

## DETERMINATION OF STEM PROFILES IN PICEA ABIES

by

JESPER STAHL MADSEN  
(abstract)

The Danish Land Development Service leads an investigation of production in second generation Picea Abies on the moorland and in this connection measurements of the form (profile) of 1014 trees have been taken. These trees have been selected out of 169 stands representing different ages and various yield classes.

These data have been dealt with mathematically/statistically in order to find an expression which can describe the profile of a single tree when only a few data of the stand and tree are available.

Orthogonal polynomials have made it possible to describe known stem profiles with great accuracy. The relative diameter  $q_i$  of a tree can be described as follows

$$q_i = k_0 + k_1 P_1(x_i) + k_2 P_2(x_i) + k_3 P_3(x_i) + k_4 P_4(x_i)$$

where

$q_i = d_i/H$      $H$  is the tree height.

$k_0 - k_4$  are polynomial constants

$P_1 - P_4$  are orthogonal polynomials from degree 1 to 4

$x_i = \ln(h_i/H + 0.1)$

$d_i$  is the diameter of the tree at height  $h_i$ .

The logarithmic transformation of the relative height makes the expression useful in a description of the profile of the whole tree from root to top. This is so because the transformation "stretches" the lower part of the stem where the greatest change in shape occurs.

Orthogonal polynomials secure independent estimation of the constants  $k_0 - k_4$  which in term permit an analysis of their dependence on known data of tree measurements.

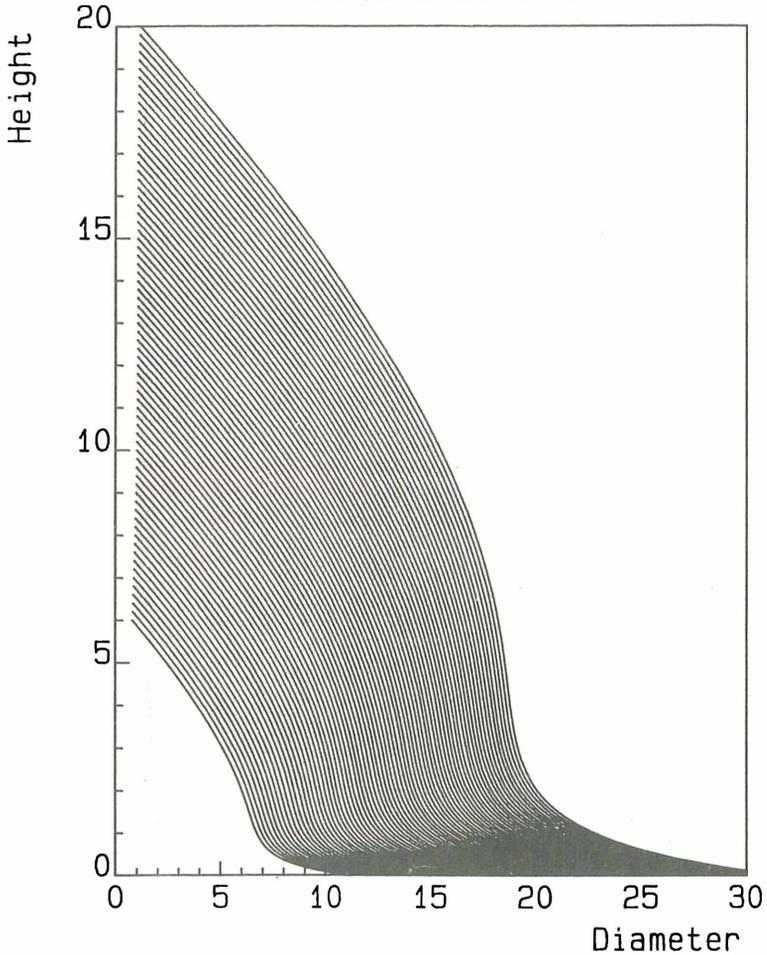
Multiple linear regression has made it possible to formulate an expression of how the five constants ( $k_0 - k_4$ ) are dependent on the height of the stand, the diameter at breast height, the age and perhaps on the standard of the stand density. Provided you have a known tree height it is possible to estimate the tree diameter at a specific height and this permits a theoretical cutting up of the trees that are felled at a planned cutting.

