STAND GROWTH AS A SYSTEM OF ACCUMULATION AND CIRCULATION

(Approaches Using Causal-Orientated Models)

by

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ABSTRACT

Mathmatical models are very important in forest research for the development of yield tables, for precalculations etc. In this paper we will show, how causal orientated growth models can be used - by means of mathematical as well as of empirical analysis - to get an idea of the structure of the dynamics behind the growth process.
First we will formulate a simple model of accumalation and circulation as the main reasons for growth of whole stands and of single trees. We will give some heuristic interpretation of the model parameters and the used differential equations, with the requirement that these parameters should be directly measurable. A partial model for thinning will be added to the basic model. Examples show that the models are reasonable but have to be modified. As a further extension we will take into account the process of nutrient intake by the rootsystem, so that the model gets a spatial structure.

ZUSAMMENFASSUNG
Waldwachstum als ein Akkumulations und Kreislaufsystem

Mathematische Modelle sind ein für die forstliche Forschung sehr bedeutsames Handwerkzeug, insb. bei der Erstellung von Ertragstafeln, zu Prognosezwecken etc. In diesem Paper soll veranschaulicht werden, wie durch kausale Modellansätze die Struktur der Wachstumsdynamik sowohl mathematisch-analytisch als auch empirisch untersucht werden kann.
Zunächst wird ein einfaches Akkumulations- und Kreislauf- modell sowohl für das Bestandes- als auch für das Einzelbaum- wachstum erläutert. Es werden anschauliche Interpretationen für die Modellparameter und die verwendeten Differential- gleichungen gegeben. Die direkte Meßbarkeit der Parameter wäre für Anwendungen erforderlich. Das Basismodell wird um einen "Durchforstungsterm" erweitert. An Beispielen wird ver- deutlicht, daß die Modellansätze sinnvoll sind, aber modifiziert werden müssen. Danach wird der Prozeß der Nährstoffversorgung über das Wurzelsystem mit einbezogen. Das Modell erhält dadurch einen "räumlichen" Charakter.

1. INTRODUCTION

In today's literature we find quite a number of attemtps to approach growth models for forestry stands and for their individual trees from a causal point of view. We shall mention here, amongst others, the works of KILMI (1967), of TAKEUCHI (1981) and of HOZUMI (1980), but also the allready classical work of MITSCHERLICH (1919) concerning some "laws of plant growth". Classical models concerning crop yields are not primarily orientated in this way, but rather, because of the specific problems they deal with, they are interested in adaption graphs by means of which data series (e.g. dia-meters, heigts, etc.) can be fitted. Approaching the modelling of growth processes causally means first of all to deal with the growth processes themselves, after which mathematical models in the form of differential or difference equations are derived. It is therfore natural that the **processes** themselves are reflected quantitatively in the mo-del parameters. This makes it easy to react to changes of these entities in the real system from inside the model. A primarily causal approach would therefore be of great importance for forestry, because it is there that the growth processes we want to observe occupy long ranges of time of up to a hundred years and more. Changes of causalities, as a result of the various natural phases that mark the growth of trees, or as a result of environmental changes because of pollution etc., play an important role and therefore ought to be incorporated into any models yet to be developed.

2. CAUSAL MODELLING WITH REGARD TO A MODEL FOREST

We shall attempt to develop a causally orientated model for growth processes in stands taken from a "model forest" which is defined in its behaviour by certain postulates, in such a way that the models which are to be discussed may be derived from them in a "natural" manner.

190

<u>Postulates</u> <u>concerning</u> <u>general</u> <u>growth</u> <u>behaviour</u>

1. The stand consists of a totality of individuals.

2. Each individual consists of three functional components:

   a) Photosyntesis apparatus (leaves)
   b) Accumulation apparatus (stem, branches)
   c) Apparatus for taking in nutrient (roots)

3. In all of the functinal components, the respective processes behave "proportionally", which make it sufficient to be acquainted quantitatively with one process only.

