Rotating Electric Dipole Domains as a Loss-Free Model for the Earth's Magnetic Field

By

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Summary

In the search for a model with non-dissipative currents for the excitation of the Earth's magnetic field, stationary dipole domains are found to exist under high pressure. With such domains in an onion-like structure as a model for the iron-rich core, the Earth's magnetic field can be calculated with acceptable accuracy. The magnetic dipole moment is predicted to be proportional to the angular velocity, the core-radius to the fourth power and to the square root of the pressure at the core radius. Field reversal may be explained by a low-loss reversal of dipole orientation, keeping the stationary energy level constant. An application of the theory to other planets yields magnetic fields in the observed order of magnitude with the exception of Mars and Venus, whose magnetic fields are grossly overestimated; in the case of Mars presumably caused by a drop in energy content of the core, in the case of Venus possibly due to the extremely slow rotation causing hydrodynamic effects to prevent the synchronous rotation of the charges. A tough demand deriving from the variety of planets is met by the theory: for planets differing in mass by a factor of *5755*, such as Jupiter and Mercury, the theory predicts magnetic dipole moments spanning eight orders of magnitude with acceptable accuracy.

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Introduction

Current theories of the earth-magnetic field are based on the generator hypothesis (Jeanloz [1], Kuang and Bloxham [2], Merrill and McElhinny [3]). They are rather intricate but not really convincing. There are two major points of criticism. Firstly, generator efficiencies, even if they are as large as in optimized technical devices, require an abounding primary source of energy and lead to a continuous increase of entropy. This appears to be incompatible with the relative sustenance of the earth magnetic field. Secondly, the reversal of polarity, which has happened many times in the earth history and become evident from geological data, could not find a simple explanation. Or as Kuang and Bloxham [2] formulated: "The mechanism by which the Earth and other planets maintain their magnetic fields against ohmic decay is among the longest standing problems in planetary science. Although it is widely acknowledged that these fields are maintained by dynamo action, the mechanism by which the dynamo operates is in large part not understood."

Since the magnetic fields of celestial bodies are linked with their rotation, the first point of criticism could be removed by abandoning the generator hypothesis and assuming stable electric charges which rotate with the body, thus creating a magnetic field without dissipative electric current flow (ohmic decay). This obviously is in contradiction to the observation that the Earth appears, at least on its surface, as an electrically neutral body. Furthermore, the reversal of magnetic-field polarity could only be explained by a reversal of charge polarity, which is extremely hard to visualize. The discrepancies are removed, however, if a dipole-structure is assumed for the rotating charges. Rotating dipoles produce a magnetic field, whose polarity depends just on the dipole orientation. Thus magnetic field reversal may also find a plausible explanation.

In the following, it should be investigated whether stationary dipole structures may exist in a conducting medium, particularly under high pressure. Furthermore, the magnetic field resulting from a rotating dipole structure should be calculated for data given by the structure of the Earth and other planets and compared with observed values.

1. Dipole Domains

Various models for dipole-domains in a conducting medium were investigated. A first model suggests itself from the knowledge, that a current-free electrode inserted into a plasma is, in thermal equilibrium, negatively charged and shielded towards the plasma interior by a Debye-cloud of

positive charges. With onion-like quasi-periodic spherical shells of alternating solid conducting material and plasma, sequences of dipoles can be visualized, which produce magnetic fields by rotation. However, the magnetic field of such a structure is strongly impaired by an improper alignment of the dipoles: each dipole is followed by a dipole of reversed polarization, diminishing the resulting magnetic field to values far below the Earth's fields.

A second model consists of a single-dipole domain in a plasma with a width given by the Debye-distance, derived as a rigorous solution of Poisson's equation and the force equations for the particles in a stationary state. This model suffers from the property of infinite energy content (poles in particle densities, potential and electric field) leading upon rotation to unlimited magnetic fields.

These drawbacks are avoided by a third model, in which two thin layers of conducting media with different work functions are considered. Upon contact, the material with the lower work function delivers electrons across the contact and, if its thickness is about equal to the Debye-distance, depletes itself completely from electrons. Thus the Fermi level will lie in a forbidden band of the depleted layer and a potential barrier will exist which prevents the electrons from backdiffusion.

The model actually used assumes complete charge separation, so that in the layer enriched by electrons no positive charges are present; this allows a straightforward analytical treatment, in which the effect of all physical parameters on the magnetic field can be followed.