Reference:

- Madsen, J. S. 1982  
Formundersøgelse for rødgran på heden.  
Royal Vet. and Agricultural University.  
Faculty of Forestry.  
Thorvaldsensvej 57  
DK-1871 Copenhagen V  
Denmark

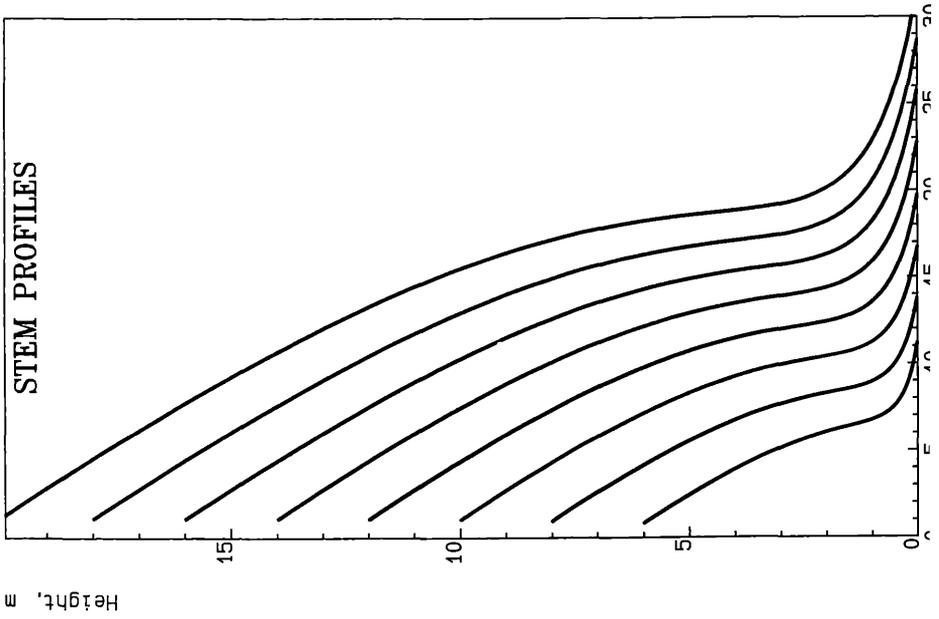
PRESENTED POSTERS,  
see the following pages.

# DETERMINATION OF STEM PROFILES IN PICEA ABIES



Stud. lic. agro.  
Jesper Stahl Madsen

Det Danske Hedeselskab  
Klostermarken 12  
8800 Viborg  
Danmark



## Introduction

- The main topic of the investigation is to find an expression describing stem profiles in *Picea Abies* when only a few stand- and tree-data are available.
- An expression like that is an important tool in planning the cutting, as the economic best way of cutting the trees can then be calculated.
- The investigation has been made as an examination project at the Faculty of Forestry in Denmark.

## Data

- The data has been collected in connection with an investigation of productivity in 2. generation of *Picea Abies* on the moorland established by The Danish Land Development Service.
- The stem profiles of 1014 trees in 169 stands have been measured. The chosen stands represent trees with ages from 26 to 76 years and heights from 6 to 24 metres.
- To eliminate the actual size of the trees the "relative trees" has been calculated in this way

