or three summers of practical experience in field forestry and take the course as an elective rather than a required subject. Average yearly enrollment is approximately 12 students.

TYPES OF GROWTH MODELS

It is important for students to recognize that the type of growth model designed will depend on its ultimate objective. With rare exceptions, all growth models have a common objective; that is to produce at some point or points in simulated time, summary tables which indicate the state of a forest stand on a per plot or a per hectare basis in much the same manner as do conventional inventory statistics. Such figures as volume, basal area and number of stems per unit area are common to all models. Smith and Williams (1980) have suggested that the structure of a model be formally defined as

$$X(t + 1) = M\{X(t)\}$$

where X is a set of stand variables at time t and M represents the functional relationships (e.g. growth, competition, etc.) incorporated in the model.

They suggest that the stand variable X can generally be partitioned into decision variables, those which are controlled directly; and state variables, those which fluctuate in response to decisions or remain constant but uncontrolled. Typical state variables are site, density, age, or average height. Typical decision variables are management actions such as harvesting, spacing or fertilizing.

State and decision variables may be classified as continuous or discrete. They may or may not be stochastic. The nature of the variables, the way in which they interact and the way in which the interaction of components behaves, constitutes what is identified as the structure of the model.

Growth models can be classified according to their state variables. I first suggested a model classification system at a meeting of this group in Vancouver in 1973 (Munro, 1974). There I proposed to distinguish models on the basis of two features: intertree dependency status and primary unit parameter requirements.

Three modelling "philosophies" were identified to incorporate these two features. The first philosophy assumes that the primary unit of stand modelling is the single tree and that intertree distance is a necessary parameter. Such a philosophy dictates that each tree be located in the model by a stacial coordinate system. Simulation with real data requires stem charts. The second philosophy assumes that the primary unit of stand modelling is the single tree, but that intertree distance is not a necessary parameter. Simulation with real data does not require stem charts. The third philosophy assumes that the primary unit of stand modelling is the stand and that information concerning individual trees is not necessary. Simulation with real data requires neither stem charts nor single tree information. In 1980, Smith and Williams extended this classification (figure 1) to provide additional information about distance independent whole stand models.

single tree / distance dependent
single tree \ distance independent

whole stand - distance independent / *diameter free*
whole stand - distance independent - *diameter function*
whole stand - distance independent \ *diameter class* — *fixed dbh class*
diameter class \ *fixed cohort*

Figure 1. Three basic forest stand modelling philosophies (MUNRO, 1974).
(italicized print are Smith and Williams (1980) extensions).

Their extension differentiates between stand models on the basis of level of aggregation (single tree or whole stand), spatial information requirements (distance dependent or distance independent) and representation of stem dimension (diameter list, diameter distribution function, diameter class or no diameter representation). They suggest that the classification system can be extended if other state variables are considered such as crown measurements, height measurements, or, in extreme cases, individual branch measurements.

In their paper Smith and Williams summarize and classify 26 growth models published between 1964 and 1980. Their paper is required introductory reading for all students taking this course.

Regardless of the type of structure of the model, it can be designed or cast in a particularly identifiable mathematical structure. For example, if it is cast in a mathematical structure that makes it amenable to manipulation with algebra or calculus it is said to be analytically tractable. Analytically tractable models lead to decision problems which may be often solved with classical optimization techniques. When a model structure does not conform with any readily identifiable classical mathematical structure it may only be analysed with repeated experiments of the "what if" kind.

Equations used in the development of the model may be selected so that their parameters have particularly significant biological meaning or they may be selected with little or no thought given to biologically meaningful parameters. Examples of the former are models of Pienaar and Turnbull (1973), Clutter (1963), and Bailley and Ware (1982). Examples of the latter are models of Newnham (1964), Hegyi (1974), and Lin (1974).

It is vitally important that all students learning the elements of growth modelling be aware of the different classifications, structures, and mathematical approaches. They must also be acutely aware of the advantages, disadvantages, and limitations of particular model types. John Moser (1980) recently prepared an excellent paper on the history of the development of modern growth and yield theory. All students learning to model should read and be familiar with John's paper.