The applicability of a simple plasma model to analyse the electrical property of the liquid core had been questioned by critical readers of the manuscript. However, a simple plasma model is quite adequate to
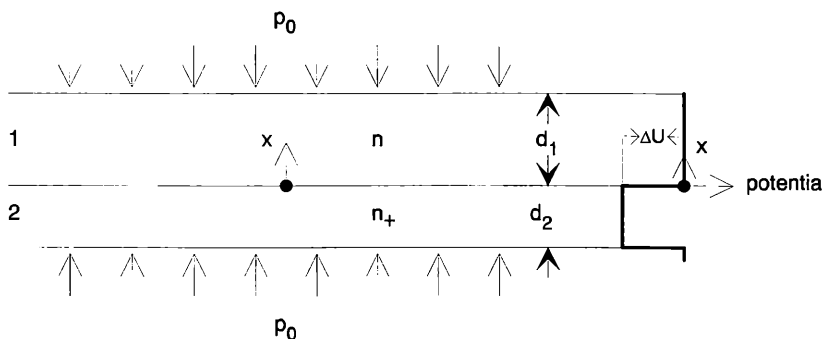

Figure 1. Planar dipole-domain formed under external pressure p_0 and a potential barrier $-\Delta U$ with the electrons of density n restricted to region 1 and the ions of density n_+ restricted to region 2

estimate electrical conductivity of metals, and the only extension tried here is to predict charge separation under high pressure, a piezo-effect.

Fig. 1 depicts such a one-dimensional plasma model. With a potential barrier $\Delta U \gg kT/e$, with k as Boltzmann-constant, T as absolute temperature and e as elementary charge, the charge separation is approximately complete, and $e\Delta U$ is a kind of binding energy for a cluster of positively charged ions. In region 1 the electrons are distributed with a density $n(x)$, in region 2 the ions with a density $n_+(x)$. This dipole structure is under the external pressure p_0. In the electron region, Poisson's equation for the electric field E in x-direction reads

$$\frac{dE}{dx} = -\frac{en}{\varepsilon} \tag{1}$$

with ε as dielectric constant of vacuum. The balance of forces requires

$$-enE - kT\frac{dn}{dx} = 0 \tag{2}$$

where for the equation of state of the electron gas thermal equilibrium is assumed leading to a pressure nkT. Gravitational forces are taken into account by a proper choice of the pressure p_0; the magnetic Lorentz force and the centrifugal force are many orders of magnitude below the forces considered here and are disregarded. A magnetic force may play a role in the alignment of the dipoles in the onion-model discussed in the next chapter.

Equations (1) and (2) can be reduced to a complete differential

$$\frac{d}{dx}\left(\frac{\varepsilon E^2}{2} - kTn\right) = 0 \tag{3}$$

which yields the integral

$$\frac{\varepsilon E^2}{2} - kTn = -p_0 \tag{4}$$

with p_0 as the pressure at $x = d_1$, where $E(d_1) = 0$ according to the condition of neutrality towards the outside.

Equations (1) and (4) are reduced to the nonlinear differential equation

$$\frac{dE}{dx} + \frac{1}{\varepsilon U_T}\left(\frac{\varepsilon E^2}{2} + p_0\right) = 0 \tag{5}$$

with

$$U_T = kT/e \tag{6}$$

as abbreviation (temperature voltage). Equation (5) can easily be solved by separating the variables, resulting in

$$E = -\left(\frac{2p_0}{\varepsilon}\right)^{1/2} \tan(x - d_1)\left(\frac{p_0}{2\varepsilon U_T^2}\right)^{1/2} \quad 0 \leq x \leq d_1 \quad (7)$$

and, from Eq. (4),

$$n = \frac{p_0/eU_T}{\cos^2(x - d_1)\left(\dfrac{p_0}{2\varepsilon U_T^2}\right)^{1/2}} \quad 0 < x < d_1. \quad (8)$$

The corresponding solutions for region 2 with $E(-d_2) = 0$ read

$$E = \left(\frac{2p_0}{\varepsilon}\right)^{1/2} \tan(x + d_2)\left(\frac{p_0 Z^2}{2\varepsilon U_T^2}\right)^{1/2} \quad -d_2 \leq x \leq 0 \quad (9)$$

$$n_+ = \frac{p_0/eU_T}{\cos^2(x + d_2)\left(\dfrac{p_0 Z^2}{2\varepsilon U_T^2}\right)^{1/2}} \quad -d_2 < x < 0. \quad (10)$$

The boundary conditions at $x = 0$ requests continuity of the pressure and is met if

$$d_2 = d_1/Z. \quad (11)$$

It is the barrier which reflects the majority of both carriers and thus allows a balance of electron and ion pressures, introducing a pronounced anisotropy in conductivity. Due to Eq. (4) and its counterpart for the ions, the electric field is also continuous except for the singularity caused by the potential barrier.