$$q_j = d_j / H \quad \text{relative diameter}$$

$$x_j = h_j / H \quad \text{relative height}$$

where

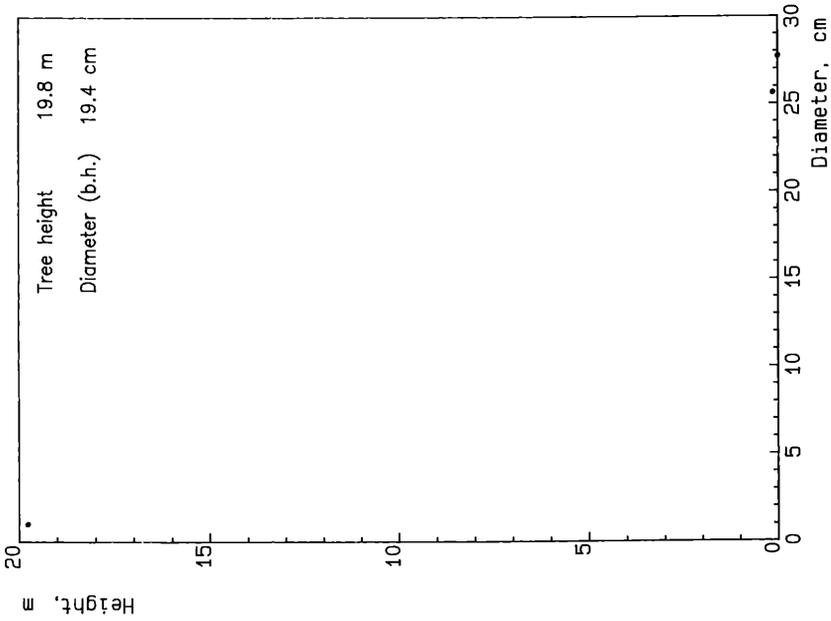
$d_j$  is the tree diameter in height  $h_j$

$H$  is the total height of the tree.

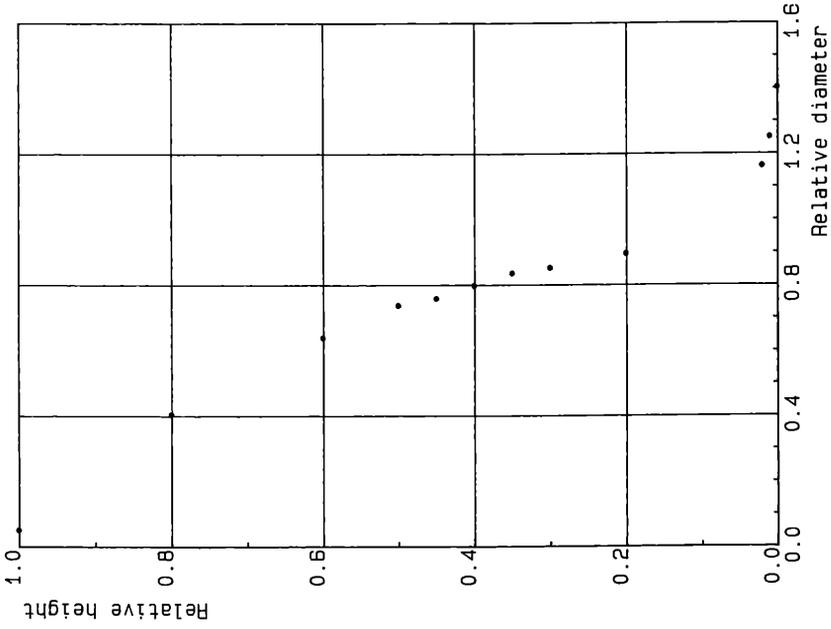


Jylland  
Danmark

Location of the stands from which the data has been collected.



Measured stem profile of one tree.



The same tree in relative measure.

# Model

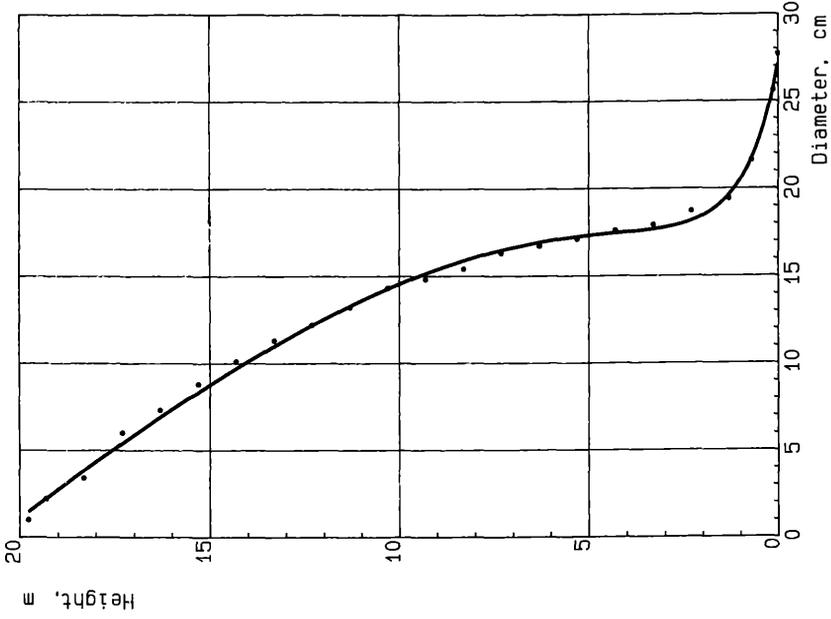
- A sum of orthogonal polynomials has been used to describe the relative diameter as a function of the relative height.
- Before using the relative height this value is transformed logarithmic which "stretches" the lower part of the profile with respect to the upper part.
- This results in the following expression:
 