C L A S S   A S S I G N M E N T S

I am thoroughly convinced that the only way to learn to growth model is to practice, starting with the most simple kind and progressing as ability improves to more complicated models. I have found it is very useful to have available a sophisticated interactive growth model with which to play. The combined activities of building elementary growth models and at the same time having the opportunity to carry out experiments on complicated models provides an incentive for students to work quickly and efficiently in developing their own models.

I have developed a series of eight assignments beginning with simple, elementary diameter growth equations and graphs, progressing through simple whole stand models whose coefficients have no biological meaning to an elementary model based on the Chapman-Richards generalization of von Bertalanffy's equation (Pienaar and Turnbull, 1973) to which a biological meaning can be attached to each coefficient, to an academic model (Goulding, 1972) developed at the University of British Columbia. I follow this with exercises in the development of competition indices which are an essential component of all distance dependent models. In the remainder of this paper I set out briefly the nature and purpose of these assignments.

For those who are particularly interested in the details of the eight assignments they are available upon request in their entirety.

Assignment #1. DBH Increment Estimation.

The purpose of this assignment is twofold:

a. To give the students practice in graphing and mathematically fitting DBH increment functions, and

b. To emphasize the importance of matching the data and the equation developed with the purpose or objective of the function.

Two sets of data are used. The first set is essentially a stand table from a 30-year-old stand of western hemlock showing the number of trees per hectare in each diameter class and in addition, average ten-year diameter increment at breast height for each diameter class.

The second set of data is from a normal yield table for western hemlock (Barnes, 1962). It shows for ages from 30 to 120 years average diameters for 10 year intervals. For each data set students are asked to graph the relationship between average DBH increment and average DBH (figure 2).

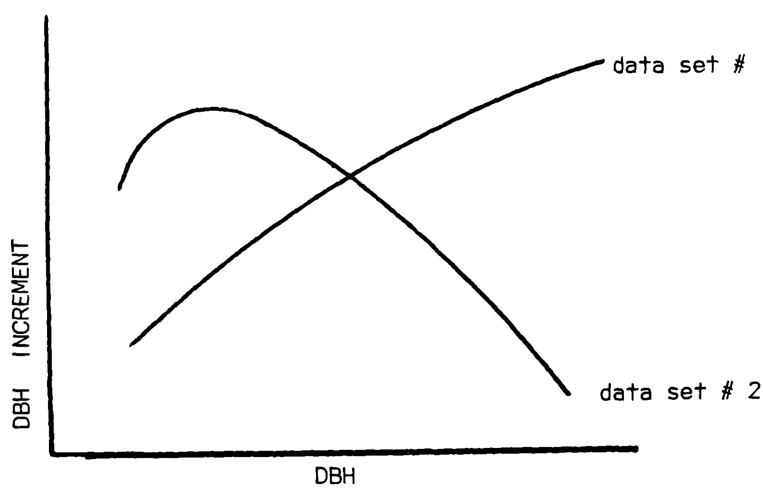

Figure 2. Graphs of average DBH increment for two data sets.

They are asked to explain why the two curves are different in shape and slope. The explanation of course, is that one of the data sets is confounded with time. This exercise is fundamental in emphasizing to the students the importance of knowing the data base, the collection method, and the objective and application of the increment function they develop.

Assignment #2. Development of Simple Yield Models.

This is the first assignment which involves the actual development of a very simple growth and yield model. It is primarily an exercise in mathematical curve fitting to assure myself and the students that they have the ability to manipulate mathematical equations sufficiently well to be able to continue with model development. In this assignment the simple mechanics of initializing, predicting and updating are introduced (figure 3). As background the concept of compatibility (Clutter, 1963) is introduced and the alternatives of developing a cumulative equation which can be differentiated to give increment or a difference equation which can be integrated to give yield are examined. The data set is from normal yield tables for Douglas-fir (McArdle & Meyer and Bruce, 1949). Most students have little difficulty in developing an equation model to duplicate these tables. They are encouraged to use an incrementing model rather than a cumulative model, primarily because the incrementing or differential system is easier to develop.

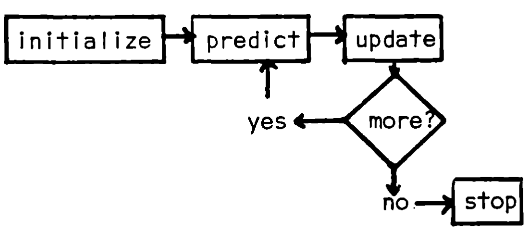

Figure 3. Basic looping for growth models.