Thus a stationary dipole domain may exist under external pressure with electric fields which are positive within the domain and zero outside. Dipole orientation may easily be reversed by turning the model of Fig. 1 upside down, which leads to negative electric fields within the domain, maintaining neutrality towards the outside.

The densities within the domain are determined by the parameter

$$\alpha = d_1\left(\frac{p_0}{2\varepsilon U_T^2}\right)^{1/2} \quad (12)$$

which, when approaching $\pi/2$, leads to infinite density. If it were arbitrary, any magnetic field may be predicted. A selection criterion is offered by the field energy stored within the domain. In the electron region 1, where

$$N = \int_0^{d_1} n\,dx = \frac{1}{e}(2\varepsilon p_0)^{1/2}\tan\alpha \tag{13}$$

electrons per unit area are present, the stored energy per unit area is

$$W = \int_0^{d_1} \varepsilon E^2/2 \; dx = U_T(2\varepsilon p_0)^{1/2}(\tan\alpha - \alpha). \tag{14}$$

The corresponding values for the ion region 2 are

$$N_+ = \frac{1}{Ze}(2\varepsilon p_0)^{1/2}\tan\alpha \tag{15}$$

$$W_+ = \frac{U_T}{Z}(2\varepsilon p_0)^{1/2}(\tan\alpha - \alpha). \tag{16}$$

Thus the energy stored per particle is the same in both regions

$$\frac{W}{N} = \frac{W_+}{N_+} = eU_T\left(1 - \frac{\alpha}{\tan\alpha}\right). \tag{17}$$

One may from this observation consider the existence of one additional degree of freedom, allocating to it, according to the equipartition principle of thermodynamic equilibrium, the energy $eU_T/2$. From this condition $(1 - \alpha/\tan\alpha = 0,5)$

$$\alpha = 1,165 \tag{18}$$

is selected.

Note from Eq. (12) and (18), that the distances $d_{1/2}$ are − not unexpectedly − closely related to the well-known Debye-distance λ_D of a slightly-perturbed quasi-neutral plasma of electron density n_0 by

$$d_1 = d_2 Z = 2,33\lambda_D(1 + Z)^{1/2},$$
$$\lambda_D = \left(\frac{\varepsilon U_T}{en_0(1 + Z)}\right)^{1/2} \tag{19}$$

where n_0 is taken to be equal to the average electron density N/d_1 from Eq. (13).

So far the analysis has demonstrated that charge separation is plausible under pressure, and results in the formation of dipole domains. The separation will not be total, not only due to the limited depth of the postulated potential wall, but also since the balance of force, Eq. (2), and its

equivalent for the ions, result from statistical averaging. A state with stacked-up dipoles is not likely, however, since its energy content is, for a given pressure, higher than for a neutral state, caused by the electric energy, of course. The field energy accounts for $kT/2$ per particle, and potential energy, as will be shown later (Eq. (36)), for kT per particle, so that the total stored energy per domain and area would be $3kTN(1 + 1/Z)$, which, for a given pressure, is double the value of the neutral plasma state. The dipole state can therefore only be achieved if the work done on the system is of such a level that the system is unable to heat up and condense further and has to evade into electrical energy – field energy and potential energy – the latter equivalent to binding energy (chemical energy). The dipole domain resembles a giant two-dimensional molecule, which can only exist under pressure. This state appears possible in the liquid core of the earth.