4. The individual tree accumulates various nutritional substances in fixed proportions.

5. Part of the substances (or substance conglomerates) accumulated is returned to the environment (soil etc.) by the individual tree.

In the long run the circuit system defined by postulate 5. leads to a certain kind of stability, as can be observed for instance in natural forests.

<u>Special</u> <u>postulates</u> <u>concerning</u> <u>growth</u> <u>dynamics</u>

6. At a time t, the stand takes in a quantity a(t) of a substance or conglomerate per unit time.

7. Of this quantity, a(t), an individual tree receives a proportional share.

8. The individual tree returns to the soil a certain percentage b(t) of the substance or conglomerat accumulated.

In a real forest, of course, these postulates will never be fulfilled with such rigidity. In order to use them in a concrete application, those parts of the model implied by the postulates would have to be changed or adapted accordingly. For forming a causal model, however, the assumptions about the real system that formally determine the causality are indispensible.

3. MODEL CONSTRUCTION

Postulates 6., 7., and 8. in particular, are suited for the construction of the following two differential equations for stand growth and for individual tree growth:

(stand) $$\frac{\partial G(t)}{\partial t} = a(t)-b(t)G(t)$$

(individual i) $$\frac{\partial F_i(t)}{\partial t} = \frac{a(t)F_i(t)}{\sum_j F_j(t)} - b(t)F_i(t)$$

Because $G(t) = \sum_j F_j(t)$ we arrive at the two equations
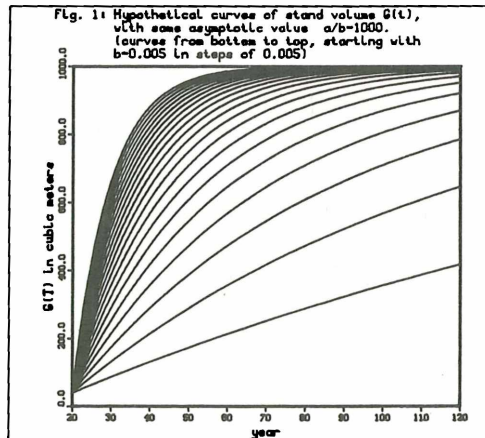
$$\frac{\partial G(t)}{\partial t} = a(t)-b(t)G(t)$$

$$\frac{\partial F_i(t)}{\partial t} = [\frac{a(t)}{G(t)}-b(t)]F_i(t)$$

These two linear differential equations of first order have the following analytic solutions if $a(t)=a$ and $b(t)=b$

$$G(t) = \frac{a}{b} - e^{-b(t-t_o)}[\frac{a}{b} - G(t_o)]$$

$$F_i(t) = \frac{F_i(t_o)}{G(t_o)} G(t)$$

G(t) approximates asymptotically the constant   a/b. Fig. 1 presents some of the solutions for G(t) for a/b=1000. and various values of a corresponding to given values of b (G(20)=40.).

Fig. 1: Hypothetical curves of stand volume G(t), with some asymptotic value a/b=1000. (curves from bottom to top, starting with b=0.005 in steps of 0.005)

As we will see in fig. 2, stand growth will behave in a man-

ner similar to that of G(t). The solutions for volume growth
of individual trees, however, do not correspond to obser-
vations. They grow nearly exponentially, an effect which we
will try to explain now.

4. INTRODUCTION OF A THINNING TERM

Indeed not all single trees continue to grow indefinitely,
but an elimination of trees takes place, partly by natural
death, partly by thinnings etc. Therefore we now introduce
into the stand equation a thinning term D(t):

$$\frac{\partial G(t)}{\partial t} = a - D(t) - bG(t)$$

The number of stems in a stand frequently develops in a
negative-exponential manner, i.e.

$$N(t) = Ae^{-Bt} \qquad \frac{\partial N(t)}{\partial t} = -BN(t)$$

where N(t) is the number of stems at time t, with A
corresponding to the number of stems at time t=0. B
corresponds to the proportion of N(t) taken away per unit
time in the course of the "thinning".