$$q_i = k_0 + k_1 z_1 + k_2 z_2 + k_3 z_3 + k_4 z_4$$
 where
 
$$z_1 = a_0 + a_1 y^1$$

$$z_2 = b_0 + b_1 y^1 + b_2 y^2$$

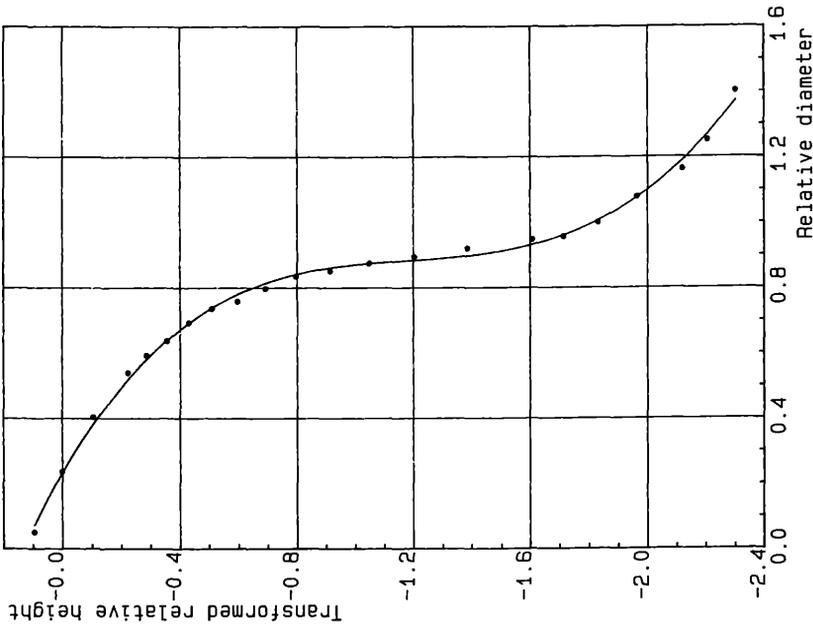
$$z_3 = c_0 + c_1 y^1 + c_2 y^2 + c_3 y^3$$

$$z_4 = d_0 + d_1 y^1 + d_2 y^2 + d_3 y^3 + d_4 y^4$$
 and  $y = \ln(x_i + 0.1)$
- When using orthogonal polynomials the estimates of the polynomial constants  $k_0 - k_4$  are being independent.
- This allows an investigation of the constants' dependence of the well-known stand- and tree-data.
- Multiple linear regression has been used to find the relationship between the polynomial constants and well-known tree measurement data such as height of the stand, stand diameter in breast height and, stand age.

$k_0 - k_4$  are polynomial constants.



Polynomial fit to the original data.



Polynomial fit to the transformed data.  
Note the stretching of the lower part of the tree.

# Results

- The result of this investigation leads to a number of expressions which can determine the five polynomial constants from different tree measurement data.
- Knowing more about the stand from which the trees comes makes the determination of the constants more precise.
- When only the stand height (H) and diameter in breast height (D) are used to determine the constants, the expression on the following sheet of paper can be used.