Before applying the model to magnetic-field generation, caution must be called for as to the applicability of a continuum theory and statistical mechanics for the case under study. The distances $d_{1,2}$ prove for data of the Earth's core to be extremely small (compare the data in Table 1 of chapter 3). They range from about $d_1 = d_2 = 109$ Picometer for $Z = 1$ to $d_1 = 25,6$ Picometer and $d_2 = 3,2$ Picometer for $Z = 8$. This is to be compared with the average distance between electrons or ions, which range from 167 Picometer for $Z = 1$ to 101 Picometer for $Z = 8$. It is well known from the theory of electrolytic conductivity (Falkenhagen [4]) that even with only one particle, mostly of opposite polarity, in the sphere of radius λ_D around an ion, the continuum theory yields excellent results. The reason lies in ergodic behaviour, in the fact that in many snapshots of the positions of the ions within the Debye sphere around the ion in the center, the charge distribution of the continuum theory is well represented by averaging the charge distributions of the snapshots. The Debye cloud of the continuum theory proves to be an excellent model even in such extreme cases. A similar situation exists in very thin (thickness approximately λ_D) semiconductor layers designed to obtain negative differential conductivity by space transfer of hot electrons (Pacha and Paschke [5]). Again the average electron distance is about equal to the Debye-distance, which corresponds to the thickness of the layer, but the continuum theory remains powerful.

In the present model, only $Z = 1$ corresponds to the aforementioned cases and deserves confidence. For a larger number of elementary charges on the ions serious doubts are indicated whether continuum theory and statistical mechanics may be extrapolated to these cases.

With these doubts in mind the author proceeds to calculate the magnetic field produced by rotating stacks of dipole domains.

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2. The Magnetic Field of the Earth Resulting from Rotating Dipole Domains (Onion Model)

Consider a sphere of radius a rotating with an angular velocity Ω on which there is a surface charge σ uniformly distributed. The surface current

$$s_\varphi = \sigma a \Omega \sin \vartheta \tag{20}$$

produces a magnetic field outside of the sphere $r > a$, which in spherical coordinats r, ϑ, φ reads (Fano, Chu and Adler [6])

$$H_r = \frac{m_H}{2\pi} \cdot \frac{\cos \vartheta}{r^3} \tag{21a}$$

$$H_\vartheta = \frac{m_H}{2\pi} \cdot \frac{\sin \vartheta}{2r^3} \tag{21b}$$

with the magnetic moment m_H given by

$$\frac{m_H}{2\pi} = \frac{2a^4 \Omega \sigma}{3}. \tag{22}$$

Now the magnetic moment of one single dipole domain can be calculated:

$$\frac{m_{H1}}{2\pi} = \frac{2\Omega e}{3} \left(-\int_{r=a}^{a+d_1} r^4 \quad n \cdot \left(\frac{a}{r}\right)^2 \cdot dr + Z \int_{r=a-d_2}^{a} r^4 \quad n_+ \quad \left(\frac{a}{r}\right)^2 \cdot dr \right) \tag{23}$$

where $r = a$ is placed into the interface $x = 0$ of Fig. 1. The factor $(a/r)^2$ accounts for the transition from the planar model to a spherical model. The method is not rigorous but ensures the maintenance of external neutrality of the domain despite the curvature and appears acceptable since

$$d_1 \ll a.$$

Thus a Taylor expansion

$$r^2 \approx a^2 + 2ax \tag{24}$$

is justified, leading with the Eq. (8) and (10) to

$$\frac{m_{H1}}{2\pi} = \frac{2\Omega p_0}{3U_T} \left(-\int_{x=0}^{d_1} \frac{(a^4 + 2a^3 x)dx}{\cos^2 \alpha \left(\dfrac{x}{d_1} - 1\right)} + Z \int_{x=-d_2}^{0} \frac{(a^4 + 2a^3 x)dx}{\cos^2 \alpha \left(\dfrac{x}{d_2} + 1\right)} \right) \tag{25}$$

Upon integration

$$\frac{m_{H1}}{2\pi} = \frac{8}{3} a^3 \varepsilon \Omega U_T \left(1 + \frac{1}{Z}\right) \ln \cos \alpha. \qquad (26)$$

Since $\ln \cos \alpha < 0$, $m_{H1} < 0$, so that a magnetic south pole appears at the geographic northpole ($\vartheta = 0$). Note that with a change in dipole orientation, the same result, Eq. (26), is obtained with opposite sign, so that a magnetic northpole would coincide with the geographic northpole.

Consider now a whole stack of dipole domains forming an onion-like structure. The proper alignment of the dipole orientations may be caused or at least supported by the magnetic field, or comes about through stability, which will be discussed later.