Now think of a steady thinning process where medium-sized
trees of volume G(t)/N(t) are removed. This means that per
unit time a quantity of accumulated substance

$$\frac{G(t)}{N(t)} BN(t)$$

is taken from the total stand. If we make the additional
assumption that smaller trees are eliminated, rather than
medium sized trees, one obtains

$$D(t) \quad c(t)BG(t) \quad \text{where} \quad c(t) < 1$$

Let us again suppose that a(t)=a, b(t)=b and c(t)=c are
constant with respect to time. We then get the differential
equations

$$\frac{\partial G(t)}{\partial t} = a - [cB+b]G(t)$$

$$\frac{\partial F_i(t)}{\partial t} = [\frac{a}{G(t)} - b]F_i(t)$$

and the corresponding analytic solutions

$$G(t) = \frac{a}{cB+b} - e^{-(cb+b)(t-t_o)}[\frac{a}{cB+b} - G(t_o)]$$

$$F_i(t) = \frac{F_i(t_o)}{G(t_o)} G(t)e^{cB(t-t_o)}$$

Thus the solution of G(t) has not changed its form. But $F_i(t)$ now grows exponentially for large values of t - a phenomenon that could be observed in nature, if the trees did not age.

In order to visualize the theoretical results, we have chosen the following parameter values for model graphs, on the basis of WIEDEMANN's yield tables (fir, moderate thinning, 1st yield class) (1)

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    a     25.
    b     0.08 to 0.005 (from bottom to top
                         in steps of 0.01, last step 0.005)
    c     0.7
    B     0.02
 G(20)    40.
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Fig. 2: Hypothetical curves of stand volume in the case of thinning, with 9 different parameter sets, and 3 volume curves from yield tables (WIEDEMANN and GUTTENBERG)
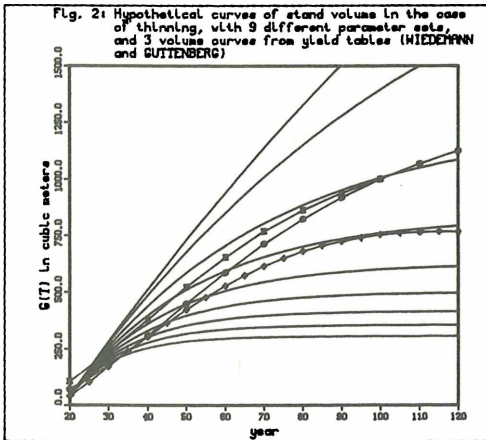

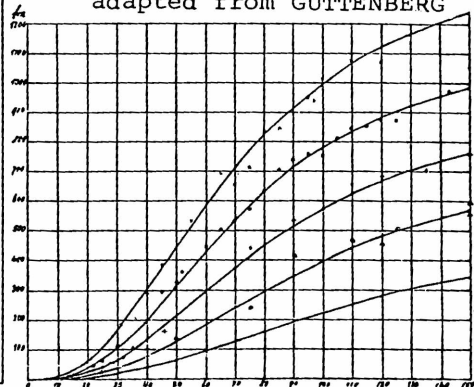Fig. 3: stand model curves adapted from GUTTENBERG

Fig. 5. Holzmassen pro Hektar.

The bottom marked curve in fig. 2 depicts the yield table values of cubic metre volume of the remaining stand from the yield table mentioned for b=0.03. The model graph does indeed approximate asymptotically the yield table values. The other two marked curves refer to stand values from GUTTENBERG

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    1) in: SCHOBER (1975)
194

(1915) (2). Fig. 3 shows model curves used by GUTTENBERG.