Assignment #3. Elementary Interactive Yield Model.

This is a relatively short assignment. Its purpose is to make the model developed in Assignment #2 interactive so that it can be stopped and started at various stages during the growing cycle and to make provision for elementary and arbitrary changing of the state variable values to simulate thinning. Assignment 3 is based on the flow chart in figure 4.

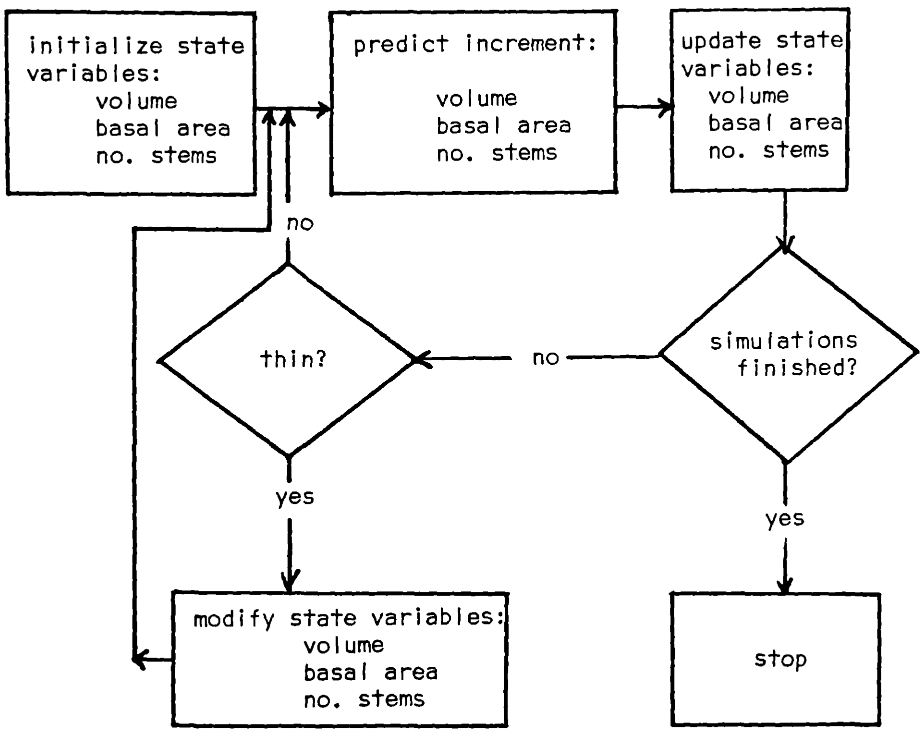

Figure 4. Flow chart for simple interactive growth models.

Following completion of assignment 3, the students are usually completely engrossed in the concept of designing a more complicated growth model and, in fact, usually begin to work ahead on their own through the remainder of the assignments. The excitement which comes from developing your own interactive model with which to test your ideas and concepts about forest growth and management never fails to excite the enthusiasm.

Assignment #4. B.C. Forest Service - VAC model

This assignment introduces equations with coefficients to which a biological meaning can be attached. A good function is the Chapman-Richards generalization of von Bertalanffy's growth model (Pienaar and Turnbull, 1973) which takes the form:

$$V = b_1 \left\{ 1 - e^{b_2 (t - b_4)} \right\}^{b_3}$$

where "V" = volume at time "t" and b_1, b_2, b_3 and b_4 are coefficients. Included with this assignment is a working example of an interactive growth model which can be used for practise. The students are encouraged to modify it and to criticize it technically for its lack of biological realism. Here b_1 represents the maximum volume per hectare that can be attained on the site, b_2 governs the general shape of the curve, b_3 shortens or lengthens the time necessary for the curve to culminate and b_4 is a scaling value. In the absence of data b_3 can be assumed to be 3, the value originally assigned by von Bertalanffy. An interesting property of the Chapman-Richards function is that the culmination of current annual increment can be shown to occur at an age (time) equal to $b_2/\ln|b_3|$ (b_2 divided by the absolute value of the natural logarithm of b_3). Once the students have grasped this concept they begin to develop an understanding and enthusiasm for developing models without an extensive data base from which to work. For example, if one has some silvicultural and ecological familiarity with the species,one can estimate the maximum volume per hectare (b_1), assume the b_3 value to be 3, and calculate b_2 from the formula

$$b_2 = t_i \, (\ln |b_3|)$$

where t_i= age at which current annual increment culminates assuming b_4 is zero. Students particularly enjoy working with this function. They can test the response of various parameters and build growth models in mathematical terms in much the same way the ecologist talks about growth in descriptive terms.