Within a radial distance Δa, there are

$$\frac{\Delta a}{d_1 + d_2} = \frac{\Delta a}{d_1} \cdot \frac{Z}{1 + Z} \qquad (27)$$

domains. When adding up the magnetic moments of the very small domains, summation may be replaced by integration, so that in a spherical shell limited by a_1 and $a_2 < a_1$

$$\frac{m_H}{2\pi} = \int_{a_2}^{a_1} \frac{8\Omega}{3} \left(\frac{p_0 \varepsilon}{2}\right)^{1/2} \cdot \frac{\ln \cos \alpha}{\alpha} \cdot a^3 da. \qquad (28)$$

Note that this result is independent of Z, which is important for regaining confidence in the theoretical approach in view of the doubts expressed at the end of the last chapter.

Now, due to gravitational forces, the pressure p_0 definitely is a fuction of a, ranging from approximately $p_0(a_1) = 1, 6.10^{11} \ N/m^2$ at the edge of the core at $a_1 \dot{=} 3, 5.10^6 \ m$ to maybe $p_0(0) = 4, 0.10^{11} \ N/m^2$ in the center (Jeanloz [1]).

In a crude parabolic approximation

$$p_0(a) = p_0(0) - \left(\frac{a}{a_1}\right)^2 (p_0(0) - p_0(a_1)), \qquad (29)$$

which is not rigorous since the mass density is not constant. But the approximation enables an integration of Eq. (28), and the exact distribution of pressure is of little influence on the result. For the factor $\ln \cos \alpha / \alpha$ it is assumed that in each dipole domain the domain geometry,

given by d_1, adjusts itself to the corresponding pressure and temperature to yield the equilibrium value of Eq. (18). Thus

$$\frac{ln \cos \alpha}{\alpha} \doteq -0,8 \qquad (30)$$

independent of p_0.

Now the integration of Eq. (28) is without problems. Including the inner core ($a_2 = 0$)

$$\frac{m_H}{2\pi} = -\frac{8\Omega}{3} \cdot \left(\frac{\varepsilon p_0(a_1)}{2}\right)^{1/2} \cdot 0,8 a_1^4$$

$$\cdot \frac{\frac{1}{3}\left[1 - \left(\frac{p_0(a_1)}{p_0(0)}\right)^{3/2}\right] - \frac{1}{5}\left[1 - \left(\frac{p_0(a_1)}{p_0(0)}\right)^{5/2}\right]}{\left(\frac{p_0(a_1)}{p_0(0)}\right)^{1/2} \cdot \left(1 - \frac{p_0(a_1)}{p_0(0)}\right)^2} \cdot \qquad (31)$$

With $\Omega \doteq 7,27 \ 10^{-5} \, s^{-1}$ and the pressures estimated above, the value of magnetic induction μH_r is calculated at the geographic northpole $\vartheta = 0, r \doteq 6,32 \ 10^6 \, m$; here μ is the permeability of vacuum. Equations (31) and (21a) yield

$$\mu H_r(r,0) = -0,3 \, Gauss \qquad (32)$$

which falls short by a factor of 2 to observed data, but – for various reasons – appears quite satisfactory. Firstly, because the model is rather sensitive to the choice of the radius a_1: extending the region of dipole domains by about 1000 km into the solidified region to $a_1 \approx 4,5 \ 10^6 \, m$ would yield $-0,6 \, Gauss$. Secondly, the model is also sensitive to deviations from thermal equilibrium: If $0,86 kT$ (corresponding to $\alpha = 1,475$) would be assigned to the electric-field energy per particle instead of the equilibrium value $0,5 kT$ (corresponding to $\alpha = 1,165$), the magnetic field would double. In the author's judgement it is the critical radius which should be corrected since there is no reason why the dipole-domain region should be restricted to the liquefied region – it is charge mobility which counts.

The aberration of the magnetic pole axis from the rotational axis of the Earth may find an explanation by a slight deviation of the body containing dipole domains from an ideal sphere: local protrusions and indentations of the sphere lead to a tilt and offset of the magnetic pole-axis.