In fig. 4 and 5, three unmarked single tree model curves are shown in each figure in comparision to the empirical data (marked curves). The four empirical sets of data in fig. 4 are from stem analyses by GUTTENBERG (1915) (3). The two empirical sets of data in fig. 5 were taken by ATHARI (1980) from a yield category similar to the yield table mentioned above.
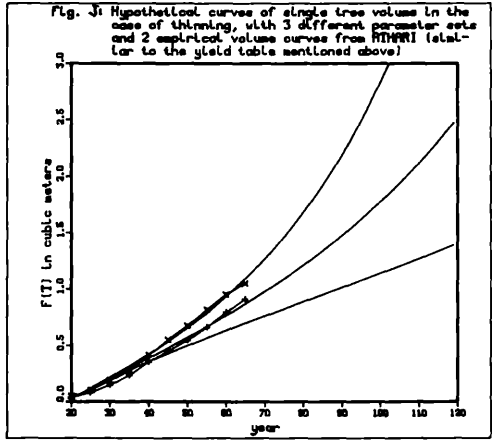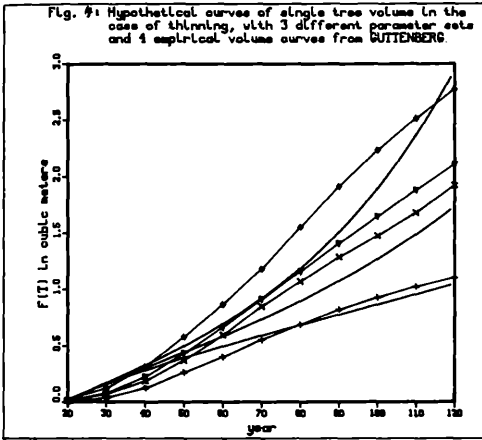
From all comparisons carried out so far it becomes clear that in the first few years the model curves in question are steeper than the empirical ones. To obtain satisfactory models, the parameters a,b and c should probably not be handled as constants but rather as functions of time. Possible reasons for that would for instance be a not yet fully developed root system (intake rate a(t) would initially be a monotonically growing function) and a high crown-percentage (high "return percentage", decreasing) in young stands etc.

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2) pp. 45
3) p. 85 (Beilage 2)

5. SUGGESTION FOR ROOT SYSTEM MODELLING

Finally we shall therfore put forward some ideas, modelling the developement of the root volume as intake apparatus. Let $x \in X$ be a point in the root space X of a certain stand. Call $f_x^i(t)$ the amount of roots per unit volume of soil that the i-th tree in x has at its disposal. Let $g_x(t)$ be the corresponding total amount of root apparatus of the stand at x. Furthermore let the development of the roots of the i-th tree at x be proportional to size $F_i(t)$ of the i-th tree and, for example, reversly proportional to the square of the distance $r_i(x)$ between x and the i-th tree. We now want to model the development of the total amount of root material g_x (t) as a function of root density, thus for example arriving at a system of four differential equations of the following kind:

$$\frac{\partial g_x(t)}{\partial t} = [a_x - b_x g_x(t)] g_x(t)$$

$$\frac{\partial f_x^i(t)}{\partial t} = \left[\frac{a_x g_x(t) \dfrac{F_i(t)}{r_i(x)^2}}{\sum_j \dfrac{F_j(t)}{r_j(x)^2} f_x^j(t)} - b_x g_x(t) \right] \cdot f_x^i(t)$$

$$\frac{\partial F_i(t)}{\partial t} = \int_X f_x^i(t) dx - b F_i(t)$$

$$\frac{\partial G(t)}{\partial t} = \int_X g_x(t) dx - b G(t)$$

Supposing that root turnover takes place very fast compared to the developement of the volumes $F_i(t)$ of individual trees (in which case the $F_i(t)$ may be considered constant provided that the time intervalls are sufficiently small). The second equation, for instance, leads us to realize that individual

196

trees have a tendency to form well-defined disjunct "root
districts". Unfortunately we do not up til now know of any
such root system models and hence lack appropriate data. This
is why the suggestions we have put forward here should be
seen as a stimulus for further discussion.

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