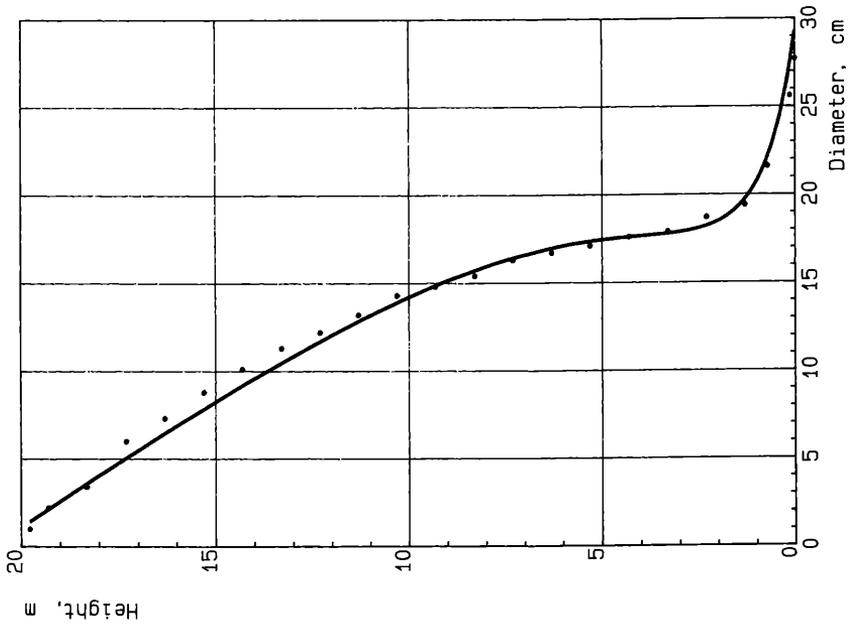
$$\begin{aligned}
 k_0 = & 0.8540 \\
 & - 0.0007 * H^2 \\
 & + 0.0006 * D^2 \\
 & + 8.0721 * 1/H \\
 & - 7.8803 * 1/D
 \end{aligned}$$

$$\begin{aligned}
 k_1 = & 3.6295 \\
 & + 0.1958 * H \\
 & - 0.3008 * D \\
 & + 0.0028 * D^2 \\
 & - 3.8187 * H/D \\
 & -40.2347 * 1/H \\
 & +34.7075 * 1/D
 \end{aligned}$$

$$\begin{aligned}
 k_2 = & - 1.3968 \\
 & + 0.0516 * D \\
 & - 0.0010 * D^2 \\
 & + 0.3755 * H/D \\
 & + 3.7268 * 1/H
 \end{aligned}$$

$$\begin{aligned}
 k_3 = & - 0.9156 \\
 & - 0.0002 * H^2 \\
 & + 0.4727 * H/D
 \end{aligned}$$

$$\begin{aligned}
 k_4 = & - 0.2575 \\
 & + 0.3262 * H/D \\
 & + 4.1452 * 1/H \\
 & - 4.6794 * 1/D
 \end{aligned}$$



**Stem profile determined by means of the described method.**

- When the polynomial constants have been found, the relative diameter corresponding to a relative height can be calculated by means of the polynomial.
- The constants  $a, b, c$  and  $d$  in the model are fixed. They only depend on the way in which the measured data have been used in the calculations.
- When the relative diameter corresponding to a relative height are known the actual diameter and height can be found by multiplying the relative measures with the tree height.
- Doing this gives the stem profile on the next sheet of paper.

The values of the constants a to d

$$a_0 = 0.28564$$

$$a_1 = 0.28283$$

$$b_0 = 0.30673$$

$$b_1 = 1.03174$$

$$b_2 = 0.46296$$

$$c_0 = 0.22105$$

$$c_1 = 1.85056$$

$$c_2 = 2.32237$$

$$c_3 = 0.71089$$

$$d_0 = 0.07680$$

$$d_1 = 2.64248$$

$$d_2 = 6.77854$$

$$d_3 = 5.10071$$

$$d_4 = 1.16383$$

# ZOBODAT - [www.zobodat.at](http://www.zobodat.at)

Zoologisch-Botanische Datenbank/Zoological-Botanical Database

Digitale Literatur/Digital Literature

Zeitschrift/Journal: [Mitteilungen der forstlichen Bundes-Versuchsanstalt Wien](#)

Jahr/Year: 1983

Band/Volume: [147\\_1983](#)

Autor(en)/Author(s): Stahl Madsen Jesper

Artikel/Article: [Determination of stem profiles in Picea Abies 263-273](#)