3. Further Properties of the Onion Model

The first subject to be commented upon is mass density and temperature. If the electron mass is neglected as compared to the ion mass m, the mass density is, averaged over the domain,

$$\rho_m = \bar{n}_+ m \cdot \frac{d_2}{d_1 + d_2}. \tag{33}$$

With $\bar{n}_+ d_2 = N_+$ given by Eq. (15), the definition of α, Eq. (12), and considering that $\tan \alpha / \alpha = 2$,

$$\rho_m = \frac{2 m p_0}{e U_T (1 + Z)} \tag{34}$$

Taking the iron-ion mass $m \doteq 9,37 \quad 10^{-26} \, kg$, a reasonable mass density of about $10^4 \, kg/m^3$ is obtained at the edge of the core $a_1 = 3,5 \quad 10^6 \, m$, where $p_0 = 1,6 \quad 10^{11} \, N/m^2$, for all combinations of the parameters shown in Table 1.

The values of $d_{1,2}$ and the average distance between electrons or ions, which equals $(d_1/N)^{1/3}$, demonstrate, that only $Z = 1$ is a case which is comparable to successful applications of continuum theory and statistical mechanics to theories of electrolytic conductivity or semiconductor devices. At the temperatures predicted, which are 5-to 10-times higher than expected by others (Jeanloz [1]), it appears to be certain that $Z > 1$. In the light of the data given in Table 1, the extrapolation of the theoretical approach to $Z > 1$ remains doubtful, particularly for $Z \gg 1$. It should be recalled, however, that the magnetic field calculated from the theory is independent of Z and deserves confidence. The high temperatures predicted by the theory are model-immanent and to be expected - in order to balance the large Coulomb forces, the pressure gradient has to be large too. Deviations from thermal equilibrium, which are associated with

Table 1

Z	U_T/V	T/K	d_1/pm	d_2/pm	$(d_1/N)^{1/3}/pm$
1	9,36	109.020	114,7	114,7	167,3
2	6,24	72.405	76,6	38,3	146,2
3	4,68	54.252	57,3	19,1	132,8
4	3,74	43.422	45,9	11,5	123,3
5	3,12	36.170	38,3	7,7	116,0
6	2,68	31.046	32,8	5,5	110,2
7	2,34	27.126	28,7	4,1	105,4
8	2,08	24.135	25,6	3,2	101,4

Z is the valency, U_T/V the temperature voltage, T/K the temperature in Kelvin, $d_{1,2}/pm$ the electron- and ion-layer thicknesses in Picometer, and $(d_1/N)^{1/3}/pm$ is the average particle distance in Picometer

values of $\alpha > 1,165$ (above equilibrium), would lead to lower temperatures. Alternatives to the plasma model may behave differently and possibly not require such high temperatures.

A deficiency of the model should not be left unnoticed: The condition for the potential barrier $\Delta U \gg U_T$ appears difficult to meet at such high temperatures. On the other hand, nobody knows the work functions of liquid metals under pressures of $10^{11}\,N/m^2$ and high temperatures, but here may be an inconsistency of the model.

The issue of potential energy, already addressed in the first chapter, is the next subject of attention. In each domain, the potential is raised by

$$\Delta\Phi = -2U_T\left(1+\frac{1}{Z}\right)ln\cos\alpha \qquad (35)$$

(or dropped with reversed polarisation of the dipoles) as one proceeds towards the center of the core. This leads to a negative potential energy for the electrons and a positive potential energy for the ions. Taking the charge and potential distributions from Eqs. (7)–(10) into account, the analysis yields a net value of potential energy per unit area and domain

$$W_P = 2U_T(2\varepsilon p_0)^{1/2}\left(1+\frac{1}{Z}\right)(\tan\alpha - \alpha) - eZN_+\Delta U. \qquad (36)$$

The last term is a kind of binding energy for the ion-cluster. The first term cannot be allocated to particles, but rather to ion-electron pairs (dipoles) and resembles, as indicated earlier, binding energy of the dipole. It is exactly twice as much as the field energy of one domain, that is the sum of Eq. (14) and (16). It thus looks as if two more degrees of freedom (kT per particle) can be allocated to this dipole-based potential energy, which appears quite plausible and might have been expected: Charge on a sphere produce a field and potential, which lead to the ratio of field energy and potential energy of exactly 0,5.

For reversed polarisation of the domains, the same result (Eq. (36)) is obtained with the electrons as carriers of positive and the ions of negative potential energy.

The potential increase, as one proceeds from the outer edge at $r = a_l$ towards the center, can be calculated from Eq. (35) and (27)

$$\Phi(a) = -2\cdot\frac{ln\cos\alpha}{\alpha}\int_a^{a_l}\cdot\left(\frac{p_0(a)}{2\varepsilon}\right)^{1/2}da. \qquad (37)$$

Note that also this result is independent of Z.