Following the completion of assignment 4, the students have an elementary grasp of the principles of model building. They also have a basic understanding of the advantages and disadvantages of various approaches and, more importantly, have lost any fears they may have harboured about the subject.

Assignment #5. B.C. Forest Service - Variable Density Yield Projection

This short assignment requires a critique of a suggested yield projection system prepared by the Ministry of Forests in the Province of British Columbia. It is a logical exercise at this point because the system suggested by the Ministry of Forests is based on the Chapman-Richards function. In several instances the coefficients do not appear to be biological meaningful. The students enjoy this excercise, especially when they find what appear to be inconsistencies in the variable parameters through sites, ages and species. The assignment gives me a chance to assess their ability to write reports and also to assess their technical ability to constructively criticize and examine an operationally used growth model.

Assignment #6. Experiment with Stand Model TOPSY

By the time assignment 6 is reached according to the normal class schedule, most students have had considerable experience dealing with a detailed large and complicated stand model. The one I use, TOPSY, was written by Goulding in 1972. It is a distance independent individual tree model based on a large sample of permanent sample plot information and was expressly designed to carry out thinning experiments in natural stands of Douglas-fir in British Columbia and Washington. It involves a considerable level of detailed output including individual diameter lists, volumes, basal areas, tree sizes, and mean annual increments at various ages. In addition, it allows for various thinning types and cycles. It resembles a natural forest in that it has stochastic elements. This means the students are not able to duplicate results, either for themselves or for each other! The main usefulness of this excercise is to give the students a chance to carry out experiments and bring their silvicultural experience and ecological experience to bear in criticising the responses to thinning regimes that are produced by the model. It also gives them a chance to work with output from a more complicated model which requires more planning and assessment time to use efficiently. Most students spend 8 or 10 hours at a computer terminal to produce 25 or 30 simulations.

Assignment #7. Competition Indices

Students are now ready to begin to consider the essential elements involved in building a single tree distance dependent model. Although they will not progress to the stage of actually building one, they will consider the various components which are required. In doing this assignment they will become familiar with the work of Newnham (1964), Hegyi (1974), Mitchell (1969) and others who pioneered growth model construction. The importance of competition indices, the different parameters incorporated in them and the methods in which they can be incorporated into models are stressed. Each student is encouraged to do library research and all are required to prepare a paper on the subject.

Assignment #8. Parameter Estimation for Common Growth Model Functions

To conclude the formal assignments, students are to assess several functions which possess parameters to which some biological meaning can be attached. In addition to the Chapman-Richards, the Weibull (Clutter and Allison, 1974) and various forms of the e^{-k} family of functions are examined. The response of each function to changing parameter values is examined with the aid of computer graphics. A formal report with emphasis on biological interpretation is required.

CONCLUSION

At the beginning of the course the students are provided with data from a series of remeasured permanent sample plots. The number of plots totals over 100, most of which have been remeasured two or three times. The are told at the beginning of the term that they will have to design an interactive model which is capable of simulating natural and managed stand growth based on the records of these permanent sample plots. They are given no further guidance but are expected to make use of all the tools and techniques that they have learned and come in contact with during the course. A concluding term project must be presented in the form of a working computerized growth model. This must be documented and presented in report form and in addition be available in computer files for me to check personally. Each year the complexity and detail that students are able to incorporate in their own growth models increases. Always, they are able to demonstrate a high level of competence in growth model development. Hopefully they leave the course as good will ambassadors for the general field of growth modelling.

They take with them the computer programmes that they have developed and others which have been made available to them. Many find permanent work in situations where they have access to electronic computing facilities and continue to work on projects they started during this course. Others work in unrelated areas. Regardless of their employment however, they all share the knowledge that todays growth model is simply a modern version of a yield table.

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