With the values of Eq. (29) and (30), a rather strong piezo-effect is predicted:

$$
\begin{aligned}
\Phi(a) = 0,8a_1 \; & \left(\frac{p_0(a_1)}{2\varepsilon}\right)^{1/2} \left[\frac{p_0(a_1)}{p_0(0)}\left(1 - \frac{p_0(a_1)}{p_0(0)}\right)\right]^{-1/2} \\
& \cdot \left\{ arc\sin\left(1 - \frac{p_0(a_1)}{p_0(0)}\right)^{1/2} \right. \\
-arc\sin\frac{a}{a_1} & \left(1 - \frac{p_0(a_1)}{p_0(0)}\right)^{1/2} + \left[\frac{p_0(a_1)}{p_0(0)}\left(1 - \frac{p_0(a_1)}{p_0(0)}\right)\right]^{1/2} \\
-\frac{a}{a_1} & \left.\left[\left(1 - \frac{p_0(a_1)}{p_0(0)}\right)\left(1 - \left(\frac{a}{a_1}\right)^2\left(1 - \frac{p_0(a_1)}{p_0(0)}\right)\right)\right]^{1/2} \right\}
\end{aligned}
\tag{38}
$$

In the center of the core, $a = 0$, a value of $7,5 \cdot 10^{17} \, V$ is obtained. Since for polarization reversal, the result would be $-7,5 \cdot 10^{17} \, V$, it looks at first difficult to explain magnetic field reversal. However, from the viewpoint of energy, both polarizations are of equal value: Field energy (the sum of Eq. (14) and (16)) and potential energy (Eq. (36)) are the same for both states. According to the model presented here, the Earth's core looks like a huge stack of capacitors in series connection with an enormous amount of energy stored. Voltage reversal may be achieved by currents causing the charges to change place within the domains, that is within very short distances. The huge two-dimensional molecules envisaged earlier have to flip over. No energy transfer is necessary except for that caused by dissipative currents which are unavoidable for field reversal. Of course, the binding energy and parts of the field energy are temporarily lost and have to be stored elsewhere during the transition (maybe as kinetic energy), which certainly is associated with losses, but they need not be large. The model looks like a bistable system with two equal energy levels where the transitions may be caused by a temporary hydrodynamic destruction of the domains and a spontaneous reformation to one state or the other.

So far it was tacitly assumed that the shell-shaped dipole domains in the onion-like structure have equal radial orientation, either all with the negative charges on the outside (present earth magnetic field), or all with their positive charges on the outsides (reversed earth magnetic field). Are there arguments against misalignment? Not from the energy content, which is equal in all dipole states, but from stability. A true stability analysis requires a dynamic theory, which has not yet been tackled, but the stationary state may reveal those stability properties, which are associated

with very long time constants. Consider dipole stacks with periodically alternating orientation, which keep the potential variation within the stack to the value $\Delta\Phi$ given by Eq. (35), and a dipole stack with uniform orientation with the voltages of the domains adding up. In the first case, surface potential perturbations of the core by $\pm\Delta\Phi$ would eventually destroy all domains by discharges, while in the co-oriented case only one domain would be lost.

A second argument which supports co-orientation can be obtained from the magnetic Lorentz force, which is very weak as compared to the Coulomb force and the pressure gradient in the domains, but very strong as compared to the centrifugal force: in the case of domains with electrons on the outside, it yields a radial force component directed to the outside for electrons and to the inside for the ions, in the case of ions on the outside of the domain in opposite directions. It thus may be that the magnetic Lorentz force aids to eliminate disoriented domains within the stack.

4. Other Planets

The results of Chapter 2, particularly Eq. (31), may be applied to other planets, of course. Limited knowledge of the internal structure of the planets makes the results presented here rather speculative. The main reason for the uncertainties can be found in the inaccurate knowledge of the core radius which enters Eq. (31) with the fourth power.

To obtain a rough estimate the planets were analyzed by separating them into an iron-rich core of radius $r < a_1$ with an average density ρ_1 and an outer region $R > r > a_1$ with an average density of ρ_2. With plausible assumptions for $\rho_{1,2}$, the value of a_1 and the pressure distribution can be calculated using the known values of mass and planet radius. If this model is applied to the Earth with its known value of $a_1 \approx 3,5 \cdot 10^6\,m$ and an assumed value of $\rho_1 \approx 1,2 \cdot 10^4\,kg/m^3$, a value of $\rho_2 = 4,22 \cdot 10^3\,kg/m^3$ is obtained. The resulting pressure values $p(a_1) = 1,21 \cdot 10^{11}\,P$, $p(0) = 4,6 \cdot 10^{11}\,P$ deviate from the more accurate values used in chapter 2, but the magnetic dipole moment calculated from Eq. (31) with the inaccurate pressure values deviates from the correct moment only by -2%.

The values for the densities used henceforth are: For Earth and Venus: $\rho_1 = 1,2 \cdot 10^4\,kg/m^3$, $\rho_2 = 4,22 \cdot 10^3\,kg/m^3$; for Mercury and Mars: $\rho_1 = 10^4\,kg/m^3$, $\rho_2 = 3 \cdot 10^3\,kg/m^3$; for Jupiter and Saturn: $\rho_1 = 1,4 \cdot 10^4\,kg/m^3$, $\rho_2 = 0,55 \cdot 10^3\,kg/m^3$; for Uranus and Neptune: $\rho_1 = 1,3 \cdot 10^4\,kg/m^3$, $\rho_2 = 0,8 \cdot 10^3\,kg/m^3$

Masses and radii of the planets are taken from Merrill and McElhinny [3], the angular velocities from Ness [7]. The values of the magnetic moment of the various planets shown in Table 2 are related to the value obtained for the Earth. The last column shows the observed values reported by Ness [7].

The author considers it as a remarkable that for planets differing in mass by a factor of 5755, such as Jupiter and Mercury, magnetic dipole moments which differ by eight orders of magnitude are predicted with acceptable accuracy.

The overestimation of the magnetic moment of Mars may be caused by a loss of energy content of the core – there exists, aside from the high-energy dipole-domain states, also the low-energy neutral state, which may be associated with solidification of the core. In the case of Venus, the extremely slow rotation may lead to hydrodynamic perturbations preventing the synchronous charge rotation.

The large offset and tilt of the magnetic axes with relation to the rotational axes of Uranus and Neptune may be caused by a mixture of spatially unevenly distributed high-energy dipole-domain states and the low-energy neutral state. It is doubtful whether such mixed states are stationary and correspond to thermal equilibrium. It would be interesting to observe possible changes of the magnetic fields particularly of these two planets.

The present article is intended as an alternative to dynamo theories. It should be mentioned, however, that the existence of rotating dipole domains under pressure may support dynamo theories since they may provide the magnetic field which is necessary for the excitation of the dynamo. The results of the present theory, which go beyond qualitative

Table 2

	$a_1/10^6\,m$	$p(a_1)/10^{11}\,P$	$p(0)/10^{11}\,P$	Ω/Ω_E	m_H/m_{HE}^a	m_H/m_{HE}^b
Mercury	1,71	0,0633	0.513	$1,710^{-2}$	2.10^{-4}	$6,25.10^{-4}$
Venus	3,0	1,09	3.14	$4,1\ \ 10^{-3}$	$2,5.10^{-3}$	$<5.10^{-5}$
Earth	3,5	1,21	4.6	1	1	1
Mars	1,7	0,2	0.6	0,975	2.10^{-2}	$<2.5.10^{-4}$
Jupiter	26,5	8,61	200	2,41	$4,2.10^4$	2.10^4
Saturn	10,7	3,41	34.6	2,24	463	590
Uranus	8,28	1,8	17.9	1,39	74,3	47,5
Neptune	9,82	2,07	24.7	1,48	180	25

$a_1/10^6\,m$ is the core radius in 10^6 Meters, $p(a_1)/10^{11}\,P$ and $p(0)/10^{11}\,P$ are the pressures at the edge and the center of the core in 10^{11} Pascal, Ω/Ω_E the angular velocity of the planet related to the Earth's value, and m_H/m_{HE} the magnetic dipole moment related to the Earth's value
[a]Theoretical value
[b]Observed, from Ness [7]

agreement with observations, and the fundamental argument against ohmic decay speak for the alternative.

In the best case, the theory is accepted as an alternative. From the reactions received so far, this is not expected in the lifetime of the present peers and the author. In the worst case, the theory should stimulate researchers with knowledge and experience in the physics of condensed matter to look with more depth than the author into piezoelectric effects of liquid metals under extremely high pressures.

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